



**TANZANIA HEADS OF ISLAMIC SCHOOLS COUNCIL**  
**FORM SIX INTER ISLAMIC MOCK EXAMINATION**  
**ADVANCED MATHEMATICS 2**

**(For Both School and Private Candidates)**

**142/2**

**TIME: 3 HOURS**

**Monday, 10<sup>th</sup> March 2025 p.m.**

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**Instructions**

1. This paper consists of sections A and B with a total of **eight (08)** questions.
2. Answer **all** questions in section A and **two** questions from section B
3. All work done in answering each question must be shown clearly.
4. Mathematical tables and non-programmable calculators may be used.
5. Communication devices and any unauthorized materials are **not** allowed in the examination room.
6. Write your **Examination Number** on every page of your answer booklet(s).

## SECTION A (60 Marks)

Answer **all** questions in this section

1. (a) If A and B are two events chosen from a sample space S.

Prove that  $P(A) + P(B) = P(A \cup B) + P(A \cap B)$

- (b) A roulette wheel is divided into 6 sectors marked with numbers 1, 2, 3, 4, 5 and 6. The wheel is spin and X is the random variable (the number on which the wheel stops). The probability distribution of X is as follows:

| X          | 1              | 2              | 3             | 4             | 5              | 6              |
|------------|----------------|----------------|---------------|---------------|----------------|----------------|
| $P(x) = x$ | $\frac{1}{16}$ | $\frac{3}{16}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{3}{16}$ | $\frac{1}{16}$ |

Calculate:

- (i)  $E(x)$
  - (ii)  $E(x^2)$
  - (iii)  $E(3x-5)$
  - (iv)  $E(6x^2+6x-10)$
- (c) The time taken by a milk man to deliver milk to the high street is normally distributed with mean 12 minutes and variance 4 minutes. He delivers milk everyday. Estimate the number of days during the year when he takes;
- (i) Longer than 17 minutes
  - (ii) Less than 10 minutes
  - (iii) Between 9 and 13 minutes
2. (a) By using truth table determine whether  $P \wedge (q \wedge r)$  is equivalent to  $[P \rightarrow (q \vee r)] \rightarrow (P \wedge q)$ ?
- (b) Use laws of algebra of proposition to establish whether each of the following proposition is a tautology
- (i)  $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$
  - (ii)  $[(p \rightarrow aq) \wedge (r \rightarrow q) \wedge q] \rightarrow r$
- (c) Write the inverse, converse and contrapositive of the statement.
- “If Juma was born before Ally, then Aisha was born before Maganga.”

- (d) Using basic conjunctions, find the equivalent statement for k given in the truth table below, simplify it hence draw its corresponding simple network diagram

| p | q | r | k |
|---|---|---|---|
| T | T | T | T |
| T | T | F | T |
| T | F | T | T |
| T | F | F | F |
| F | T | T | F |
| F | T | F | F |
| F | F | T | F |
| F | F | F | F |

3. (a) Find a vector in the direction of vector  $5\mathbf{i} - \mathbf{j} + 2\mathbf{k}$  which has magnitude of 8 units.
- (b) Find the displacement vector  $\mathbf{r}(t)$  for the initial condition, for  $\mathbf{r}''(t) = -4\cos t \mathbf{i} - 3\sin 3t \mathbf{k}$  if  $\mathbf{r}'(0) = 3\mathbf{k}$ ,  $\mathbf{r}(0) = 4\mathbf{j}$ .
- (c) Find the work done by  $\mathbf{F} = 2\mathbf{i} + \mathbf{j} + 3\mathbf{k}$  acting on a particle if a particle is displaced from a point with position vector  $2\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$  to the point with position vector  $3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}$ .
- (d) Consider a vector of magnitude 14 in the direction of  $3\mathbf{i} + 6\mathbf{j} + 2\mathbf{k}$ . What will be the magnitude of a vector in the direction perpendicular to a plane where by  $A(0, 1, 3)$ ,  $B(-1, 0, 1)$  and  $C(1, -1, -2)$  are vertices of a mentioned plane?
4. (a) Solve the equation  $\frac{13z}{z+1} = 11 - 3i$  giving the answer in form of  $x + iy$ , where  $x$  and  $y$  are real numbers.
- (b) Given that  $x = \cos\alpha + i \sin\alpha$  and  $y = \cos\beta + i \sin\beta$  prove that  $x^m y^n - \frac{1}{x^m y^n} = 2i \sin(m\alpha + n\beta)$
- (c) If  $z = x + iy$  is a complex number satisfying the equation  $|z - (1 + i)|^2 = 2$  and  $p = \frac{2}{z}$ , find the locus of  $p$ .

## SECTION B (40 Marks)

Answer **two (2)** questions from this section

5. (a) If  $\sin A = \frac{1}{4}$  and A is in second quadrant respectively, evaluate each of the following (leaving the answer in surd form);
- $\tan \frac{A}{2}$
  - $\cos \frac{A}{2}$
- (b) Solve the  $\sin 2y + \cos y = \sin y \cos y$  for  $0 \leq y \leq 2\pi$ .
- (c) Use trigonometric identities to prove that;
- $$\cos 8A = 128 \cos^8 A - 256 \cos^6 A + 160 \cos^4 A - 32 \cos^2 A + 1$$
- (d) If  $a \cos^2 x + b \sin^2 x = c$ , prove that  $\tan^2 x = \frac{c-a}{b-c}$
6. (a) Decompose  $\frac{2x^2-3x+2}{(x-2)^3}$  in partial fraction.
- (b) Given that in the expansion of  $(1 + px)^{12}$  the coefficient of x is -q and the coefficient of  $x^2$  is 11q. Find the value of p and q, where p is non zero constant.
- (c) Suppose that r and R are the remainders when the polynomials  $x^3+2x^2-5x-7$  and  $x^3+kx^2-12x+6$  are divided by  $x+1$  and  $x-2$  respectively. Find the value of k if  $2r+R=6$ .
- (d) Prove by mathematical induction that  $2^{n+2} + 3^{2n+1}$  is divisible by 7 for all positive integers.
7. (a) Determine whether the following differential equations are homogeneous:
- $x^3 \frac{dy}{dx} - x^2 - y = 0$ .
  - $(x^2 + y^2 \sin(xy^{-1})) y' = 2xye^{x/y}$
- (b) Show that  $x+y-2 = a(x-y)^3$  is a solution to the differential equation  $\frac{dy}{dx} = \frac{x+2y-3}{2x+y-3}$
- (c) Solve each of the following differential equations:
- $dy - (xye^{2x}) dx = 0$
  - $y'' - y' - 6y = 10e^{3x}$
- (d) A certain radioactive material is known to decay at a rate proportional to the amount present. If initially there is 50 milligrams of the present after two hours it is observed that the material has lost 10 percent of its original mass. Find:
- An expression for the mass of the material remaining at any time, t
  - The mass of the material after four hours

8. (a) Given the parabola  $x^2+4x+4y+16=0$  Find:

- (i) The coordinate of vertex
- (ii) The coordinate of the focus
- (iii) The equation of the directrix

(b) If  $3x+4y=12\sqrt{2}$  is a tangent to the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{9}$  for some  $a \in \mathbb{R}$ , find the distance between the foci.

(c) Given that the length of latus rectum and transverse axis of the hyperbola

$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$  are  $6\frac{1}{2}$  and 16 units respectively. Find the equation of the hyperbola.

(d) Given the polar equation  $r = 3 + 2\cos\theta$

- (i) Draw its graph
- (ii) Find the corresponding rectangular equation.

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