TANZANIA HEADS OF ISLAMIC SCHOOL COUNCIL

FORM SIX INTER ISLAMIC MOCK EXAMINATION

BASIC APPLIED MATHEMATICS

141

MARKING GUIDE

1\. (a) (i) 1.507×10^{-2} ---- 02

(ii) 0.219 ---- 02

(b) $\bar{x} = 7.62$ ---- 02

$\delta x = 1.2472$ ---- 02

$\Sigma x^2 = 228701$ ---- 02

2 (a) $f(x) = \dfrac{ax+b}{x+1}$ $\quad f(2) = \dfrac{2a+b}{2+1} = 1$

$2a + b = 3$ ---- (i) ---- 01

also $f(1) = \dfrac{a+b}{1+1} = \dfrac{a+b}{2} = 2$

$a + b = 4$ ---- (ii) ---- 01

Solve (i) and (ii)

$$-\begin{cases} 2a+b = 3 \\ a+b = 4 \end{cases}$$

$a = -1$ ---- 01

$a + b = 4$

$-1 + b = 4$

$b = 5$ - - - - - - - 01

2 b) $g(x) = \dfrac{x}{x+2}$

x-intercept, y=0

$\cancel{x} = 0$

$(0,0)$. - - - 00½

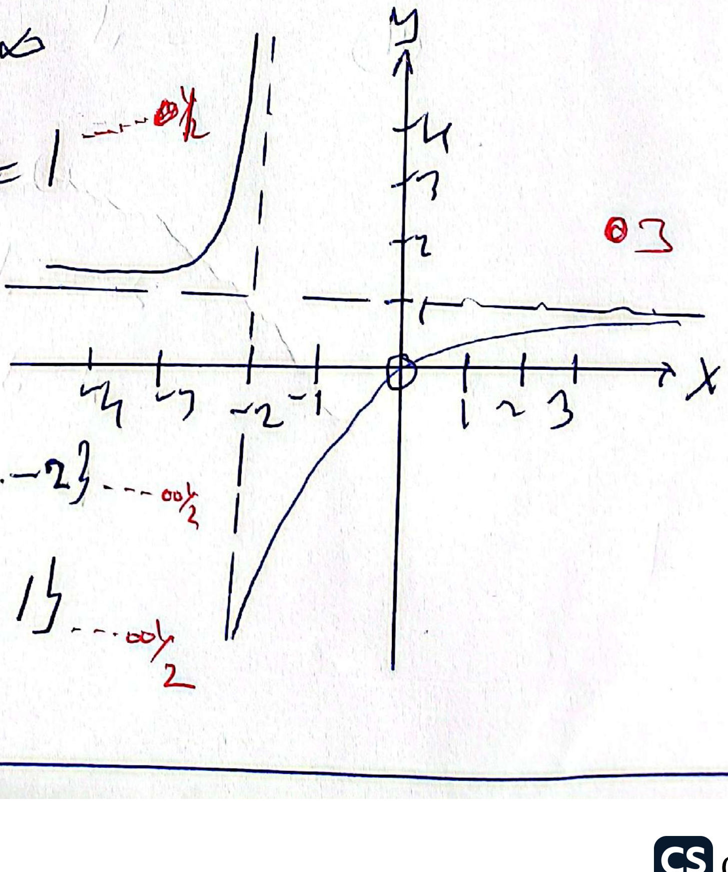
V.A: $x + 2 = 0$

$x = -2$ - - - - 00½

H.A. $y = \dfrac{x/x}{x/x + 2/x} = \dfrac{1}{1 + 2/x}$

as $x \to \infty$

$y = \dfrac{1}{1} = 1$ - - - - 0½



03

Domain = $\{x \in \mathbb{R} : x \neq -2\}$ - - - - 00½

Range = $\{y \in \mathbb{R} : y \neq 1\}$ - - - - 00½

3 (a)

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{10} = \frac{10}{2}[2a + (10-1)d] =$

$S_{10} = 5[2a + 9d)$

$= 10a + 45d = 210$ --- (1) ---01

$S_{20} = \frac{20}{2}[2a + (20-1)d]$

$S_{20} = 10[2a + 19d]$

$= 20a + 190d$.

$20a + 190d - 210 = 410$ ---01

$20a + 190d = 620$ --- (11)

solve (1) & (11)

$20a + 190d = 620$

$10a + 45d = 210$

$a = 12,$

$d = 2$

$d = 2, \quad a = 12$ ---01

3 b)

$I \propto \frac{A}{d^2}$ I = Intensity, A = Amplitude
d = distance

$I = \frac{kA}{d^2}$

$50 = \frac{k(5)}{2^2}$ - - - - 01

$\frac{4 \times 50}{5} = k$

$k = 40$ - - - - - - 01

$I = \frac{40A}{d^2}$

$A = 8, d = 4$

$I = \frac{40 \times 8}{16}$ - - - - 01

$I = 20$

The Intensity is 20 units - - - 01

4(a)

$$y = \frac{x^3-4}{2-x^2}$$

$$\frac{dy}{dx} = \frac{(2-x^2)\frac{d}{dx}(x^3-4) - (x^3-4)\frac{d}{dx}(2-x^2)}{(2-x^2)^2}$$ — — — 00½

$$= \frac{3x^2(2-x^2) - (-2x)(x^3-4)}{(2-x^2)^2}$$ — — — 00½

$$= \frac{6x^2 - 3x^4 + 2x^4 + 8x}{(2-x^2)^2}$$

$$\frac{dy}{dx} = \frac{6x^2 - x^4 - 8x}{(2-x^2)^2}$$ — — — 01

4 a) ii) $x^2y - xy^{-2} = 2x$ at $(1,-1)$.

Differentiate both side w.r.t x.

$$2xy + x^2\frac{dy}{dx} - y^{-2} + 2xy^{-3}\frac{dy}{dx} = 2.$$ - - - $01/2$

$$x^2\frac{dy}{dx} + 2xy^{-3}\frac{dy}{dx} = -2xy + y^{-2} + 2$$

$$(x^2+2xy^{-3})\frac{dy}{dx} = y^{-2} - 2xy + 2$$

$$\frac{dy}{dx} = \frac{y^{-2} - 2xy + 2}{x^2+2xy^{-3}}$$ - - - $01/2$

at $(1,-1)$.

$$\frac{dy}{dx} = \frac{(-1)^{-2} - 2(1)(-1) + 2}{(-1)^2 + 2(1)(-1)^{-3}}.$$ - - - $01/2$

$$\frac{dy}{dx} = \frac{1+2+2}{1-2}$$

$$\frac{dy}{dx} = -5$$

- - - $01/2$

$$\therefore \frac{dy}{dx} = -5.$$

4 (b) $y = 1 + \cos^2 x$

$y = 1 + (\cos x)^2$

$\frac{dy}{dx} = -2\sin x \cos x$

$= -2\sin 2x$ (03)

4 (c) $S(t) = t^3 - 2t^2 + 6t + 5$

$V = S'(t) = 3t^2 - 4t + 6$ ------01

at $t = 2$ sec.

$V = S'(2) = 3(2)^2 - 4(2) + 6$

$= 12 - 8 + 6$

$V = 10$ m/s ------01

$a = V'(t) = 6t - 4$

at $t = 2$ sec

$V'(2) = 6(2) - 4$

$= 12 - 4$ ------01

$= 8$ m/s²

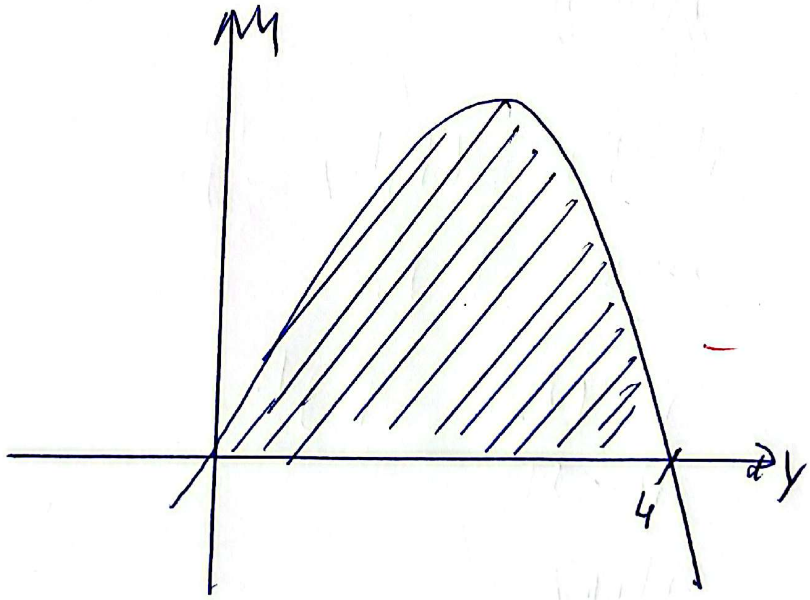
At the end of 2sec. the velocity is 10m/s. and acceleration is 8m/s²

5 (a) $\int \left(4x^3 + 7x^2 - \frac{3}{2}\cos x\right) dx$

$= x^4 + \frac{7x^3}{3} + \frac{3}{2}\sin x + c$ - - - - 02

(b) $y = 4x - x^2$, $y = 0$, $4x - x^2 = 0$

$x(4 - x) = 0$, $x = 0$, or $x = 4$ - - - - 001/2



- - - - 001/2

$$\text{Area} = \int_0^4 (4x - x^2)\,dx \quad \text{------------} \quad 00\tfrac{1}{2}$$

$$\left[2x^2 - \frac{x^3}{3}\right]_0^4 \quad \text{------------} \quad 00\tfrac{1}{2}$$

$$= \left(2(4)^2 - \frac{4^3}{3}\right) - 0 \quad \text{------------} \quad 01$$

$$32 - \frac{64}{3}$$

$$96 - \frac{64}{3}$$

$$\text{Area} = \frac{32}{3} \text{ Sq. units} \quad \text{------} \quad 01$$

5 (c)

$$\frac{ds}{dt} = 4m/s$$

$$ds = 4\,dt$$

$$\int ds = \int 4\,dt$$

$$s = 4t + c \quad \text{------------} \quad 01$$

$$5 = 4 \times 2 + c$$

$C = 5 - 8$

$C = -3$ ------ 01

$s = 4t - 3$

$s = \int_1^3 (4t-3)\,dt$

$s = 2t^2 - 3t \Big]_1^3$ ------ 01

$s = (18-9) - (2-3)$

$9 + 1$

$s = 10m$ ------ 01

Distance = 10m

6

Marks %	f	x	d = x - A	d^2	fd	
20-29	26	24.5	-20	400	-520	10400
30-39	36	34.5	-10	100	-360	3600
40-49	44	44.5	0	0	0	0
50-59	42	54.5	10	100	420	4200
60-69	28	64.5	20	400	560	11200
70-79	24	74.5	30	900	720	216,000
		00½	00½	00½	820 00½	510,000 00½

$$\text{Mean} = A + \frac{\sum fd}{N}$$ ---- 00½

$$= 44.5 + \frac{820}{200}$$

$$= 48.6$$ ------- 01

Variance $$\frac{\sum fd^2}{N} - \left(\frac{\sum fd}{N}\right)^2$$ ---- 00½

$$\frac{51000}{200} - \left(\frac{820}{200}\right)^2$$ ---- 00½

$$38.19$$ ------- 01

b) $$\text{Median} = L + \left(\frac{\frac{N}{2} - cb}{fw}\right)c$$ ---- 00½

Median class 40 - 49. $L = 39.5$ ---- 00½

$$\text{Median} = 39.5 + \left(\frac{\frac{200}{2} - 62}{44}\right) \times 10$$

$$= 48.136364$$ ------- 01

(c) Total number of students who passed

$= 42 + 28 + 24$

$= 94$ ------ 01

% students who passed $= \frac{94}{200} \times 100$

$= 47\%$ ------ 01

7\. (a) $\frac{8!}{3!\,2!} = 3360$ ways. --- 02

(b) (i) $n(S) = {}^{10}C_3 = 120$

$n(E) = {}^{6}C_3 = 20$

$P(E) = \frac{n(E)}{n(S)} = \frac{20}{120} = \frac{1}{6}$ ---- 02

(ii) ${}^{6}C_1 \times {}^{4}C_2$ ~~n(E)~~ $=$

$n(E) = 36$

$P(E) = \frac{n(E)}{n(S)} = \frac{36}{120} = \frac{3}{10}$

p (one boy obtained) $= \frac{3}{10}$ ---- 02

7 (c) (i) T = people who like tea

C = people who like coffee

$n(T) = 65$, $n(C) = 55$

$n(T \cap C) = 30$

$n(S) = 100$

$P(T \cup C) = P(T) + P(C) - P(T \cap C)$

$= \frac{65}{100} + \frac{55}{100} - \frac{30}{100}$ ---- 01

$\frac{90}{100}$

$P(T \cup C) = \frac{90}{100}$

$= \frac{9}{10}$ ---- 01

(ii) P(like neither tea nor coffee) $= P(T \cup C)'$

$P(T \cup C)' = 1 - P(T \cup C)$ ---- 01

$1 - \frac{9}{10}$

$P(T \cup C)' = \frac{1}{10}$ ---- 01

8 (5) $\sin x = 4 \cos x$

$\frac{\sin x}{\cos x} = 4$

$\tan x = 4$ ---- 01

$x = \tan^{-1}(4)$

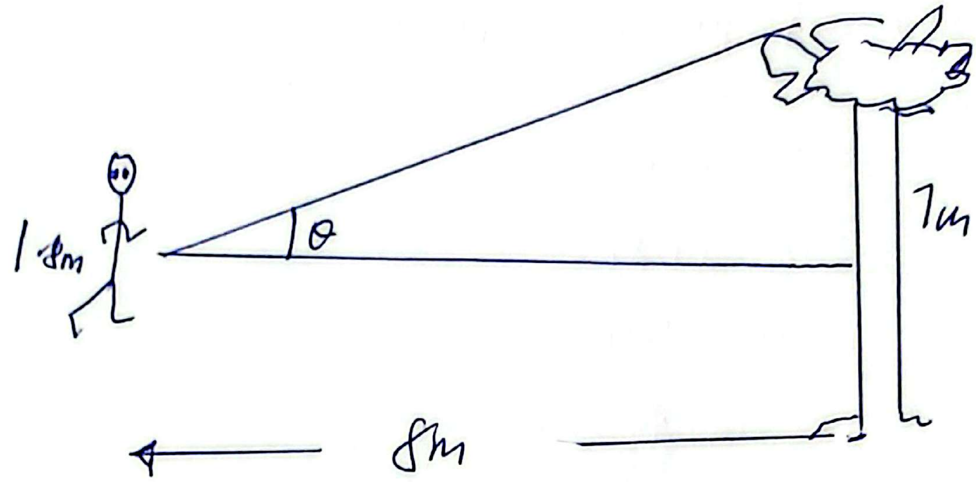
$x = 75.96^\circ$ ---- 01

$x = 180n + 75.96$

$= 180 + 75.96$

$x = 255.96$ ---- 01

$x = 75.96^\circ$, $x = 255.96^\circ$



$$\theta = \tan^{-1}\left(\frac{7m - 1.8m}{8m}\right)$$

$$= 33.02$$

$$\theta \simeq 33^\circ$$

Angle of elevation $= \underline{33^\circ}$.

8 (c) $$T = \frac{2u}{g}\left(\sin\theta - \frac{\cos\theta}{\cot\alpha}\right)$$

$$T = \frac{2u\sin(\theta - \alpha)}{g\cos\alpha}$$

b) $\frac{2u}{g}\frac{\sin(\theta-\alpha)}{\cos\alpha}$

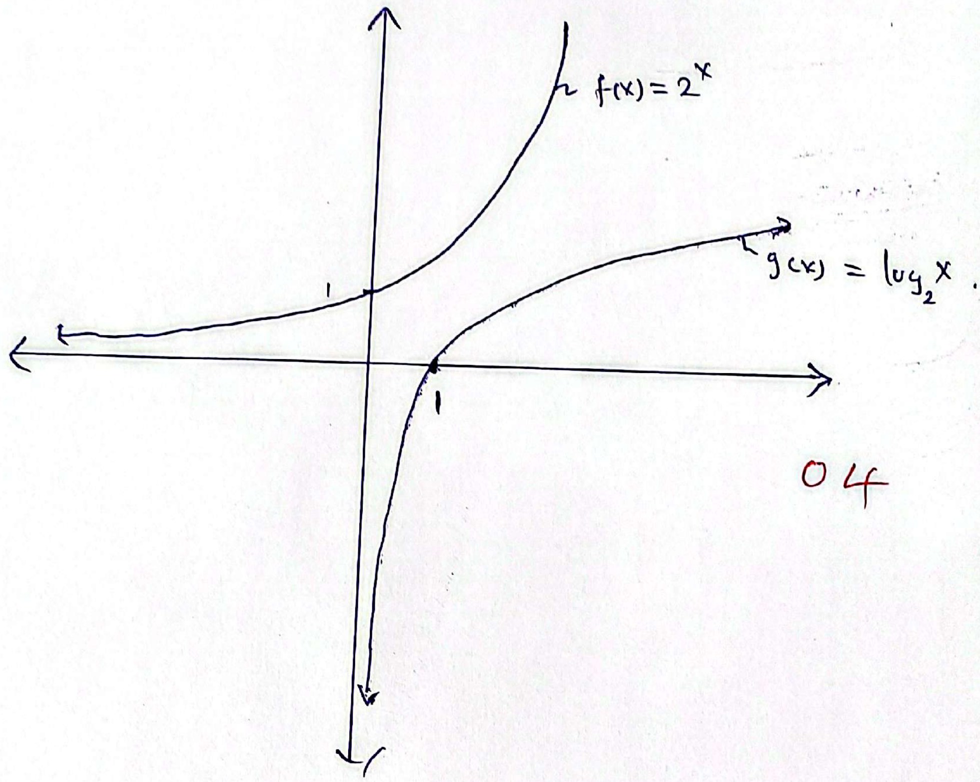
$= \frac{2u}{g}\frac{\sin\theta\cos\alpha - \sin\alpha\cos\theta}{\cos\alpha}$

$= \frac{2u}{g}(\sin\theta - \cos\theta\cdot\tan\alpha)$

$= \frac{2u}{g}\left(\sin\theta - \frac{\cos\theta}{\cot\alpha}\right)$ - - - - 03

$\therefore \frac{2u}{g}\frac{\sin(\theta-\alpha)}{\cos\alpha} = \frac{2u}{g}\left(\sin\theta - \frac{\cos\theta}{\cot\alpha}\right)$.

9 a)



04

b) Given

i) $N = 50 + 30e^{t/100}$

At the start, $t = 0$

$N = 50 + 30e^{0/100}$ ---- 01

$N = 50 + 30$ ---- 01

$N = 80$.

∴ The size of polutation at the start of observation is 80 insects. ---- 01

.9 ii) Given $N=120$

$$120 = 50 + 30e^{t/100} \quad - - - - - - 01$$

$$e^{t/100} = \frac{120-50}{30}$$

$$e^{t/100} = 7/3.$$

$$t/100 = \ln(7/3). \quad - - - - 01$$

$$t = 100\ln(7/3).$$

$$\simeq 85. \quad - - - - -.01$$

$\therefore$ It will take 85 days to reach 120 infect

·10 a) Given

$$A = \begin{bmatrix} 1 & k & 2 \\ 3 & 4 & 5 \\ 6 & 7 & 8 \end{bmatrix}$$

for singular matrix

Determinant = 0

$$|A| = 0$$

$$\begin{vmatrix} 1 & k & 2 \\ 3 & 4 & 5 \\ 6 & 7 & 8 \end{vmatrix} = 0 \quad \text{- - - - 01}$$

$$\begin{vmatrix} 4 & 5 \\ 7 & 8 \end{vmatrix} - k\begin{vmatrix} 3 & 5 \\ 6 & 8 \end{vmatrix} + 2\begin{vmatrix} 3 & 4 \\ 6 & 7 \end{vmatrix} = 0 \quad \text{- - - - 01}$$

~~32-8x5~~

$$-3 - k(-6) + 2(-3) = 0 \quad \text{- - - - 01}$$

$$-9 + 6k = 0$$

$$6k = 9 \quad \text{- - - - 01}$$

$$k = 9/6 = 3/2.$$

$$\therefore \quad k = 3/2 \quad \text{- - - - 01}$$

10 b) let

x be the number of table manufactured ---00½

y be the number of chairs manufactured --00½

Maximize

$$f(x, y) = 90x + 25y \quad ------ 01$$

Subject to;

$$8x + 2y \leq 400 \Rightarrow 4x + y \leq 200 \quad --- 01$$

$$2x + y \leq 120 \quad ----- 0½$$

$$x \geq 0 \quad ------ 00½$$

$$y \geq 0 \quad ------ 00½$$