



TANZANIA HEADS OF ISLAMIC SCHOOL COUNCIL
FORM SIX INTER ISLAMIC MOCK EXAMINATION
ADVANCED MATHEMATICS 1

(For Both School and Private Candidates)

142/1

TIME: 3 HOURS

Tuesday, 4th March 2025 a.m.

Instructions

1. This paper consists of ten (10) compulsory questions.
2. All work done in answering each question must be shown clearly.
3. Mathematical tables and Non-programmable calculators may be used.
4. Cellular phones and any unauthorized materials are **not** allowed in the examination room.
5. Write your **Examination Number** on every page of your answer booklet(s).

1. Use a non-programmable scientific calculator to:

(a) Find the value of
$$\left[e^{\left[\frac{\int_1^2 \ln x \times dx + \sum_{x=0}^2 \log(x+1)}{\cosh e - \sin 3\frac{\pi}{2}} \right]} \right]^{\frac{1}{2}}$$

(b) Find the value of E in the equation

$$q = \frac{6\pi n}{E} \sqrt{\frac{9}{2} \left(\frac{\eta v_1}{\rho - \delta g} \right)} (v_1 + v_2)$$

Given that:

$$\rho = 0.820, \delta = 0.0421, g = 9.8, \eta = 14.588, \pi = 3.14, q = 1 \times 10^{-3}, V_1 = 5, V_2 = 15.$$

(c) Compute the mean and standard deviation of the data: $\ln 5, \sqrt{2}, 8, \cosh^{-1} 3, e, \pi$. Correct to three decimal places.

2. (a) Prove that $\tanh^{-1} \left(\frac{x^2 - a^2}{x^2 + a^2} \right) = \ln \left(\frac{x}{a} \right)$

(b) Define the function $\cosh X$ and $\sinh X$. From your definition prove that:

(i) $\cosh 2x = 2 \cosh^2 x - 1$

(ii) $\sinh (x+y) = \sinh X \cosh y + \cosh X \sinh y$

(c) Find:

$$\int \frac{dx}{\sqrt{x^2 + 2x + 10}}$$

3. (a) The labour cost for two professional tailors A and B are Tsh 80,000 and Tsh 100,000 per day respectively. Tailor A can stitch 6 shirts 10 shirts and 4 pairs of trousers per day while tailor B can stitch 10 shirts and 4 pairs of trousers per day. The tailors intend to produce at least 60 shirts and 32 pairs of trousers. Formulate a linear programming problem which minimizes the labour cost involved.

(b) The daily meat requirements of three hotels L, M and N is 70kg, 40kg and 60kg respectively, two meats suppliers X and Y have capacities of supplying 90kg and 80kg of meat respectively. The cost of transportation in Tanzania shillings are given in the table below:

From/To	X	Y
L	3000	2000
M	2000	3000
N	400	4000

How many kilograms of meat should be transported from each supplier to each hotel in order to minimize the transportation cost?

4. (a) The mean and standard deviation of 6 observations are 8 and 4 respectively. If each observation is multiplied by 3, find the new mean and new standard deviation of the resulting observations.

- (b) An original frequency table with mean 11 and variance 9.9 was lost but the following table derived from it was found.

Construct the original table

Value (d)	-2	-1	0	1	2
Frequency (f)	1	6	7	4	2

5. (a) Prove the identity $(A \cup B)' \cap (A \cap B)' = (A \cup B)'$ by using laws of algebra of set.

- (b) Given set $A = \{x \in \mathbb{R} : x \geq 3 \text{ and } x \leq 4\}$, $B = \{x \in \mathbb{R} : x > 3\}$ and $C = \{x \in \mathbb{R} : -1 \leq x < 3\}$

Represent on a real number line the following set

(i) $(A \cup B)'$

(ii) $(A \cup B)' \cap C$

- (c) In a survey of 100 local newspapers was found that 28 people read mipasho, 30 people read mgeni, 42 people read maprosoo, 8 people read mipasho and mgeni, 10 people read mipasho and maprosoo, 5 people read mgeni and maprosoo and maprosoo, 3 people read all three newspapers. Find:-

- (i) The number of people who read maprosoo only
(ii) The number of people who read none of the three newspapers

6. (a) (i) Given that $f(x) = 10x$ and $g(x) = x + 3$. Find $fg(x)$ and $(fg)^{-1}(x)$ and hence verify that if $b = fg(a)$ then $(fg)^{-1}(b) = a$.

- (ii) The domain of the function f is $\mathbb{R}$.

$$\text{Given } \begin{cases} f: x \rightarrow 1, & \text{if } x \in \mathbb{Z} \\ f: x \rightarrow 2, & \text{if } x \notin \mathbb{Z} \end{cases}$$

Write down $f(5)$, $f(-1)$, $f(0)$, $f(3,4)$, $f(\sqrt{2})$ and $f(\pi)$

- (b) Sketch the graph of the function given $f(x) = 3 - 2(x+1)$ from the graph sketched, find

- (i) Domain and Range
(ii) Asymptotes of the graph

7. (a) Calculate the maximum possible error in $Y = \frac{4mgL}{\pi \Delta^2 s}$ where;

Mass $M = 100\text{g}$ original length,

$L = 200\text{cm}$, Diameter, $D = 0.075\text{cm}$

Extension, $\ell = 0.325\text{cm}$

$\Delta L = 0.1\text{cm}$

$\Delta D = \Delta \ell = 0.001\text{cm}$

g is a gravitational constant.

- (b) By using Newton's method, prove that the iterative formula for the equation $ax^2+bx+c=0$ is given by $X_{n+1} = \frac{ax_n^2 - c}{2ax_n + b}$,

Hence starting with $X_0 = 2.000$ calculate approximate root of $f(x) = x^2-2x-1$ to three decimal places. Perform two iterations.

- (c) Approximate the value of $\int_0^{10} \sqrt{\frac{1 + \cosh x}{x}} dx$ with $n=8$ sub-intervals using the Simpson's rule.

8. (a) Point A lies at co-ordinate (2, 3) and point B at (8, 7). Determine the distance AB and angle AB makes with the horizontal.

- (b) If the point P(x,6) divides externally the line segment joining the points A(-1,3) and B(4,y) in the ratio 3:2, find the values of x and y.

- (c) Determine the equation of the circle which passes through the point P (2, 3) has its centre on the line $x + y = 4$ and intersects orthogonally the circle $x^2 + y^2 - 4x + 2y - 3 = 0$.

9. (a) Find:

$$\int \frac{\cot x}{\sqrt{1 - \cos 2x}} dx$$

- (b) Show that $\int_{\frac{\pi}{2}}^{2\pi} \frac{3\cos x - 5\sin x}{4\cos x - \sin x} dx = \ln 4 + \frac{3}{2}\pi$

- (c) Find the volume of the solid generated by rotating about x-axis the area bounded by $y^2 = x^2 + 1$ and the a-axis between $x = 1$ and $x = 2$.

10. (a) Differentiate $(\ln(2x-1))^2$ with respect to x.

- (b) A particle moves along a straight line $\overline{OB}$ so that it is S metres from O in t seconds, where $S = t(2t-3)(t-4)$. Deduce the expressions for its velocity and acceleration in terms of t.

- (c) Verify that $\frac{dy}{dx} = 1$, if $x = \tan^{-1}\left(\frac{2t}{1-t^2}\right)$ and $y = \sin^{-1}\left(\frac{2t}{1+t^2}\right)$

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