TANZANIA HEAD OF ISLAMIC SCHOOL COUNCIL (TAHISCO)

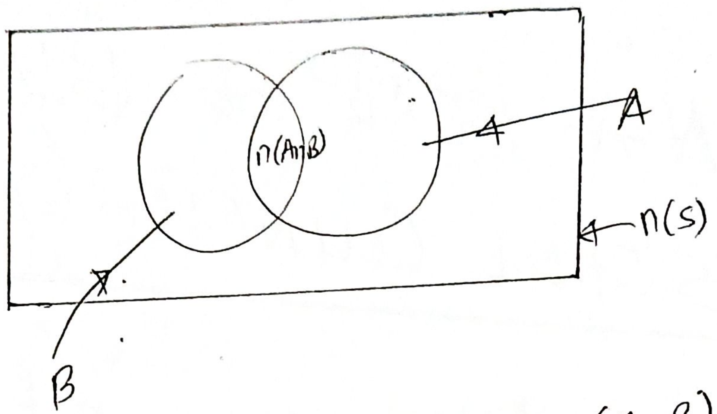
MAKING SCHEME OF ADVANCED MATHEMATICS 02

YEAR - 2025

SECTION A

01. (a) Solution.

Consider The Venn diagram,



from $n(A\cup B) = n(A) + n(B) - n(A\cap B)$ ---- (01 mark)

$n(A\cap B) + n(A\cup B) = n(A) + n(B)$

Divide by $n(S)$

$\frac{n(A\cap B)}{n(S)} + \frac{n(A\cup B)}{n(S)} = \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)}$ --- (01 mark)

But $P(E) = \frac{n(E)}{n(S)}$ --- (01 mark)

$\Rightarrow P(A\cap B) + P(A\cup B) = P(A) + P(B)$

Hence shown!

1 (b)(i) $E(X) = \sum_{X=1}^{6} X P(X)$ ------ (00½ mark)

$E(X) = (1)(1/16) + 2(\frac{3}{16}) + 3(1/4) + 4(1/4) + 5(\frac{3}{16}) + 6(1/16)$ ------ (01 mark)

$E(X) = 3.5$ ------ (00½ mark)

(ii) $E(X^2) = \sum_{X=1}^{6} X^2 P(X)$

$E(X^2) = 1^2(1/16) + 2^2(3/16) + 3^2(1/4) + 4^2(1/4) + 5^2(3/16) + 6^2(1/16)$ ------ (01 mark)

$\therefore E(X^2) = 14$

1(b) (iii) $E(3X-5) = E(3X) - E(5)$ ---------- (00½ mark)

$= 3E(X) - 5$

But $E(X) = 3.5$ and $E(5) = 5$.

$\Rightarrow E(3X-5) = 3 \times 3.5 - 5 = 5.5$ ---------- (01 mark)

$\therefore \underline{E(3X-5) = 5.5}$

(iv) $E(6X^2 + 6X - 10) = ?$

$E(6X^2 + 6X - 10) = 6E(X^2) + 6E(X) - 10.$ ---------- (00½ mark)

But $E(X^2) = 14$, $E(X) = 3.5$

$= 6(14) + 6(3.5) - 10 = 95.$

$\therefore \underline{E(6X^2 + 6X - 10) = 95}$ ---------- (01 mark)

1(c) Solution;

Let X be the random variable "time taken to deliver the milk to the high street".

Then $X \sim N(12, 2^2)$

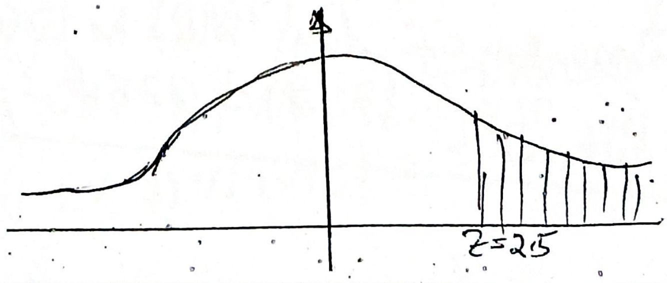
$\mu = 12$

$$Z = \frac{X-\mu}{\sigma} = \frac{X-12}{2}$$

(i) $P(X > 17) = ?$

$P(X > 17) = P\left(\frac{X-12}{2} > \frac{17-12}{2}\right) = P(Z > 2.5) = Q(2.5)$

$= Q(2.5) = 0.00621$ ---------- (01 mark)



(c) from $P(E) = \frac{n(E)}{n(S)} \Rightarrow n(E) = n(S)\,P(E)$ ———— (00½ marks)

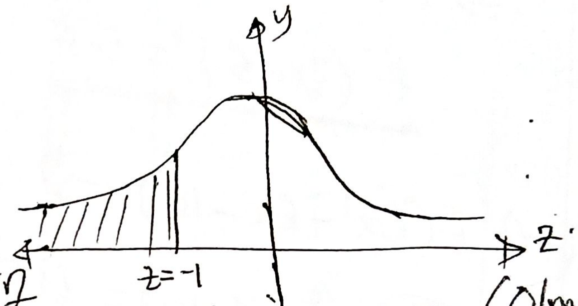
Then; the number of days when he takes longer than 17 minutes is

$365(0.00621) = 2.27 \simeq 2$ days. ———— (00½ marks)

(c)(ii) $P(X < 10) = ?$

$P(Z) = P\left(Z < \frac{10-12}{2}\right)$

$P(Z < -1) \Rightarrow Q(-1) = 0.1587$ ———— (01 mark)



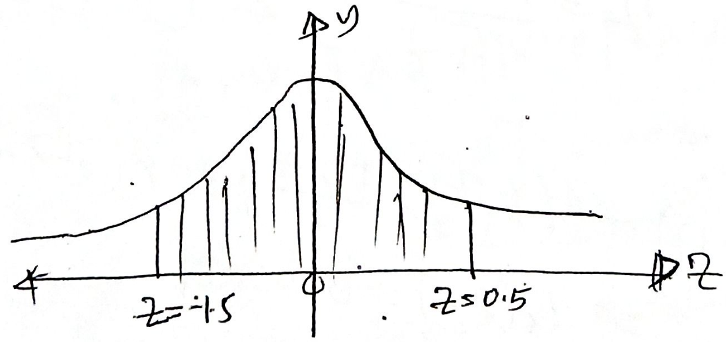
Then; the number of days when he takes less than 10 minutes is $= 365(0.1587) = 57.9 \simeq 58$ days. ———— (01 mark)

(iii) $P(9 < X < 13) = P\left(\frac{9-12}{2} < Z < \frac{13-12}{2}\right)$

$= P(-1.5 < Z < 0.5)$

$= 1 - (Q(1.5) - Q(0.5)) = 0.6247$

———— (01 mark)



Now, the number of days when he takes between 9 and 13 minutes is $365(0.6247) = 228$ days. ———— (01 mark)

2(a)

p	q	r	$q\vee r$	$p\wedge(q\vee r)$	$(q\vee\sim r)$	$p\to(q\vee\sim r)$	$p\wedge q$	$[p\to(q\vee\sim r)]\to(p\wedge q)$
T	T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T	T
T	F	T	T	T	F	F	F	T
T	F	F	F	F	T	T	F	F
F	T	T	T	F	T	T	F	F
F	T	F	T	F	T	T	F	F
F	F	T	T	F	F	T	F	F
F	F	F	F	F	T	T	F	F
						T	F	F

--- 02 marks

Since $p\wedge(q\vee r)$ has the same truth values as $[p\to(q\vee\sim r)]\to(p\wedge q)$ hence they are equivalent.

(b) (i) $[(p\to q)\wedge(q\to r)]\to(p\to r)$

$\sim[(\sim p\vee q)\wedge(\sim q\vee r)]\vee(\sim p\vee r)$ — Implication law

$[(p\wedge\sim q)\vee(q\wedge\sim r)]\vee(\sim p\vee r)$ — De-morgans law

$(p\wedge\sim q)\vee[(q\wedge\sim r)\vee(\sim p\vee r)]$ — Associative law

$(p\wedge\sim q)\vee[(\sim r\vee(q\wedge\sim r)\vee\sim p]$ -- Associative law

$(p\wedge\sim q)\vee[(r\vee r)\wedge(r\vee q)\vee\sim p]$ — Distributive law

$(p\wedge\sim q)\vee[T\wedge(q\vee r)\vee\sim p]$ — Negation law

$(p\wedge\sim q)\vee((q\vee r)\vee\sim p)$ — Identity law

$[\sim p\vee(p\wedge\sim q)]\vee(q\vee r)$ — Associative law.

$[(\sim p\vee p)\wedge(\sim p\vee\sim q)]\vee(q\vee r)$ — Distributive law

$(T\wedge(\sim p\vee\sim q)\vee(q\vee r)]$ -- Negation law.

$(\sim p \vee \sim q) \vee (q \vee r)$ —— Identity law.

b(i) $(\sim p \vee r) \vee (\sim q \vee q)$ - - - Associative law

$(\sim p \vee r) \vee T$ - - - - Negation/complement law.

T - - - - Identity law

- - - - (03marks)

$\therefore$ It is Tautology

(b)(ii) $[(p \rightarrow \sim q) \wedge (r \rightarrow q) \wedge q] \rightarrow r$ —— Given.

$\sim[(\sim p \vee \sim q) \wedge (\sim r \vee q) \wedge q] \vee r$ —— Implication law.

$\sim[(\sim p \vee \sim q) \wedge q] \vee r$ —— Absorption law

$\sim[(q \wedge \sim p) \vee (q \wedge \sim q)] \vee r$ —— Distributive law

$\sim[(q \wedge \sim p) \vee F] \vee r$ —— Negation law

$\sim[(q \wedge \sim p)] \vee r$ —— Identity law

$(\sim q \vee p) \vee r$ —— De-morgan's law

$(\sim q \vee p) \vee r$.

- - - (03marks)

$\therefore$ It is not tautology.

2(c) It's converse;
"If Aisha was born before maganga then Juma was born before Ally." - - - - (01mark)

It's inverse of converse (Contrapositive):
"If Aisha was not born before maganga then Juma was not born before Ally" - - - (01mark)

It's Inverse:

"If Juma was not born before Ally then Aisha was not born before Kaganga" ———— (01 mark)

(d) $K = (p \wedge q \wedge r) \vee (p \wedge q \wedge \sim r) \vee (p \wedge \sim q \wedge r)$ ———— (01 mark)

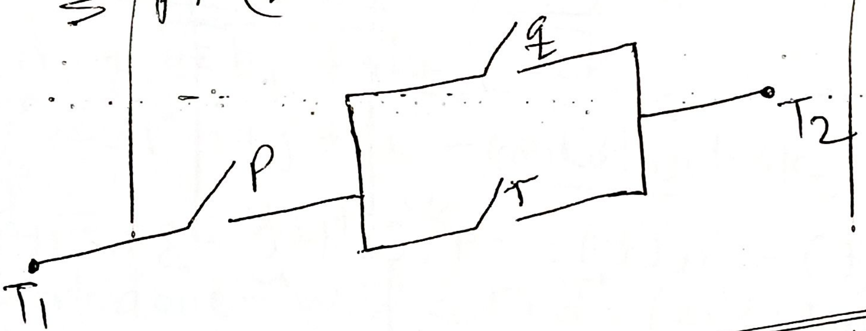
$= [(p \wedge q) \wedge (r \vee \sim r)] \vee (p \wedge \sim q \wedge r)$ ———— Distributive law

$= (p \wedge q) \wedge (p \wedge \sim q \wedge r)$ ———— Negation & Identity laws.

$= p \wedge ((q \vee (\sim q \wedge r))$ ———— Distributive law

$= p \wedge ((q \vee \sim q) \wedge (q \vee r))]$ ———— Distributive law (02 marks)

$= p \wedge (q \vee r)$ ———— Negation & Identity laws.



———— (01 mark)

3(a) solution

Given vector let $\underset{\sim}{a} = 5i - j + 2k$ - - - - $00\frac{1}{2}$ mark

$|b| = 8$

but $\hat{\underset{\sim}{a}} = \hat{\underset{\sim}{b}}$

$\Rightarrow \frac{\underset{\sim}{a}}{|a|} = \frac{\underset{\sim}{b}}{|b|}$ - - - - - - 01 mark

$\underset{\sim}{b} = |b| \times \frac{\underset{\sim}{a}}{|a|}$

$\underset{\sim}{b} = 8\left(\frac{5i - j + 2k}{\sqrt{5^2 + (-1)^2 + (2)^2}}\right) = \frac{8}{\sqrt{36}}(5i - j + 2k)$ - - - 01m

$\therefore$ The vector is $\frac{40}{\sqrt{30}}i - \frac{8j}{\sqrt{30}} + \frac{16}{\sqrt{30}}k$ - - - - $00\frac{1}{2}$ mark

solution

3(b) Given: $r''(t) = a(t) = -4\cos t\,\underset{\sim}{i} - 3\sin 3t\,\underset{\sim}{k}$

To find $r(t)$

$v(t) = \int r''(t) = \int (-4\cos t\,\underset{\sim}{i} - 3\sin 3t\,\underset{\sim}{k})\,dt$ - - - - 01m

$v(t) = r'(t) = -4\sin t\,\underset{\sim}{i} + \cos 3t\,\underset{\sim}{k} + C$

Given $r'(0) = 3k$

$3\underset{\sim}{k} = -4\sin 0°\,\underset{\sim}{i} + \cos 3(0)\,\underset{\sim}{k} + C$ - - - - 01 mark

$C = 3k - k = 2k$

$C = 2\underset{\sim}{k}$

3(b) then

$r'(t) = v(t) = -4\sin t\, \underset{\sim}{i} + \cos 3t\, \underset{\sim}{k} + 2\underset{\sim}{k}$

$r'(t) = -4\sin t\, \underset{\sim}{i} + (2 + \cos 3t)\, \underset{\sim}{k}$

then $\int r'(t) = r(t) = \int (-4\sin t\, \underset{\sim}{i} + (2 + \cos 3t)\underset{\sim}{k})\, dt$ - - - 01 mar

$r(t) = 4\cos t\, \hat{i} + (2t + \frac{1}{3}\sin 3t)\, \underset{\sim}{k} + D$

but; $r(0) = 4\hat{j}$

$4\hat{j} = 4\cos(0)\underset{\sim}{i} + (2(0) + \frac{1}{3}\sin(3(0)))\hat{k} + D$

$D = 4j - 4i$

There fore

$r(t) = 4\cos t\, \underset{\sim}{i} + (2t + \frac{1}{3}\sin 3t)\hat{k} + 4j - 4i$ - - 01 ma

$\therefore r(t) = (-4 + 4\cos t)\underset{\sim}{i} + 4\hat{j} + (2t + \frac{1}{3}\sin 3t)\underset{\sim}{k}$

solution

3(c) Given $F = 2i + j + 3k$

$\vec{d} = 3i + 4j + 5k - (2i + 2j + 3k)$ - - - - 01 mark

$\vec{d} = i + 2j + 2k$

then workdone $= w.d = F.\vec{d} = (2i + j + 3k).(i + 2j + 2k)$ - 01 ma

$w.d = 2 + 2 + 6 = 10$ units - - - - - 01 mark.

$\therefore$ Then workdone is 10 units - - - - - 01 mark.

(d) soln

let $\underset{\sim}{a}$ be a vector

$|\underset{\sim}{a}| = 14$.

So, its unit $\hat{a}$ in the direction of $3i + 6j + 2k$ is

$\hat{a} = \frac{14(3i + 6j + 2k)}{\sqrt{3^2 + 6^2 + 2^2}} = 6i + 12j + 4k$ - - - 01 mark.

3(d) Given; plane ABC

now, $\overline{AB} = \dot{O}B - OA = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} = -i - j - 2k$

$A\bar{C} = OC - OA = \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} = i - 2j - 5k$ ---- 01 mark

then $\vec{AB} \times \vec{AC} = \left| \begin{pmatrix} i & j & k \\ -1 & -1 & -2 \\ 1 & -2 & -5 \end{pmatrix} \right| = i - 7j + 3k$ ---- 01/2 mark

Then: To find the magnitude of vector $\underset{\sim}{a}$ in the direction perpendicular to plane ABC is given as:

$\Rightarrow \dfrac{\underset{\sim}{a} \cdot (AB \times AC)}{|AB \times AC|} = \dfrac{(6i + 12j + 4k) \cdot (i - 7j + 3k)}{\sqrt{1^2 + (-7)^2 + 3^2}}$ ---- 01 mar

$= \dfrac{6 - 84 + 12}{\sqrt{59}} = |-8.592|$

---- 01/2 mark

$\therefore$ The magnitude is 8.592 units

4 (a) $\frac{13z}{z+1} = 11-3i$

$13z = (11-3i)(z+1)$ ---------- (01 mark)

$13z = 11z + 11 - 3iz - 3i$

$2z + 3iz = 11 - 3i$

$z(2+3i) = 11-3i$

$z = \frac{11-3i}{2+3i}$ ---------- (02 marks)

Rationalize by Denominator

$z = \frac{(11-3i)(2-3i)}{(2+3i)(2-3i)}$ ---------- (01 mark)

$z = \frac{13-39i}{4+9} = \frac{13-39i}{13}$

---------- (01 mark)

$\underline{z = 1-3i}$

(b) $x = \cos\alpha + i\sin\alpha$ and $y = \cos\beta + i\sin\beta$.

$x^m y^n = (\cos\alpha + i\sin\alpha)^m \cdot (\cos\beta + i\sin\beta)^n$

$= (\cos(m\alpha) + i\sin(m\alpha))(\cos(n\beta) + i\sin(n\beta))$ ---------- (01 mark)

$= \cos(m\alpha + n\beta) + i\sin(m\alpha + n\beta)$

Also $\frac{1}{x^m y^n} = (x^m y^n)^{-1}$

$= \cos(m\alpha + n\beta) + i\sin(m\alpha + n\beta)^{-1}$ ---------- (01 mark)

$\frac{1}{x^m y^n} = \cos(m\alpha + n\beta) - i\sin(m\alpha + n\beta)$

---------- (01 mark)

Now; $x^m y^n - \frac{1}{x^m y^n} = [\cos(m\alpha + n\beta) + i\sin(m\alpha + n\beta)] - [\cos(m\alpha + n\beta) - i\sin(m\alpha + n\beta)]$

4(b) $\therefore x^m y^n - \frac{1}{x^m y^n} = 2i\sin(m\alpha + n\beta)$ ———— (01 mark)

(c) Given $z = x + iy$

$|z-(1+i)|^2 = 2$

$|x+iy-(1+i)|^2 = 2$

$(x-1)^2 + (y-1)^2 = 2$

$x^2+y^2-2x-2y+2 = 2$, ———— (01 mark)

$x^2+y^2 = 2(x+y)$ ———— (i)

let $P = u+iv$.

$u+iv = \frac{2}{z} = \frac{2}{x+iy}$

$u+iv = \frac{2}{x+iy}$

$u+iv = \frac{2(x-iy)}{2(x+iy)(x-iy)}$, Rationalize by Denominator.

$u+iv = \frac{2(x-iy)}{x^2+y^2}$

$u+iv = \frac{2x}{x^2+y^2} - \frac{2yi}{x^2+y^2}$ ———— (02 mark)

$u = \frac{2x}{x^2+y^2}$ and $v = \frac{-2y}{x^2+y^2}$

(01 mark)

4(c) $u-v = \frac{2x}{x^2+y^2} + \frac{2y}{x^2+y^2}$

$u-v = \frac{2(x+y)}{x^2+y^2}$ — — — — (01 mark)

But $x^2+y^2 = 2(x+y)$ from eqn(i)

$u-v = \frac{2(x+y)}{2(x+y)}$

$u-v = 1$ — — — — (01 mark)

$\therefore$ The Locus is $u-v=1$.

SECTION B:

5(a) solution:-

$\sin A = 1/4$.

$\cos A = -\sqrt{1-\sin^2 A} \Rightarrow \cos A = -\sqrt{1-(1/4)^2}$

$\cos A = \frac{-\sqrt{15}}{4}$ — — — (00½ mark)

(i) $\tan(A/2) = ?$ — — — (00½ mark)

from $\cos A = 1-2\sin^2 A/2$

$\frac{-\sqrt{15}}{4} = 1-2\sin^2 A \Rightarrow 2\sin^2 A/2 = 1+\frac{\sqrt{15}}{4}$

$\sin A/2 = \sqrt{\frac{4+\sqrt{15}}{8}}$

$\cos A/2 = \sqrt{\frac{4-\sqrt{15}}{8}}$ — — — (00½ mark)

5(a) (i) $\tan A/2 = \frac{\sin A/2}{\cos A/2}$

$$= \pm \frac{\sqrt{\frac{4+\sqrt{15}}{8}}}{\sqrt{\frac{4-\sqrt{15}}{8}}} = \pm\sqrt{\frac{4+\sqrt{15}}{4-\sqrt{15}}} \text{ --- (01 mark)}$$

$\therefore \tan A/2 = \pm\sqrt{\frac{4+\sqrt{15}}{4-\sqrt{15}}}$ (2½)

5(a) (ii) $\cos A/2$

from $\cos A = 2\cos^2 A/2 - 1$ --- (00½ mark)

$-\frac{\sqrt{15}}{4} = 2\cos^2 A/2 - 1$

$\cos^2 A/2 = \frac{4-\sqrt{15}}{8}$ --- (01 mark)

$\cos A = \pm\sqrt{\frac{4-\sqrt{15}}{8}}$ (By squaring roots both sides)

$\therefore \cos A = \pm\sqrt{\frac{4-\sqrt{15}}{8}}$. --- (01 mark) (2½)

Solution;

5(b)

$\sin 2y + \cos y = \sin y \cos y$

$\sin 2y + \cos y = \frac{\sin 2y}{2}$, since $\sin 2y = 2\sin y\cos y$. --- (00½ mark)

$2\sin 2y + 2\cos y - \sin 2y = 0$

$\sin 2y + 2\cos y = 0$

7(b) $\sin 2y + 2\cos y = 0$ — — — — — (00½ mark)

$2\sin y\cos y + 2\cos y = 0$

$2\cos y(\sin y + 1) = 0$

Either $2\cos y = 0$ or $\sin y + 1 = 0$.

$\Rightarrow y = \cos^{-1}0$ or $y = \sin^{-1}(-1)$. — — — (01 mark)

$y = 2\pi n \pm \cos^{-1}0$ or $y = \pi n + (-1)^n \sin^{-1}(-1)$ — — (01 mark)

where $n = \pm 1, \pm 2, ---$

then $y = \pi/2,\ 3/2\pi$ — — — — (01 mark)

$\therefore y = \pi/2$ and $3/2\pi$.

(4) Marks

5(c) Solution;

To prove $\cos 8A = 128\cos^8 A - 256\cos^6 A - 32\cos^2 A + 1 + 160\cos^4 A$.

By taking LHS

$\cos 8A = \cos(4A + 4A)$ — — — — — (01 mark)

$\cos 8A = \cos^2 4A - \sin^2 4A$

$= (2\cos^2 2A - 1)^2 - (2\sin 2A\cos 2A)^2$, since; $\cos 2A = 2\cos^2 A - 1$, $\sin 2A = 2\sin A\cos A$.

$= 4\cos^4 2A - 4\cos^2 2A + 1 - 4\sin^2 2A\cos^2 2A$

$= 4(2\cos^2 A - 1)^4 - 4(2\cos^2 A - 1)^2 - 4(2\sin A\cos A(2\cos^2 A - 1))^2$ — — (02 mark)

but $(a-b)^4 = a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4$

$= 64\cos^8 A - 128\cos^6 A + 96\cos^4 A - 32\cos^2 A + 4 - 16\cos^4 A - 4 + 1 - 16(8\cos^6 A - 5\cos^4 A - 4\cos^5 A + \cos$

$\cos 8A = 64\cos^8 A - 128\cos^6 A + 80\cos^4 A - 16\cos^2 A - 128\cos^6 A + 80\cos^4 A + 64\cos^8 A - 16\cos^2 A + 1$

— — — (02 marks)

$\therefore \cos 8A = 128\cos^8 A - 256\cos^6 A + 160\cos^4 A - 32\cos^2 A + 1$ --- (01 mark)

Hence proved!

(6 Marks)

5(d) Solution.

If $a\cos^2 x + b\sin^2 x = c$ to prove $\tan^2 x = \frac{c-a}{b-c}$

From $a\cos^2 x + b\sin^2 x = c$

But $\cos^2 x = 1 - \sin^2 x$.

$a(1-\sin^2 x) + b\sin^2 x = c$ ------ (00½ mark)

$a - a\sin^2 x + b\sin^2 x = c$

$a + (b-a)\sin^2 x = c$

$(b-a)\sin^2 x = c-a$

$\frac{(b-a)\sin^2 x}{b-a} = \frac{c-a}{b-a}$ ----- (01 mark)

$\sin^2 x = \frac{c-a}{b-a}$ ----- (00½ mark)

From $\cos^2 x = 1 - \sin^2 x$ ---- (01 mark)

$\cos^2 x = 1 - \left(\frac{c-a}{b-a}\right) = \frac{b-c}{b-a}$

$\tan^2 x = \left(\frac{c-a}{b-a}\right) \div \left(\frac{b-c}{b-a}\right)$; As, $\tan^2 x = \frac{\sin^2 x}{\cos^2 x}$ --- (01 mark)

$\tan^2 x = \left(\frac{c-a}{b-a}\right) \times \left(\frac{b-a}{b-c}\right) = \frac{c-a}{b-c}$

Hence proved that $\tan^2 x = \frac{c-a}{b-a}$. --- (01 mark)

(5 Marks)

6(a) Soln

$\frac{2x^2-3x+2}{(x-2)^8}$, let $u = x-2$

$\Leftrightarrow x = u+2$

$$\frac{2x^2-3x+2}{(x-2)^8} = \frac{2(u+2)^2-3(u+2)+2}{u^8}$$ ---- 01 marks

$$= \frac{2(u^2+4u+4)-3u-6+2}{u^8}$$

$$= \frac{2u^2+8u+8-3u-4}{u^8}$$ --- 01 marks

$$= \frac{2u^2+5u+4}{u^8}$$

$$= \frac{2}{u^6} + \frac{5}{u^7} + \frac{4}{u^8}$$ --- 01 marks

$$= \frac{2}{u^6} + \frac{5}{u^7} + \frac{4}{u^8}$$

but $u = x-2$

$$= \frac{2}{(x-2)^6} + \frac{5}{(x-2)^7} + \frac{4}{(x-2)^8}$$ ---- 01 mark

$$\frac{2x^2-3x+2}{(x-2)^8} = \frac{2}{(x-2)^6} + \frac{5}{(x-2)^7} + \frac{4}{(x-2)^8}$$ — 01 m

6(b) Given

$$(1+px)^{12} = 1 + 12(1)^{11}(px) + \frac{(12)(11)}{2}(px)^2 + \ldots \quad \text{--- 01 mark}$$

$$= 1 + 12px + 66p^2x^2$$

$$12p = -q$$

$$66p^2 = 11q$$

$$6p^2 = q \quad \text{--- 01 mark}$$

$$12p = -6p^2$$

$$6p^2 + 12p = 0$$

$$6p(p+2) = 0 \quad \text{--- 01 mark}$$

$$p = 0, \quad p = -2$$

p is non-zero

$$p = -2, \quad q = -12p \quad \text{--- 01 mark}$$

$$q = -12(-2)$$

$$q = 24$$

$\therefore p = -2$ and $q = 24$. — 01 mark

6(c) when $x^3 + 2x^2 - 5kx - 7$ is divided by $x+1$

$$p(-1) = (-1)^3 + 2(-1)^2 - 5k(-1) - 7 \quad \text{--- 01 mark}$$

$$r = 3 + 5k - 7$$

$$r = 5k - 4 \quad \text{--- 01 mark}$$

when $x^3 + kx^2 - 12x + 6$ is divided by $x - 2$

$$p(2) = 8 + 4k - 24 + 6$$

$$p(2) = 4k - 10$$

$$R = 4k - 10 \quad \text{--- 01 mark}$$

from $2r + R = 6$

$$2(5k - 4) + 4k - 10 = 6$$

$$10k - 8 + 4k - 10 = 6$$

$14k = 6 + 18$ ----- 01 mark

6(c) $14k = 24$

$k = \frac{24}{14}$

$k = \frac{12}{7}$ ----- 01 mark

6(d) $2^{n+2} + 3^{2n+1} = 7m$

when $n = 1$

$2^{n+2} + 3^{2n+1} = 7m$

$2^3 + 3^3 = 7m$

$(7)5 = 7m$

Hence it is true for $n = 1$ ----- 01 mark

Assume it is true for $n = k$

$2^{k+2} + 3^{2k+1} = 7m$

$2^{k+2} = 7m - 3^{2k+1}$ ----- 01 mark

To prove for $n = k+1$

$2^{k+1+2} + 3^{2(k+1)+1} = 2^{k+3} + 3^{2k+3}$ ----- 01 mark

$= (2^{k+2})2 + 3^{2k+3}$

$= 2(7m - 3^{2k+1}) + 3^{2k+3}$

$= 14m - 2(3^{2k+1}) + (3^{2k+1})9$

$= 14m + 7(3^{2k+1})$ ----- 01 mark

$2^{(k+1)+2} + 3^{2(k+1)+1} = 7(2m + 3^{2k+1})$

Hence it is true for $n = k+1$ – 01 mark

$\therefore$ The statement is true for all positive integers.

7(c)

Solution:

(i) $x^3\frac{dy}{dx} - x^2 - y = 0$

Test for Homogeneous differential equation.

If $f(x,y) = f(tx,ty)$, then the given differential equation is homogeneous

$$x^3\frac{dy}{dx} - x^2 - y = 0$$

$$\frac{dy}{dx} = \frac{x^2+y}{x^3} = f(x,y)$$ (0$\frac{1}{2}$ mark)

$$f(tx,ty) = \frac{t^2x^2+ty}{t^3x^3}$$ (0$\frac{1}{2}$ mar)

Therefore $x^3\frac{dy}{dx} - x^2 - y = 0$ is not homogeneous, since

$$f(x,y) \neq f(tx,ty).$$

(ii) Solution:

$$(x^2+y^2\sin(xy^{-1}))y' = 2xye^{x/y}$$

$$y' = \frac{2xye^{x/y}}{x^2+y^2\sin(x/y)} \Rightarrow f(x,y)$$ (0$\frac{1}{2}$ mark)

$$f(tx,ty) = \frac{(2tx)(ty)e^{\frac{tx}{ty}}}{(tx)^2+(ty)^2\sin\left(\frac{tx}{ty}\right)} = \frac{2t^2xye^{x/y}}{t^2(x^2+y^2\sin(x/y))}$$ (0$\frac{1}{2}$ mark)

7(a)(ii) $f(tx, ty) = \dfrac{2t^2 xy e^{x/y}}{t^2(x^2+y^2\sin(x/y))}$ — — — (02½ mark)

$\therefore f(tx, ty) = \dfrac{2xy e^{x/y}}{x^2+y^2\sin(x/y)}$ — — — (02½ mark)

The differential equation is homogeneous since $f(x,y) = f(tx,ty)$

7(b) Solution,

Taking $x+y-2 = a(x-y)^3$ is the solution of $\dfrac{dy}{dx} = \dfrac{x+2y-3}{2x-y-3}$?

Differentiate;

$1+\dfrac{dy}{dx} = 3a(x-y)^2\left(1-\dfrac{dy}{dx}\right)$ — — (01 mark)

But; $a = \dfrac{x+y-2}{(x-y)^3}$

$1+\dfrac{dy}{dx} = 3(x-y)^2\left(\dfrac{x+y-2}{(x-y)^3}\right)\left(1-\dfrac{dy}{dx}\right)$ — — (01 mark)

$(x-y)+(x-y)\dfrac{dy}{dx} = (3x+3y-6)\left(1-\dfrac{dy}{dx}\right)$

then $(4x-2y-6)\dfrac{dy}{dx} = 2x+4y-6$ — — (01 mark)

$\dfrac{dy}{dx} = \dfrac{2(x+2y-3)}{2(2x-y-3)}$

$\dfrac{dy}{dx} = \dfrac{x+2y-3}{2x-y-3}$, Hence shown! — — (01 mark)

7 (c) (i) Solution:

$dy - (xye^{2x})dx = 0$

$dy = xye^{2x}dx$ --- (0½ mark)

By separating Variables.

$\frac{dy}{y} = xe^{2x}dx$ --- (0½ mark)

Apply $\int$ both sides

$\int \frac{dy}{y} = \int xe^{2x}dx$ --- (01 mark)

$\ln y = \frac{x}{2}e^{2x} - \frac{e^{2x}}{4} + c$, since $\int xe^{2x}dx = \frac{x}{2}e^{2x} - \frac{e^{2x}}{4}$. By using By Parts. (01 mark)

$\therefore \ln y = \left(\frac{x}{2} - \frac{1}{4}\right)e^{2x} + c$, where c = Constant.

7(c) (ii) Solution:

$y'' - y' - 6y = 10e^{3x}$; $y(0) = 3$ and $y'(0) = 6$.

Consider $y'' - y' - 6y = 0$

let $y = e^{mx} \Rightarrow y' = me^{mx} \Rightarrow y'' = m^2e^{mx}$.

$(m^2 - m - 6)e^{mx} = 0$ --- (0½ mark)

$m^2 - m - 6 = 0 \Rightarrow m_1 = -2$ or $m_2 = 3$.

(c) (ii) $\therefore y'(0) = -2A + 3B + 2 = 6$

$-2A + 3B = 4$ ——— (iii) --- (00 $\frac{1}{2}$ mark)

Solving eq (i) and (ii) simultaneously. --- (00 $\frac{1}{2}$ mark)

$A = 1$ and $B = 2$.

The solution for a given differential equation is

$y = e^{-2x} + 2e^{3x} + 2xe^{3x}$. --- (00 $\frac{1}{2}$ mark)

7(d)(i)

Solution;

Let R be the amount of mass of the material at time, t.

$\frac{dR}{dt} \propto R$

$\frac{dR}{dt} = -KR$

$\int_{R_0}^{R} \frac{dR}{R} = -\int_0^t K dt$ --- (00 $\frac{1}{2}$ mark)

$\ln\left(\frac{R}{R_0}\right) = -Kt$

$\Rightarrow R = R_0 e^{-Kt}$ --- (00 $\frac{1}{2}$ mar

7(d) (i) After 2hr material lost 10% = 5mg ------ (0.5 mark)

$$45 = 50e^{-2K}$$

$$K = \frac{\ln(0.9)}{2} = -0.05268$$

$$\therefore R = R_0 e^{-0.05268t}$$ ------ (01 mark)

(ii) from

$$R = R_0 e^{-0.05268t}$$

Data given

$R_0 = 50mg$

$t = 4hrs$

$$R = 50e^{-0.05268 \times 4}$$ ------ (0.5 mark)

$$R = 40.5004177 \approx 40.5$$

$$R = 40.5mg$$

------ (01 mark)

$\therefore$ The Mass of Material after 4hours is 40.5milligrams.

8(a) $x^2+4x+4y+16=0$

From $(x-h)^2=4b(y-k)$

$x^2+4x+4=-4y-16+4$ ——— (01 mark)

$(x+2)^2=-4(y+3)$ ——— (01 mark)

$h=-2,\ k=-3$ and $b=-1$ ——— 01 mark

(i) vertex, $v(h,k)=v(-2,-3)$

ii Focus, $F(h,b+k)=F(-2,-1,-3)$

$=F(-2,-4)$ ——— (01 mark)

Focus $(-2,-4)$

iii The directrix $y=-b+k$

$y=1-3$

$y=-2$ ——— (01 mark)

$\therefore y=-2$

(5 Marks)

b Given tangent

$y=-\frac{3}{4}x+3\sqrt{2}$

from eqn of tangent to ellipse

$y=mx+\sqrt{a^2m^2+b^2}$ ——— (01 mark)

given $b^2=9$

$y=mx+\sqrt{m^2a^2+9}$

we have $c^2=18$ and $m=-\frac{3}{4}$

$\left(\sqrt{a^2(-\frac{3}{4})^2+9}\right)^2=18$

$a^2\left(\frac{9}{16}\right)+9=18$

$\frac{9a^2}{16}=9$

$a^2=16$ ——— 01 mar

$a=-4$ or $4 \Rightarrow a=4$

∴ NOW

$$e^2 = 1 - \frac{b^2}{a^2} = 1 - \frac{9}{16} = \frac{7}{16}$$ (01 mark)

$$\therefore e = \sqrt{7}/4.$$

From distance between foci $= 2ae$

$$= 2.(4)\left(\frac{\sqrt{7}}{4}\right)$$

(5 Marks) $= 2\sqrt{7}$ units (02 mark)

c From length of latus rectum of $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$, is $\frac{2a^2}{b}$

$$\frac{2a^2}{b} = \frac{13}{2}$$

$$a^2 = \frac{13b}{4}$$ (01 mark)

The length of transverse axis is $2b$.

$$16 = 2b$$ (01 mark)

$$b = 8 \quad \text{---- ii}$$

sub (ii) into i

$$a^2 = \frac{13}{4}(8)$$ (01 mark)

$$a^2 = 26$$

The eqn will be $\frac{y^2}{64} - \frac{x^2}{26} = 1$ (01 mark)

(4 Marks)

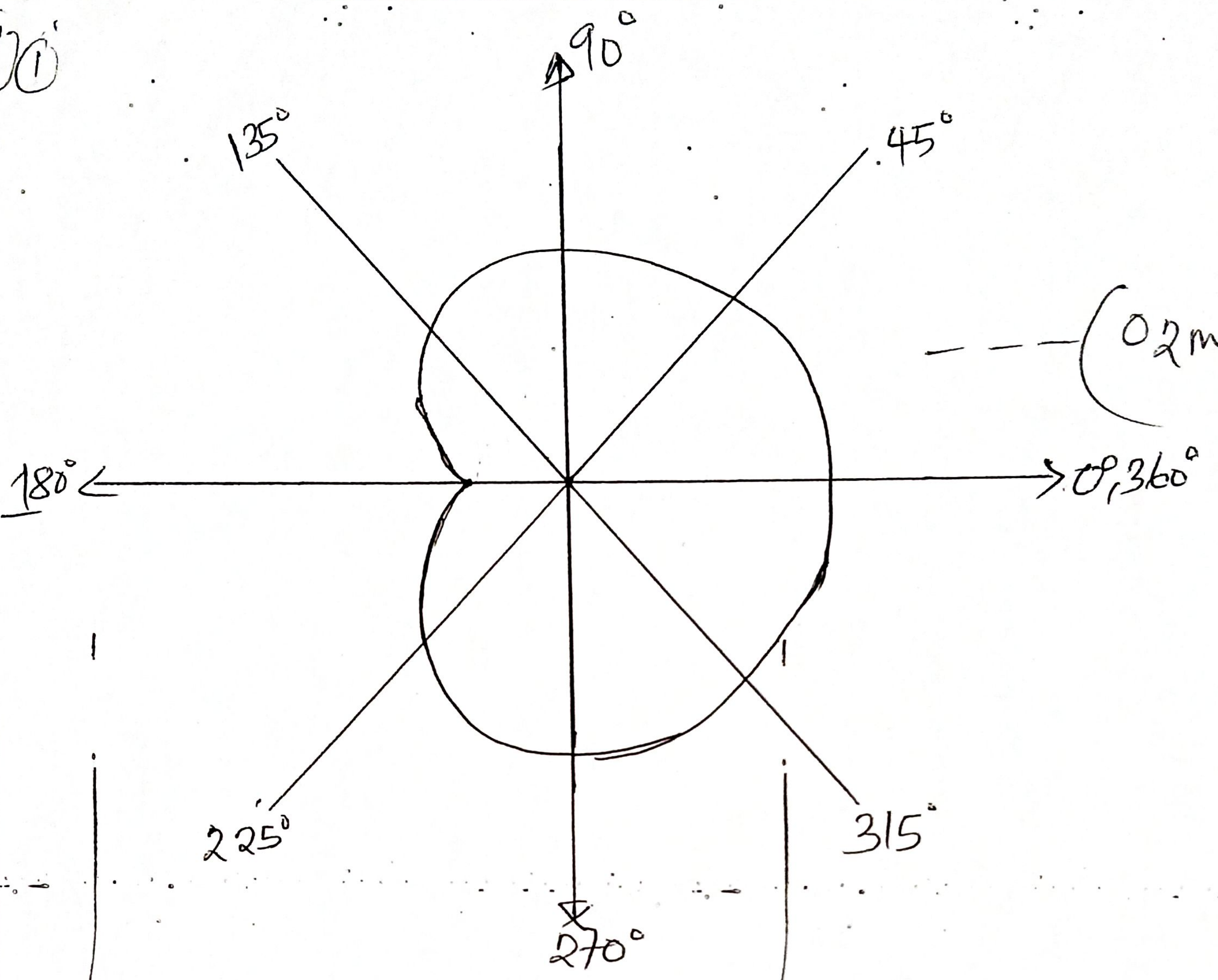
(d)(i). $r = 3 + 2\cos\alpha$

α	0°	45°	90°	135°	180°	225°	270°	315°	360°
$r = 3 + 2\cos\alpha$	5	4.4142	3	1.5858	1	1.5858	3	4.4142	5

(02

GRAPH OF $r = 3 + 2\cos\theta$

8(4)(i)



--- (02 mark)

(ii) $r = 3 + 2\cos\theta$

$r = 3 + 2\left(\frac{x}{r}\right)$ ------ (01 mark)

$r^2 = 3r + 2x$

But $r^2 = x^2 + y^2$ and $r = \sqrt{x^2 + y^2}$.

$x^2 + y^2 = 3\sqrt{x^2 + y^2} + 2x$. ------ (01 mark)

(6 Marks)