

TANZANIA HEAD OF ISLAMIC SCHOOL
COUNCIL

FORM SIX INTER ISLAMIC MOCK
EXAMINATION

142/2

ADVANCED MATHEMATICS 2

MARKING GUIDE

$$1 \quad a) \quad {}^4C_2 = \frac{4!}{(4-2)!2!} = \frac{4!}{2!2!} = \frac{4 \times 3 \times 2 \times 1}{2 \times 2} = 6 \quad \text{or}$$

$$b) \quad E(x) = 1$$

$$\frac{1}{6} + \frac{1}{3} + \frac{1}{4} + x + y = 1 \quad \text{or}$$

$$x + y = \frac{1}{4} \quad \text{--- (i) or}$$

$$\sum x P(x) = \frac{14}{3}$$

$$2\left(\frac{1}{6}\right) + 3\left(\frac{1}{3}\right) + 5\left(\frac{1}{4}\right) + 7x + 9y = \frac{14}{3} \quad \text{or}$$

$$7x + 9y = \frac{25}{12} \quad \text{--- (ii) or}$$

$$\text{(i) + (ii)}$$

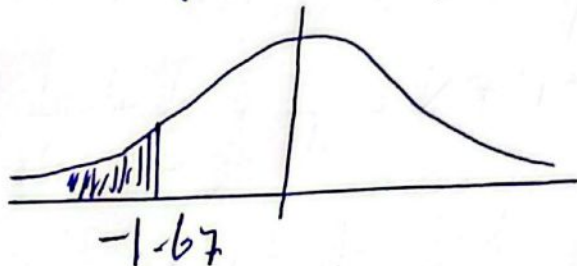
$$x = \frac{1}{12}, \quad y = \frac{1}{6} \quad \text{or}$$

$\frac{1}{20}$

es (i) $P(\text{less than } 750)$

$$P(x < 750) = P\left(z < \frac{750 - 753}{1.8}\right) \quad \checkmark$$

$$P(z < -1.67)$$



$$P(z < -1.67) = 0.0475 \quad \checkmark$$

For the batch of 1500 bottles, it is likely that

$$1500 \times 0.0475 = 71 \text{ bottles.} \quad \checkmark$$

will contain less than 750 ml.

$$\text{ii) } z_1 = \frac{751 - 753}{1.8} \quad \text{and} \quad z_2 = \frac{754 - 753}{1.8} \quad \checkmark$$

$$-1.11 \quad \text{and} \quad 0.556 \quad \checkmark_{1/2}$$

$$P(751 < x < 754) = P(-1.11 < z < 0.556)$$

$$0.3663 + 0.6123 \quad \checkmark$$

$$0.9786$$

$\checkmark_{2/2}$

For 1500 bottles it is likely that
 $1500 \times 0.5788 = 868 \text{ bottles}$ ✓

2. (i) $(p \vee q) \vee r$ -

Either both today is Friday and Asha
is clever or you will come tomorrow. ✓

(ii) $(p \vee q) \wedge r$

Both either today is Friday or Asha is clever
and you will come tomorrow. ✓

(iii) $(p \vee r) \rightarrow \neg p$

If either today is Friday or you won't
come tomorrow then Asha is not clever
won't come tomorrow. ✓

(iv) $(q \rightarrow p) \vee (p \rightarrow \neg r)$

Either if Asha is clever then today is
Friday or if today is Friday then you
will not come tomorrow. ✓

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b) $P \leftrightarrow T$ $\frac{1}{2}$
 $Q \leftrightarrow T$ $\frac{1}{2}$
 $R \leftrightarrow F$ $\frac{1}{2}$

i) $(P \wedge Q) \vee R$ $\frac{1}{2}$
 $(T \wedge T) \vee F$ $\frac{1}{2}$
 $T \vee F$ $\frac{1}{2}$
 T $\frac{1}{2}$

ii) $(P \vee R) \wedge R$ $\frac{1}{2}$
 $(T \vee F) \wedge F$ $\frac{1}{2}$
 $T \wedge F$ $\frac{1}{2}$
 $False$ $\frac{1}{2}$

iii) $(Q \rightarrow P) \vee (P \rightarrow \neg R)$
 $(T \rightarrow T) \vee (T \rightarrow \neg F)$ $\frac{1}{2}$
 $T \vee (T \rightarrow T)$ $\frac{1}{2}$
 $T \vee T$ $\frac{1}{2}$
 T $\frac{1}{2}$

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- 2 (c) let p = the borrower mathematics' book
 q = the will remain in school ✓
 r = He is punished

$$[(p \rightarrow q) \wedge (r \vee \neg q) \wedge \neg r] \rightarrow \neg p$$

(a) (b) c

p	q	r	$\neg p$	$\neg q$	$\neg r$	$p \rightarrow q$	$r \vee \neg q$	ans	$C \rightarrow \neg p$
T	T	T	F	F	F	T	T	T	F
T	T	F	F	F	T	T	F	F	T
T	F	T	F	T	F	F	T	F	T
T	F	F	F	T	T	F	T	F	T
F	T	T	T	F	F	T	T	F	T
F	T	F	T	F	T	T	F	F	T
F	F	T	T	T	F	T	T	F	T
F	F	F	T	T	T	T	T	T	T

It is not tautology ✓

3 (a) $\cos 2A + \cos 2B + \cos 2C = -1$

$\cos 2A = 2\cos^2 A - 1$ --- (i) ✓

$1 + \cos 2B = 2\cos^2 B$ ---

$\cos 2B = 2\cos^2 B - 1$ --- (ii) ✓

Add: $1 + \cos 2C = 2\cos^2 C$

$\cos 2C = 2\cos^2 C - 1$ --- (iii) ✓

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Subst- (i), (ii), (iii) into (i)

$$\cos 2A + \cos 2B + \cos 2C \quad \checkmark$$

$$2\cos^2 A - 1 + (2\cos^2 B - 1) + (2\cos^2 C - 1)$$

$$2\cos^2 A + 2\cos^2 B + 2\cos^2 C - 3$$

$$2(\cos^2 A + \cos^2 B + \cos^2 C) = 3 \quad \checkmark$$

Sum of the squares of direction cosines = 1

$$2(1) = 3$$

$$2 = 3$$

$\checkmark$

$$\cos 2A + \cos 2B + \cos 2C = -1$$

$$R \cdot F = F_1 + F_2 + F_3$$

$$\vec{R} = \begin{pmatrix} 3 \\ -4 \\ 5 \end{pmatrix} + \begin{pmatrix} 2 \\ 6 \\ -9 \end{pmatrix} + \begin{pmatrix} 2 \\ 3 \\ -6 \end{pmatrix} \quad \checkmark$$

$$\vec{R} = \begin{pmatrix} 7 \\ 5 \\ -10 \end{pmatrix} \quad \checkmark$$

6/12/20

$$\underline{d} = 7x \left(\frac{-2\underline{i} + 10\underline{j} + 11\underline{k}}{\sqrt{2^2 + 10^2 + 11^2}} \right) \quad \checkmark$$

$$= \frac{7}{15} (-2\underline{i} + 10\underline{j} + 11\underline{k}) \quad \checkmark$$

$$W-dome = \underline{F} \cdot \underline{d}$$

$$= \frac{7}{15} \begin{pmatrix} 7 \\ 5 \\ -10 \end{pmatrix} \cdot \begin{pmatrix} -2 \\ 10 \\ 11 \end{pmatrix} \quad \checkmark$$

$$= \frac{7}{15} (-14 + 50 - 110)$$

$$= \frac{7}{15} (-74)$$

$$= \left| \frac{-518}{15} \right| \quad \checkmark$$

$$W-d = \frac{518}{15} \text{ Nm}$$

$$= 34.533 \text{ joules}$$

$$67.28 \quad \checkmark$$

$$3 \text{ c) } \underline{v} = 6t^2 \underline{i} + 4e^{-2t} \underline{j} + 8 \cos 4t \underline{k} \quad \text{or } \frac{1}{2}$$

from

$$\underline{v} = \frac{d\underline{r}}{dt}$$

$$\underline{r} = \int \underline{v} dt$$

$$\underline{r}(t) = \int (6t^2 \underline{i} + 4e^{-2t} \underline{j} + 8 \cos 4t \underline{k}) dt$$

$$\underline{r}(t) = \left[2t^3 \underline{i} - 2e^{-2t} \underline{j} + 2 \sin 2t \underline{k} \right] + \underline{c} \quad \text{or}$$

$$\text{at } \underline{r}(0) = \underline{i} + \underline{j} + \underline{k}$$

$$\underline{i} + \underline{j} + \underline{k} = 2(0) \underline{i} - 2e^{-2(0)} \underline{j} + 2 \sin 2(0) \underline{k} + \underline{c} \quad \text{or}$$

$$\underline{i} + \underline{j} + \underline{k} = -2 \underline{j} + \underline{c}$$

$$\underline{c} = \underline{i} + 3 \underline{j} + \underline{k} \quad \text{or}$$

from

$$\underline{r}(t) = 2t^3 \underline{i} - 2e^{-2t} \underline{j} + 2 \sin 2t \underline{k} + \underline{c}$$

$$\underline{r}(t) = 2t^3 \underline{i} - 2e^{-2t} \underline{j} + 2 \sin 2t \underline{k} + \underline{i} + 3 \underline{j} + \underline{k} \quad \text{or}$$

$$\therefore \underline{r}(t) = (1 + 2t^3) \underline{i} + (3 - 2e^{-2t}) \underline{j} + (1 + 2 \sin 2t) \underline{k} \quad \text{or}$$

2/28

$$4 \text{ (a)} \quad \frac{x-2 + (y-3)i}{1+i} = 1-3i$$

$$\frac{(x-2) + (y-3)i(1-i)}{1+i} = 1-3i \quad \checkmark_{1/2}$$

$$\frac{x-2 - i(x-2) + i(y-3) - i^2(y-3)}{1+i} = 1-3i \quad \checkmark_{1/2}$$

$$(2x = x^2 + y^2)$$

$$\frac{(x-2+y-3) + i(y-3-x+2)}{1+i} \quad \checkmark_{1/2}$$

$$x+y-5=2$$

$$x+y=7 \quad \text{--- (1)} \quad \checkmark_{1/2}$$

$$-x+y-1 = -6$$

$$-x+y = -5 \quad \text{--- (2)} \quad \checkmark_{1/2}$$

solve (1) and (2)

$$x=6, y=1$$

$$4 \text{ (b)} \quad p(x) = x^4 - 4x^3 + 12x^2 + 4x - 13$$

$x=2-3i$ when $x=2+3i$ is also a root
the equation formed by $2-3i$ and $2+3i$

8/28

$$\begin{array}{r}
 x^2 - 1 \\
 \hline
 x^2 - 4x + 13 \mid x^4 - 4x^3 + 12x^2 + 4x - 13 \quad \text{or } 2 \\
 \underline{x^4 - 4x^3 + 13x^2} \\
 -x^3 + 4x - 13 \\
 -x^3 + 4x - 13 \\
 \hline
 \\
 \quad \text{or } 1
 \end{array}$$

$$x^2 - 1 = 0$$

$$x = \pm 1$$

The other roots are $-2 + 3i$, 1 , and -1
or $\frac{1}{2}$

9/24

4 c) $|z+1| = 2 |z-1+3i|$

let $z = x+iy$. $\frac{1}{2}$

$|x+iy+1| = 2 |x+iy-1+3i|$ $\frac{1}{2}$

$|x+1+iy| = 2 |x-1+(y+3)i|$. $\frac{1}{2}$

$\sqrt{(x+1)^2+y^2} = 2 \sqrt{(x-1)^2+(y+3)^2}$ $\frac{1}{2}$

Squaring both sides.

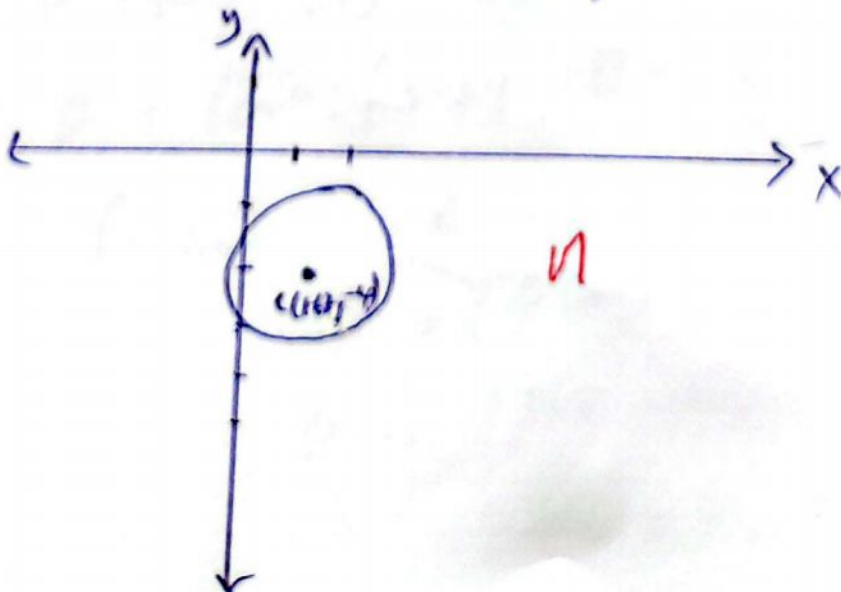
$(x+1)^2+y^2 = 4((x-1)^2+(y+3)^2)$. $\frac{1}{2}$

$x^2+2x+1+y^2 = 4(x^2-2x+1+y^2+6y+9)$. η

$x^2+2x+1+y^2 = 4x^2-8x+4+4y^2+24y+36$ η

$3x^2+3y^2-10x+24y+39 = 0$.

centre = $(\frac{5}{3}, -4)$ η



5 (a) $10 \sec^2 \theta - 3 = 17 \tan \theta$

$$10(1 + \tan^2 \theta) - 17 \tan \theta - 3 = 0$$

$$10 \tan^2 \theta - 17 \tan \theta + 7 = 0 \quad \text{VI}$$

$$\tan \theta = \frac{17 \pm \sqrt{(-17)^2 - 4 \times 10 \times 7}}{2 \times 10}$$

$$\tan \theta = \frac{17 \pm \sqrt{9}}{20}$$

$$\tan \theta = \frac{17 \pm 3}{20} \quad \text{VI}$$

$$\tan \theta = \frac{20}{20} = 1$$

$$\tan \theta = \frac{1}{10}$$

$$\theta = \tan^{-1}(1) \quad \theta = 45^\circ$$

$$\theta = \tan^{-1}(0.1) \quad \theta = 34.9^\circ \quad \text{VI}$$

General sol.

$$\theta = 150^\circ + 32^\circ$$

$$\theta = 150^\circ + 45^\circ \quad \text{VI}$$

$$\theta = 150^\circ + 34.9^\circ \quad \text{VI}$$

W/128

$$5 \quad b) \quad \frac{\tan x - \tan y}{\sec x - \sec y} = \tan x \tan y$$

L.H.S

$$\frac{\tan x - \tan y}{\sec x - \sec y} = \frac{\tan x - \tan y}{\frac{1}{\cos x} - \frac{1}{\cos y}} \quad 01$$

$$\begin{aligned} \frac{\tan x - \tan y}{\frac{\tan y - \sec y}{\tan y \tan y}} &= \frac{\tan x \tan y}{(\tan y - \sec y)} \times (\tan y - \sec y) \quad 01 \\ &= \tan x \tan y \end{aligned}$$

$$5 \quad c) \quad h = 4 \ln 2x + 3 \sin 2x$$

$$= R \ln(2x + \alpha) \quad 01$$

$$= R \ln 2x \ln \alpha + R \ln \alpha \cos 2x \quad 01$$

$$R \ln \alpha = 3$$

$$R \ln 2 = 4 \quad 01$$

$$\tan \alpha = \frac{4}{3}, \quad \alpha = \tan^{-1}\left(\frac{4}{3}\right) \quad 01$$

1/28

$$\alpha \approx 53.1^\circ$$

$$R = \sqrt{4^2 + 3^2} = \sqrt{25} = 5 \quad \text{or}$$

For maximum

$$R \sin(\alpha + \alpha) = 1 \quad \text{or}$$

$$h = 5 \text{ units}$$

$$\text{Maximum height} = 5 \text{ units} \quad \text{or}$$

5 $\Rightarrow \sin^{-1}\left(\frac{2x}{1+x^2}\right) = 2 \tan^{-1} x$

Let

$$\sin^{-1}\left(\frac{2x}{1+x^2}\right) = \tan^{-1} x \quad \text{or}$$

$$\sin^{-1}\left(\frac{2x}{1+x^2}\right) = \sin^{-1}\left(\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta}\right) \quad \text{or}$$

$$\sin^{-1}\left(\frac{2 \tan \theta}{1 + \tan^2 \theta}\right) = \sin^{-1}(\sin 2\theta) = 2\theta$$

$$\sin^{-1}(\sin 2\theta)$$

$$= 2 \tan^{-1} x$$

$$= 2\theta$$

$$= 2 \tan^{-1} x \quad \text{or}$$

17/20

5 d) (ii)

$$\sin^{-1} x = 2 \tan^{-1} y$$

let $A = \tan^{-1} y$.

~~$\tan A = y$~~ $\Rightarrow \tan A = y$. 01

from

$$\sin^{-1} x = 2 \tan^{-1} y$$

$$\sin^{-1} x = 2A$$

$$x = \sin 2A$$

01

but $\sin 2A = \frac{2 \tan A}{1 + \tan^2 A}$.

So

$$x = \frac{2 \tan A}{1 + \tan^2 A}$$

1/2

$$x = \frac{2y}{1 + y^2}$$

1/2

proved.

13/12

6 Q7 $\frac{x^3 - 2x^2 - 4x - 4}{x^2 + x - 2}$

$$\begin{array}{r} x-3 \\ x^2+x-2 \overline{) x^3 - x^2 - 4x - 4} \\ \underline{x^3 + x^2 - 2x} \\ -3x^2 + 2x - 4 \\ \underline{-3x^2 - 3x + 6} \\ x - 10 \end{array}$$

$$\frac{x^3 - 2x^2 - 4x - 4}{x^2 + x - 2} = x + 3 + \frac{x - 10}{x^2 + x - 2}$$

$$\frac{x - 10}{(x+2)(x-1)} = \frac{A}{x+2} + \frac{B}{x-1}$$

$$= \frac{A(x-1) + B(x+2)}{(x+2)(x-1)}$$

$$x - 10 = A(x-1) + B(x+2)$$

14/10

$$\frac{x-10}{(x+2)(x-1)} = \frac{4}{x+2} - \frac{3}{x-1} \quad \text{✓}$$

$$= \frac{x-3}{x+2} + \frac{4}{x+2} - \frac{3}{x-1} \quad \text{✓}$$

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6 d

$$\begin{aligned}x + y + z &= 37 \\ 2x - y + z &= -36 \\ x + 2y - z &= 11\end{aligned}$$

$$\begin{pmatrix} 1 & 1 & 1 \\ 2 & -1 & 1 \\ 1 & 2 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 37 \\ -36 \\ 11 \end{pmatrix} \quad \text{d}$$

$$z \left[\begin{array}{c|c|c} -1 & 1 & 1 \\ 2 & -1 & 1 \end{array} \right] - \left[\begin{array}{c|c|c} 1 & 2 & 1 \\ 1 & 1 & -1 \end{array} \right] + \left[\begin{array}{c|c|c} 2 & -1 & 1 \\ 1 & 2 & -1 \end{array} \right]$$

$$z(1-2) - (-2-1) + (4+1) \quad \text{d}$$

$$= 7$$

$$x = \frac{\begin{vmatrix} 37 & 1 & 1 \\ -36 & 1 & 1 \\ 11 & 2 & -1 \end{vmatrix}}{7} = \frac{49}{7} = 7 \quad \text{d}$$

$$y = \frac{\begin{vmatrix} 1 & 37 & 1 \\ 2 & -36 & 1 \\ 1 & 11 & -1 \end{vmatrix}}{7} = \frac{28}{7} = 4 \quad \text{d}$$

16/10

$$Z = \frac{1}{7} \begin{vmatrix} 1 & 1 & 37 \\ 2 & 7 & 36 \\ 1 & 2 & -11 \end{vmatrix} = \frac{182}{7} = 26 \quad \checkmark_{1/2}$$

The digit pass code is 7426 $\checkmark_{1/2}$

$$6 \text{ (c)} \quad \frac{1}{(4-x)^2} = \frac{1}{4(1-\frac{x}{4})^2} = \frac{1}{4^2(1-\frac{x}{4})^2}$$

$$\frac{1}{16(1-\frac{x}{4})^2} = \frac{1}{16(1-\frac{x}{4})^2} \quad (9)$$

$$= \frac{1}{16} (1-\frac{x}{4})^{-2} \quad \checkmark_{1/2}$$

$$\frac{1}{16} \left[1 + (-4)(-\frac{x}{4}) + \frac{(-2)(-3)}{2!} \left(\frac{x}{4}\right)^2 + \right.$$

$$\left. \frac{(-2)(-3)(-4)}{3!} \left(\frac{x}{4}\right)^3 + \dots \right] \quad \checkmark_{1/2}$$

$$= \frac{1}{16} \left(1 + \frac{x}{2} + \frac{3x^2}{16} + \frac{x^3}{16} + \dots \right) \quad \checkmark_{1/2}$$

$\checkmark_{1/2}$

It is valid for $|\frac{x}{4}| < 1$ ✓

that is $-4 < x < 4$ ✓

$$\text{Qd) } \sum_{k=1}^n 2k(k-4)(k+6) = \sum_{k=1}^n 2k^3 + 4k^2 - 48k \quad \checkmark$$

$$\sum_{k=1}^n k = \frac{n(n+1)}{2}, \quad \sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6} \quad \checkmark$$

$$\sum_{k=1}^n k^3 = \frac{n^2(n+1)^2}{4} \quad \checkmark$$

$$= \frac{n(n+1)}{6} (3n(n+1) + 4(2n+1) - 24) \quad \checkmark$$

$$= \frac{n(n+1)}{6} (3n(n+1) + 4(2n+1) - 144) \quad \checkmark$$

$$= \frac{n(n+1)}{6} (3n^2 + 11n - 140) \quad \text{Shur, } \checkmark$$

18/28

7 a) Given

$$(x-a)^2 + y^2 = a^2 \quad \text{--- (i)}$$

$$2(x-a) + 2y \frac{dy}{dx} = 0 \quad \frac{1}{2}$$

$$x-a + y \frac{dy}{dx} = 0 \quad \frac{1}{2}$$

$$a = x + y \frac{dy}{dx} \quad \text{--- (ii)} \quad \frac{1}{2}$$

Substitute eqn (ii) into (i)

$$\left(x - \left(x + y \frac{dy}{dx}\right)\right)^2 + y^2 = \left(x + y \frac{dy}{dx}\right)^2 \quad \frac{1}{2}$$

$$\left(-y \frac{dy}{dx}\right)^2 + y^2 = x^2 + 2xy \frac{dy}{dx} + y^2 \left(\frac{dy}{dx}\right)^2 \quad \frac{1}{2}$$

$$y^2 \left(\frac{dy}{dx}\right)^2 + y^2 = x^2 + 2xy \frac{dy}{dx} + y^2 \left(\frac{dy}{dx}\right)^2 \quad \frac{1}{2}$$

$$y^2 = x^2 + 2xy \frac{dy}{dx}$$

$$2xy \frac{dy}{dx} = y^2 - x^2 \quad \frac{1}{2}$$

$$\therefore \frac{dy}{dx} = \frac{y^2 - x^2}{2xy} \quad \frac{1}{2}$$

1/2

7 b) $\ln(y+1) + \left(\frac{x}{y+1}\right) \frac{dy}{dx} = \frac{1}{x(x+1)}$

This is exact.

$$\frac{d}{dx} (x \ln(y+1)) = \frac{1}{x(x+1)} \quad \checkmark$$

$$d(x \ln(y+1)) = \frac{1}{x(x+1)} dx \quad \checkmark$$

$$\int d(x \ln(y+1)) = \int \frac{1}{x(x+1)} dx$$

$$x \ln(y+1) = \int \left(\frac{A}{x} + \frac{B}{x+1} \right) dx \quad \checkmark$$

Take

$$\frac{1}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1} = \frac{A(x+1) + Bx}{x(x+1)} \quad \checkmark$$

$$A = 1, B = -1 \quad \checkmark$$

$$\frac{1}{x(x+1)} = \frac{1}{x} - \frac{1}{x+1} \quad \checkmark$$

$$x \ln(y+1) = \int \left(\frac{1}{x} - \frac{1}{x+1} \right) dx \quad \checkmark$$

$$x \ln(y+1) = \ln x - \ln(x+1) + C \quad \checkmark$$

$$\ln(y+1)^x = \ln\left(\frac{x}{x+1}\right) + \ln A \quad \checkmark$$

$$\ln(y+1)^x = \ln\left(\frac{Ax}{x+1}\right)$$

$$(y+1)^x = \frac{Ax}{x+1} \quad \checkmark$$

7 E) Given

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^{2x}$$

Consider complementary part

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0 \quad \checkmark$$

its A.P. will be

$$m^2 - 3m + 2 = 0$$

$$m = 2 \text{ \& } m = 1 \quad \checkmark$$

So

$$y_c = Ae^x + Be^{2x}$$

For particular integral part

Let

$$y = Kx e^{2x} \quad \checkmark$$

$$y' = 2Kx e^{2x} + K e^{2x}$$

$$y'' = 4Kx e^{2x} + 4K e^{2x} \quad \checkmark$$

2/28

7. c) From

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^{2x}. \quad \checkmark_2$$

$$(4kxe^{2x} + 4ke^{2x}) - 3(2kx + ke^{2x}) + 2kxe^{2x} = e^{2x}.$$

$$ke^{2x} = e^{2x}. \quad \checkmark_2$$

$$\text{So } k = 1$$

$$y_p = xe^{2x} \quad \checkmark_2$$

from

$$y = y_c + y_p.$$

$$y = Ae^x + Be^{2x} + xe^{2x}.$$

∴ The solution is $y = Ae^x + Be^{2x} + xe^{2x}$. $\checkmark$

d) Let $\Phi(t)$ be the amount of salt in tank at time t (min).

Volume of solution

$$V(t) = 10 + (3-2)t.$$

$$= 10 - t. \quad \checkmark_2$$

$$\frac{d\Phi}{dt} = \text{rate in} - \text{rate out}$$

$\checkmark_2$

7 d)

$$C_{conc} = \frac{Q}{10+t}$$

then

$$-\frac{dQ}{dt} = \frac{2Q}{10+t}$$

$\checkmark_{1/2}$

By Separating variable,

$$\frac{dQ}{Q} = -2 \int \frac{1}{10+t} dt$$

$$\int_{20}^Q \frac{-dQ}{Q} = \int_0^5 \frac{-2 dt}{10+t}$$

$\checkmark_{1/2}$

$$\ln Q \Big|_{20}^Q = -2 \left[\ln(10+t) \right]_0^5$$

$\checkmark_{1/2}$

$$\ln\left(\frac{Q}{20}\right) = -2(\ln 15 - \ln 10)$$

$\checkmark_{1/2}$

$$\ln \frac{Q}{20} = -2 \ln\left(\frac{15}{10}\right)$$

$\checkmark_{1/2}$

$$\ln\left(\frac{Q}{20}\right) = \ln\left(\frac{10}{15}\right)^2$$

$\checkmark_{1/2}$

$$\frac{Q}{20} = \left(\frac{10}{15}\right)^2 \Rightarrow Q = \frac{20 \times 100}{150}$$

$\checkmark_{1/2}$

$$Q = 8.889 \text{ kg}$$

$\checkmark_{1/2}$

$\therefore$ After 5 minutes, the tank will contain 8.89 kg of salt.

$\checkmark_{1/2}$

$\checkmark_{1/2}$

8 a) Given

$$y^2 = 8x$$

Slope of tangent, $m = 1$

From

$$y = mx + c$$

$$y = x + c$$

$\checkmark_{1/2}$

Substituting eqn of tangent into the equation of parabola.

$$(x+c)^2 = 8x$$

$\checkmark_{1/2}$

$$x^2 + 2xc + c^2 = 8x$$

$\checkmark_{1/2}$

$$x^2 + 2xc + c^2 - 8x = 0$$

$\checkmark_{1/2}$

$$x^2 + (2c-8)x + c^2 = 0$$

$\checkmark_{1/2}$

For tangent line,

$$b^2 = 4ac$$

$$(2c-8)^2 = 4(1)c^2$$

$\checkmark_{1/2}$

$$4c^2 - 32c + 64 = 4c^2$$

$\checkmark_{1/2}$

$$-32c = -64$$

$\checkmark_{1/2}$

$$c = +2$$

from

$$y = x + c$$

$\checkmark_{1/2}$

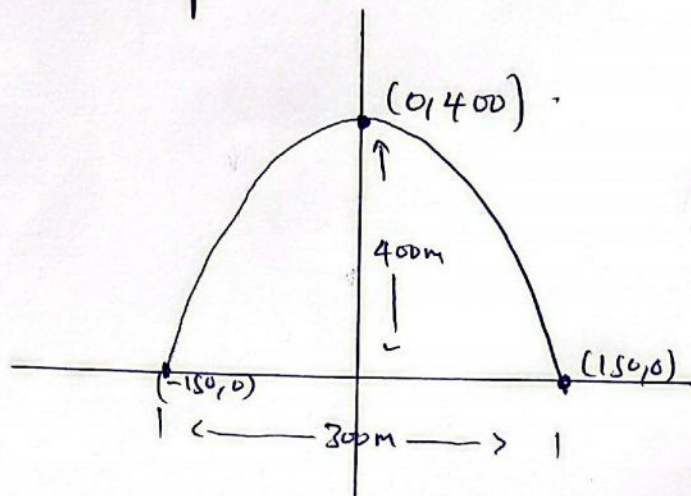
$$y = x + 2$$

$\checkmark_{1/2}$

$\therefore$ The eqn of tangent will be $y = x + 2$

§ b) Given, height = 400m
distance = 300m.

This is parabola that opens downward



from

$$(x-h)^2 = 4b(y-k)$$

$$h=0, k=400 \text{ vertex}$$

$$(h, k) = (0, 400) \Rightarrow h=0, k=400$$

$$x^2 = 4b(y-400)$$

at (150, 0)

$$150^2 = 4b(0-400)$$

$$150^2 = -1600b$$

$$b = -14.0625$$

the equation will be

$$x^2 = -4(-14.0625)(y-400)$$

$$x^2 = -56.25(y-400)$$

✓

8 c) Given

$$x^2 - 8x + 4y^2 + 12 = 0$$

$$x^2 - 8x + 16 + 4y^2 + 12 = 16 \quad \checkmark$$

$$(x-4)^2 + 4y^2 = 4$$

$$\frac{(x-4)^2}{4} + \frac{y^2}{1} = 1 \quad \checkmark$$

$$a^2 = 4$$

$$b^2 = 1 \quad \checkmark$$

$$a = 2 \text{ and } b = 1 \quad \checkmark$$

Since $a > b$,

$$b^2 = a^2(1 - e^2) \quad \checkmark$$

$$1 = 4(1 - e^2)$$

$$e^2 = 1 - \frac{1}{4} \quad \checkmark$$

$$e^2 = \frac{3}{4}$$

$$e = \frac{\sqrt{3}}{2} \quad \checkmark$$

1) Foci are $(\pm ae + h, 0)$

$$(h, k) = (4, 0) \quad \checkmark$$

$$\text{Foci} = \left(\pm \frac{\sqrt{3}}{2} \cdot 2 + 4, 0 \right) = (\pm \sqrt{3} + 4, 0) \quad \checkmark$$

$\frac{2b}{2a}$

$\therefore$ Foci are $(\sqrt{3}+4, 0)$ and $(-\sqrt{3}+4, 0)$. $\frac{1}{2}$

"> Vertices $= (\pm a, 0)$.

$$= (\pm 2, 0) \quad \frac{1}{2}$$

$\therefore$ Vertices are $(2, 0)$ and $(-2, 0)$. $\frac{1}{1}$

"> Equation of directrices, $X = \pm \frac{a}{e} + h$. $\frac{1}{1}$

$$X = \pm \frac{2/\sqrt{3}}{2} + 4. \quad \frac{1}{2}$$

$$X = \pm \frac{4}{\sqrt{3}} + 4 = \pm \frac{4\sqrt{3}}{3} + 4. \quad \frac{1}{2}$$

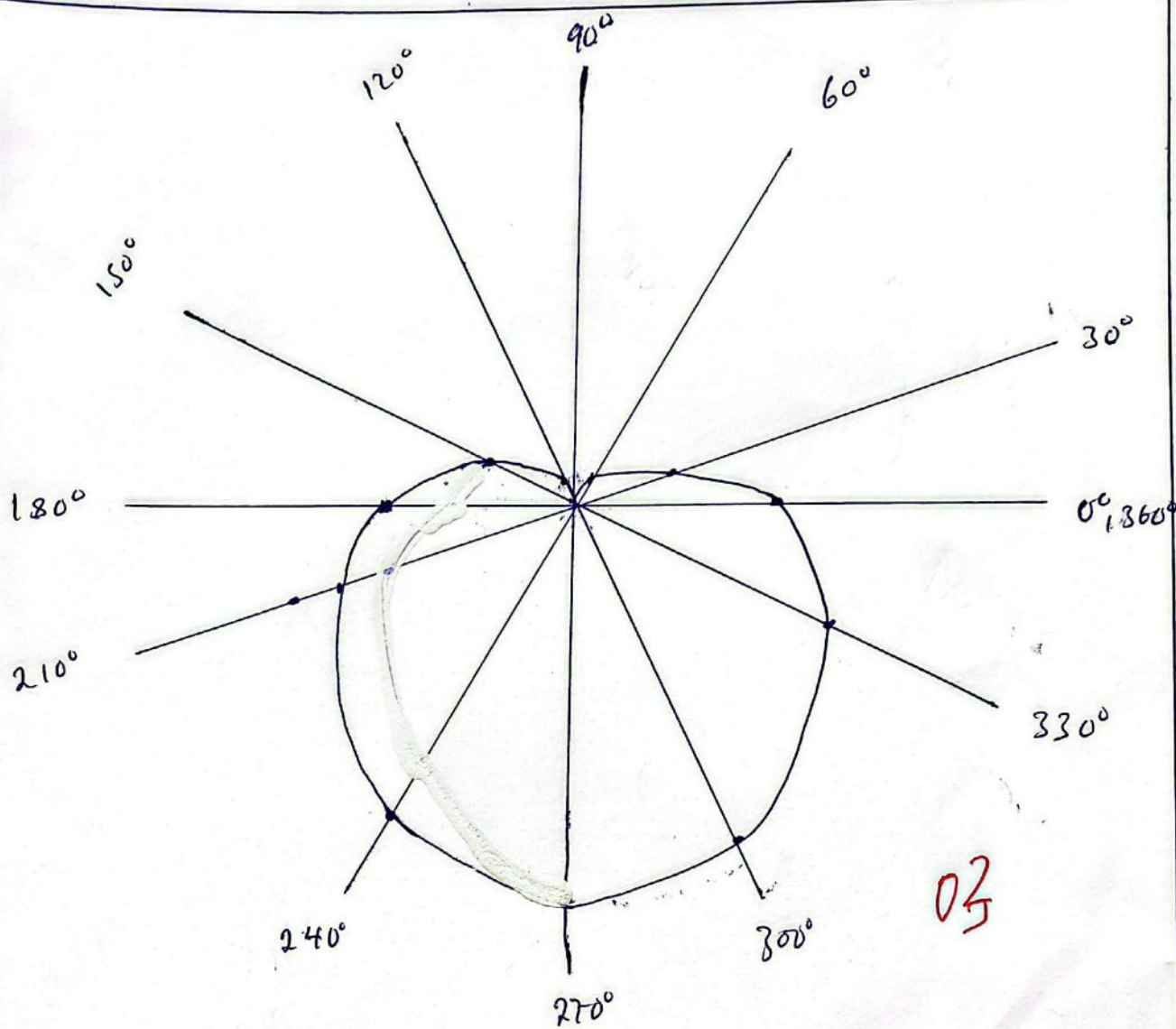
$\therefore$ Directrices are $X = \frac{4\sqrt{3}}{3} + 4$ and $X = -\frac{4\sqrt{3}}{3} + 4$. $\frac{1}{2}$

§ d) $\Rightarrow r = 3 - 3\sin\theta$

θ	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
r	3	1.5	0.4	0	0.4	1.5	3	4.5	5.6	6	5.6	4.5	3

$\frac{1}{2}$

$\frac{1}{2}$



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