TANZANIA HEADS OF ISLAMIC SCHOOL'

COUNCIL

FORM SIX INTER ISLAMIC

MOCK EXAMINATION

142/1

ADVANCED MATHEMATICS 1

MARKING GUIDE.

1 (a) i) 8.659 ---------- 03

ii) 4.877 ---------- 03

b) R = 10.0677 ---------- 04.

2 (a) Given that

$$\frac{1+\tanh x}{1-\tanh x} = e^{2x}.$$

Consider L.H.S.

$$\frac{1+\tanh x}{1-\tanh x} = \frac{1+\frac{\sinh x}{\cosh x}}{1-\frac{\sinh x}{\cosh x}} \quad \text{------ 01}$$

$$= \frac{\cosh x+\sinh x}{\cosh x-\sinh x}. \quad \text{------ 00}\tfrac{1}{2}$$

$$= \frac{e^{x}}{e^{-x}}. \quad \text{------ 00}\tfrac{1}{2}$$

$$= e^{2x}. \quad \text{------ 01}$$

$$\therefore \frac{1+\tanh x}{1-\tanh x} = e^{2x}.$$

01/31

2

b) Given

1) $x = t + \ln(\cosh t)$, $y = \sinh t$

$$\frac{dx}{dt} = 1 + \frac{\sinh t}{\cosh t} \quad \text{------} \quad 00\tfrac{1}{2}$$

$$\frac{dy}{dt} = \cosh t \quad \text{------} \quad 00\tfrac{1}{2}$$

from

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} \Rightarrow \frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt}$$

$$\frac{dy}{dx} = \cosh t \Big/ 1 + \frac{\sinh t}{\cosh t}$$

$$= \frac{\cosh^2 t}{\cosh t + \sinh t} \quad \text{------} \quad 00\tfrac{1}{2}$$

but $\cosh t + \sinh t = e^t$.

$$\frac{dy}{dx} = \frac{\cosh^2 t}{e^t} = e^{-t} \cosh^2 t$$

$$\therefore \frac{dy}{dx} = e^{-t} \cosh^2 t \quad \text{------} \quad 00\tfrac{1}{2}$$

Shown.

$01\frac{1}{3}$

2 b) Given

1) $x = t + \ln(\cosh t)$, $y = \sinh t$

$$\frac{dx}{dt} = 1 + \frac{\sinh t}{\cosh t} \quad \text{------} \quad 00\tfrac{1}{2}$$

$$\frac{dy}{dt} = \cosh t. \quad \text{----} \quad 00\tfrac{1}{2}$$

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$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}, \Rightarrow \frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt}$$

$$\frac{dy}{dx} = \cosh t \Big/ 1 + \frac{\sinh t}{\cosh t}.$$

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$$\frac{dy}{dx} = \frac{\cosh^2 t}{e^t} = e^{-t} \cosh^2 t$$

$$\therefore \frac{dy}{dx} = e^{-t} \cosh^2 t \quad \text{----} \quad 00\tfrac{1}{2}$$

Shown.

$\frac{02}{3}$

2 (iii) $x = \ln\left(\sqrt{3}\left(\frac{\sqrt{3}+1/\sqrt{3}}{2}\right)\right)$.

$= \ln\left(\frac{3+1}{2}\right)$

$= \ln 2$. ------ 00 1/2

$y = \sinh t = \sinh(\ln\sqrt{3})$.

$= \frac{e^{\ln\sqrt{3}} - e^{-\ln\sqrt{3}}}{2}$ ---- 00 1/2

$= \frac{\sqrt{3} - 1/\sqrt{3}}{2}$

$= \frac{\sqrt{3}}{3}$. ------ 00 1/2

from

$\frac{d^2y}{dx^2} = e^{-2\ln\sqrt{3}} \cosh^2(\ln\sqrt{3})\left(2\sinh\ln\sqrt{3} - \cosh\ln\sqrt{3}\right)$.

$= 0$.

Hence the point $\left(\ln 2, \frac{\sqrt{3}}{3}\right)$ is inflexion point ---- 01

$\therefore$ The curve C has point $\left(\ln 2, \frac{\sqrt{3}}{3}\right)$ which is inflexion point.

$\frac{13}{31}$

ii) $\dfrac{d^2y}{dx^2} = \dfrac{d}{dt}\left(\dfrac{dy}{dx}\right) \Big/ \dfrac{dx}{dt}$

$$\frac{d^2y}{dx^2} = \frac{d}{dt}\left(e^{-t}\cosh^2 t\right) \Big/ \frac{\cosh t + \sinh t}{\cosh t} \quad \text{------ 01}$$

$$= \frac{\left(-e^{-t}\cosh^2 t + 2e^{-t}\sinh t(\cosh t)\right)\cosh t}{\cosh t + \sinh t}$$

$$= \frac{e^{-t}\left(2\sinh t\cosh t - \cosh^2 t\right)\cosh t}{e^{t}} \quad \text{--- 00}\tfrac{1}{2}$$

$$= e^{-2t}\cosh t\left(2\sinh t - \cosh t\right).$$

$$\therefore \frac{d^2y}{dx^2} = e^{-2t}\cosh t\left(2\sinh t - \cosh t\right) \quad \text{---- 00}\tfrac{1}{2}$$

Shown.

iii) $t = \frac{1}{2}\ln 3$

$$t = \ln\sqrt{3}.$$

$$X = \ln\sqrt{3} + \ln\left(\cosh \ln\sqrt{3}\right) \quad \text{- - - 00}\tfrac{1}{2}$$

$$= \ln\left(\sqrt{3}\cosh\left(\ln\sqrt{3}\right)\right).$$

$$= \ln\left(\sqrt{3}\left(\frac{e^{\ln\sqrt{3}} + e^{-\ln\sqrt{3}}}{2}\right)\right)$$

$\frac{04}{3}$

ii) $\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right) \Big/ \frac{dx}{dt}$

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$$= \frac{\left(-e^{-t}\cosh^2 t + 2e^{-t}\sinh t(\cosh t)\right)\cosh t}{\cosh t + \sinh t}$$

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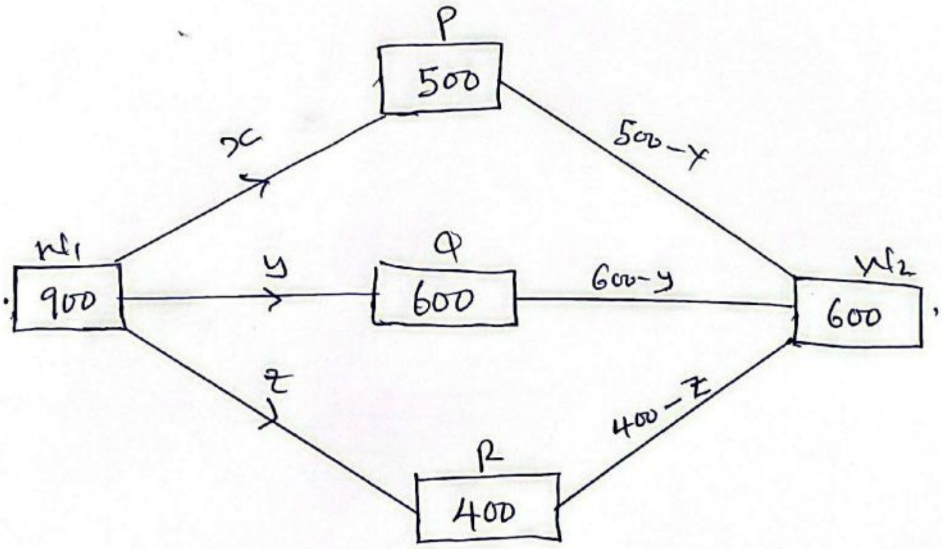
$$= \ln\left(\sqrt{3}\left(\frac{e^{\ln\sqrt{3}} + e^{-\ln\sqrt{3}}}{2}\right)\right)$$

04/3

3 a)



let

X be tones of bricks transpoted from W_1 to P. - - - 00½

y be tones of bricks transported from W_1 to Q. - - - - 00½

then

~~Minimize~~

~~$f(x,y) = 400x + 500y$ subject~~

Minimize

$$f(x,y) = 400x + 500y + 380000 \quad ---- \; 01$$

Subject to

$x \leq 500$ - - - - - 00½

$y \leq 600$ - - - - - 00½

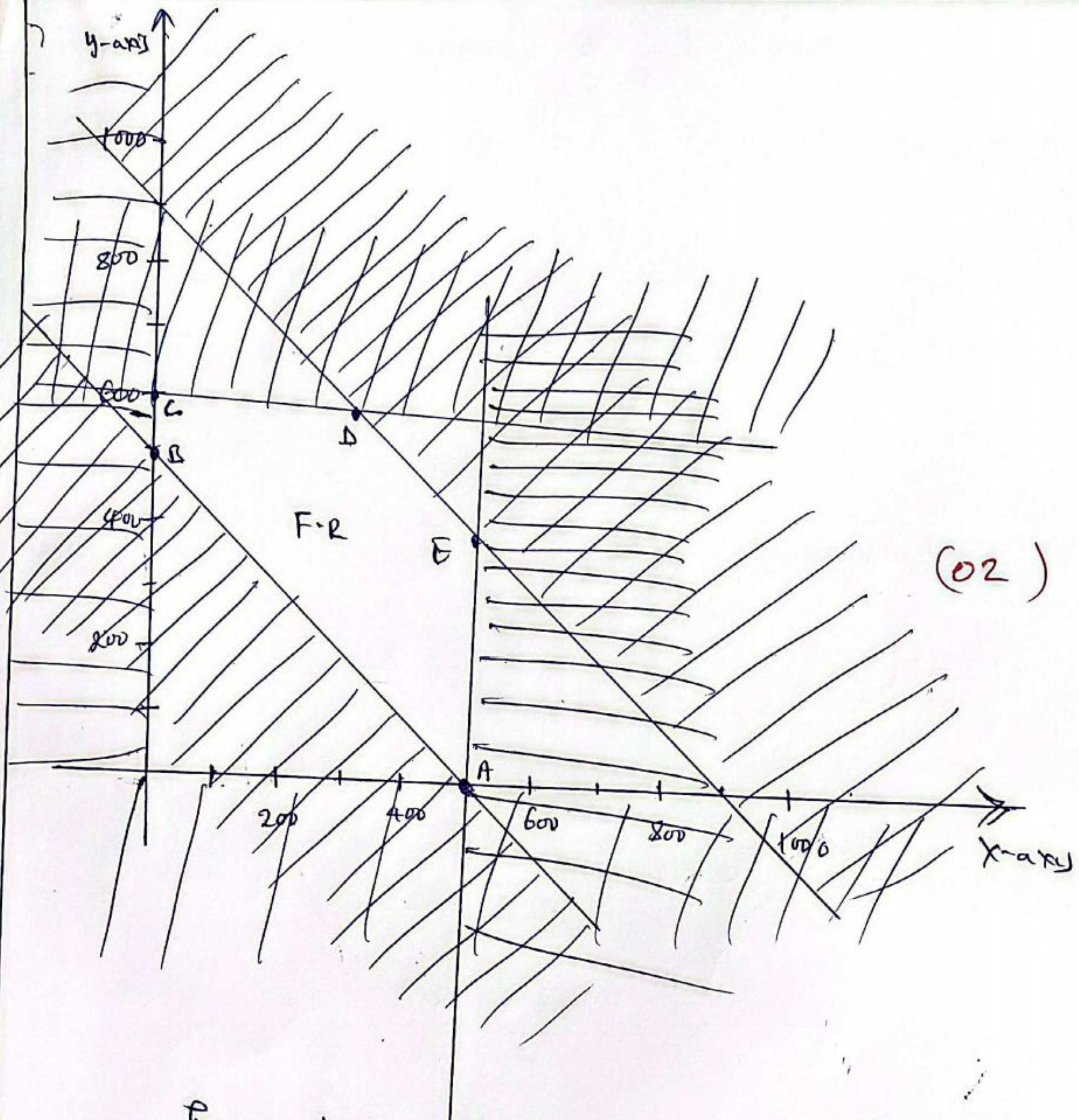
$x+y \leq 900$ - - - - 00½

$x+y \geq 500$ - - - 00½

$x \geq 0$ - - - - 00½

$y \geq 0$ - - - - 00½

05/3



(02)

From the graph

Corner points	Value of f(x,y)
A (500, 0)	580,000/=
B (0, 500)	630,000/=
C (0, 600)	680,000/=
D (300, 600)	800,000/=
E (500, 400)	780,000/=

- (07)

08/

The optimal solution is at (500,0),

Therefor inorder to minimize cost Hamisi should transport :

500 tones from W_1 to P . - - - 00½

400 tones from W_1 to R - - 00½

600 tones from W_2 to Q . - - 00½

b) The minimum cost of transportation is

580,000/= . - - - - 00½

4 a) Given

$$\mu = 50$$

$$\sigma^2 = 16$$

$$Y = \frac{X - 50}{4}.$$

$$\bar{Y} = \frac{\Sigma Y}{n}$$

$$\bar{Y} = \frac{\Sigma\left(\frac{X}{4} - \frac{50}{4}\right)}{n} = \frac{\frac{1}{4}\Sigma X - \frac{50}{4}n}{n}.$$ - - - 01

$$= \frac{1}{4}\frac{\Sigma X}{n} - \frac{50}{4}.$$

$$= \frac{1}{4}\bar{X} - \frac{50}{4}.$$

$$= \frac{1}{4}(50) - \frac{50}{4} = 0$$ - - - 01

∴ New mean, $\bar{Y} = 0$.

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(a)(ii) For standard deviation, σ

$$\sigma_x^2 = 16,$$

$$\sigma_y^2 = ?$$

from

$$Var(ax+b) = a^2 Var(X).$$

$$Var(Y) = Var\left(\frac{X}{4} - \frac{50}{4}\right)$$

$$= \left(\frac{1}{4}\right)^2 Var(X). \quad \text{------ 01}$$

$$= \left(\frac{1}{4}\right)^2 \cdot 16$$

$$= 1.$$

$$\therefore Var(Y) = 1. \quad \text{------ 01}$$

ii) Since all data are transformed by $Y = \frac{X_i - 50}{4}$.

the quartiles will also transform.

$$Q_3(Y) = \frac{Q_3(X) - 50}{4} \quad \text{--- 01}$$

$$= \frac{58 - 50}{4}$$

$$= 2.$$

$\therefore$ The new third quartile is 2. --- 01

8/31

b)

C.I	f	X	$X-\bar{X}$	$f(X-\bar{X})^2$	fX
20.5 – 20.9	3	20.7	−1.22	4.4652	
21.0 – 21.4	10	21.2	−0.72	5.1840	212
21.5 – 21.9	11	21.7	−0.22	0.5324	
22.0 – 22.4	13	22.2	0.28	1.0192	
22.5 – 22.9	9	22.7	0.78	5.4756	
23.0 – 23.4	2	23.2	1.28	3.2768	
	48			19.9532	1052.1

(02)

From

$$\text{Mean } \bar{X} = \frac{\Sigma fX}{\Sigma f}$$

$$= \frac{1052.1}{48} = 21.92$$

$$\bar{X} = 21.92 \quad \text{-----} 01$$

$$\sigma = \sqrt{\frac{\Sigma f(x-\bar{x})^2}{\Sigma f}}$$

$$= \sqrt{\frac{19.9532}{48}}$$

$$= 0.645. \quad \text{-----} 01$$

∴ Standard deviation = 0.645.

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5. a) i) $(P-Q)-(P\cup Q')$ ---- Given

$=(P\cap Q')\cap(P\cup Q')'$ ---- Definition $A-B=A\cap B'$

$=(P\cap Q')\cap(P'\cap Q)$ ---- De Morgan's law.

$=(P\cap P')\cap(Q'\cap Q)$ ---- Associative law

$=\phi\cap\phi$ ---- complement law

$=\phi$ ----- Idempotent.

(03)

$\therefore (P-Q)-(P\cup Q')=\phi$

(ii) $[A\cap(A\cup B)']'$ ---- Given.

$=A'\cup(A\cup B)$ ----- DeMorgans law.

$=(A'\cup A)\cup B$ ---- Associative law

$=\mathcal{U}\cup B$ ---- Complement law

$=\mathcal{U}$ ------- Identity law

(03)

$\therefore [A\cap(A\cup B)']'=\mathcal{U}$.

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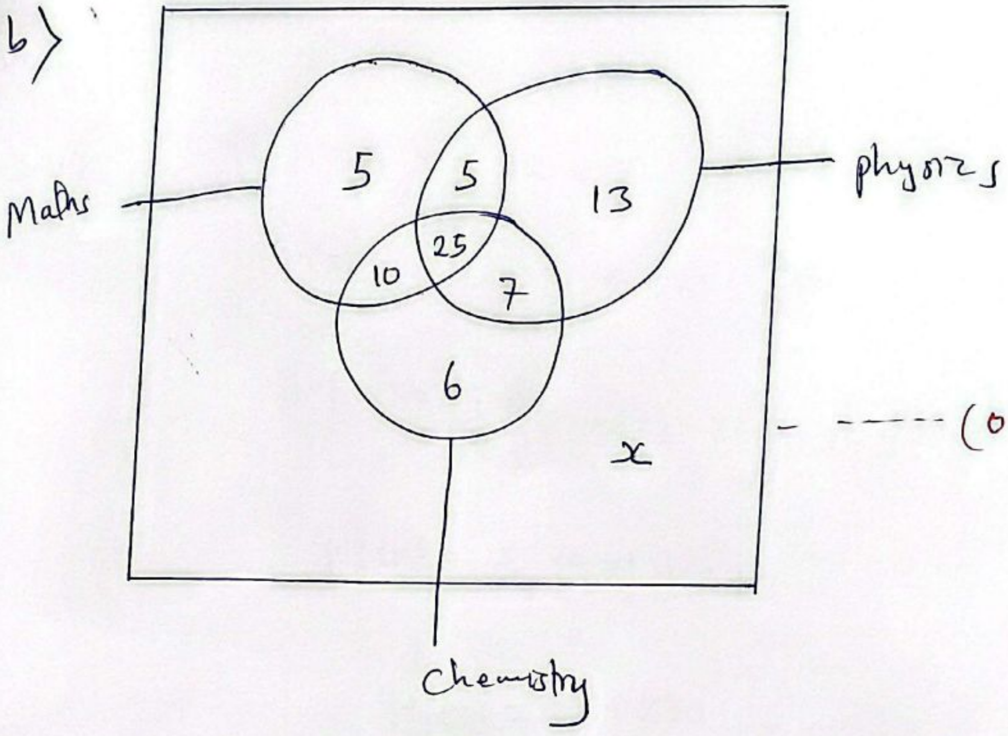
5 b)



------ (02)

i) Number of students who failed at least one subject

= 5+5+13+10+25+7+6 = 71 students. (01)

ii) Students who passed the exam

$x+71 = 150$

$x = 150-71$

$x = 79$ ------ (01)

$\therefore$ Student who passed exam = 79 students

11/31

6 a) Given

i) $f(x) = \frac{9}{5}x + 32.$

$g(x) = \frac{5}{9}(x-32) + 273.$ ------ 01

$gof(x) = \frac{5}{9}\left(\left(\frac{9}{5}x+32\right)-32\right)+273.$ ------ 01

$gof(x) = \frac{5}{9}\left(\frac{9}{5}x\right) + 273.$

$gof(x) = x+273$ ------- 01

ii) at $25°C \Rightarrow x = 25°C$

from $gof(x) = x+273$ ---- 01

$gof(25) = 25+273$ --- 01

$= 298K.$

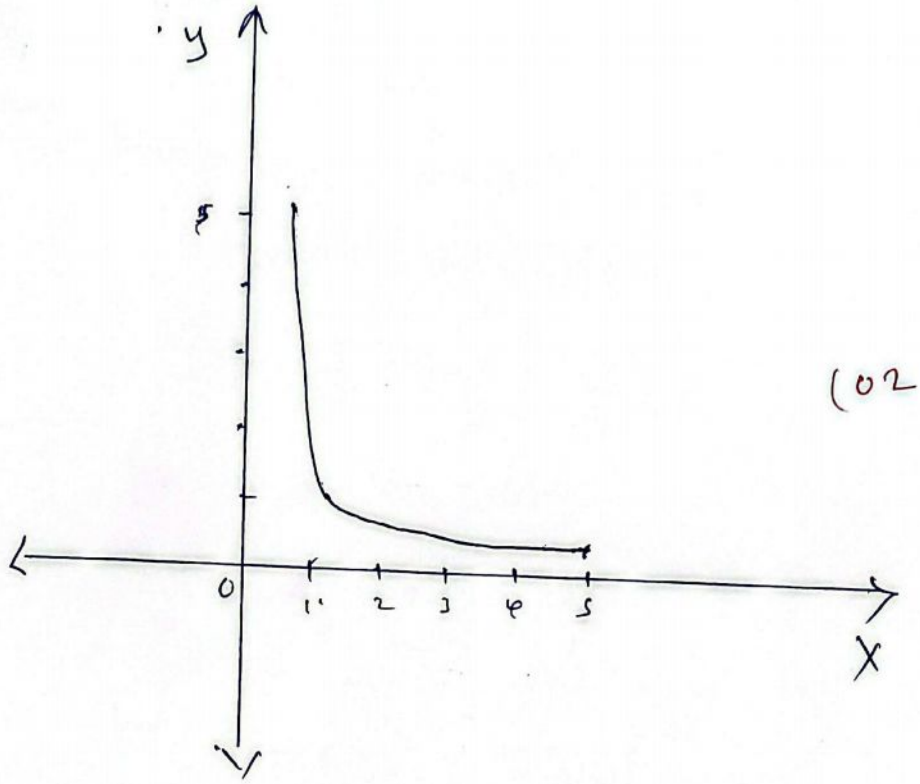
$\therefore$ The kelvin temperature is 298K. ---01

12/31

CamScanner

6 b)

i) $f(x) = 1/x$ for $0 < x \leq 5$.



(02)

(ii) Range $= \{ y \in R ; \frac{1}{5} \leq f(x) \leq \infty \}$ 01

(iii) The mapping is one to one function. ... 01

13/31

a) Given

$$u = \frac{5xy^2}{z^3}$$

Apply ln both side.

$$\ln u = \ln 5 + \ln x + 2\ln y - 3\ln z \quad \text{-----} \; 00\tfrac{1}{2}$$

Differentiate both side.

$$\frac{du}{u} = \frac{dx}{x} + 2\frac{dy}{y} - 3\frac{dz}{z} \quad \text{-----} \; 00\tfrac{1}{2}$$

For maximum error

$$\frac{\Delta u}{u} = \frac{\Delta x}{x} + 2\frac{\Delta y}{y} - \frac{3\Delta z}{z} \quad \text{-----} \; 00\tfrac{1}{2}$$

but

$$u = \frac{5xy^2}{z^3} = \frac{5(1)(1)^2}{(1)^3}$$

$$= 5 \quad \text{-----} \; 00\tfrac{1}{2}$$

$$\frac{\Delta u}{5} = \frac{0.001}{1} + \frac{2(0.001)}{1} + \frac{3(0.001)}{1} \quad \text{-----} \; 00\tfrac{1}{2}$$

$$\frac{\Delta u}{5} = 0.006$$

$$\Delta u = 0.03$$

$\text{-----} \; 00\tfrac{1}{2}$

∴ Maximum error in u is 0.03.

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b) i) Given $f(x) = x^4 - 4x + 1$

For the interval 0 to 1

$f(0) = 1$

$f(1) = 1 - 4 + 1$

$= -2$,

$f(x) = 0$ has a real root btn 0 and 1 if

$f(0) \cdot f(1) < 0$.

$f(0) \cdot f(1) = (1)(-2)$

$= -2$ - - - - - .01

$\therefore f(0) \cdot f(1) = -2$ which is less than 0.

therefore $f(x) = 0$ has the root between 0 and 1

ii) $f(x) = x^4 - 4x + 1$

$f'(x) = 4x^3 - 4$. - - - - - .01/2

from

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}.$$

$$= x_n - \frac{x_n^4 - 4x_n + 1}{4x_n^3 - 4}. \quad - - .01/2$$

$$= \frac{4x_n^4 - 4x_n - x_n^4 + 4x_n - 1}{4x_n^3 - 4}$$

$$x_{n+1} = \frac{3x_n^4 - 1}{4x_n^3 - 4}. \quad - - .01/2$$

15/3

for

$$x_1 = \frac{3x_0^4 - 1}{4x_0^3 - 4},$$

for $x_0 = 0.2$

$x_1 = 0.2508$. ---- 00½

$$x_2 = \frac{3x_1^4 - 1}{4x_1^3 - 4}$$

$x_2 = 0.2541$ ---- 00½

$$x_3 = \frac{3x_2^4 - 1}{4x_2^3 - 4}$$

$x_3 = 0.2541$ ---- 00½

∴ The root is 0.2541.

7 c) Let

$$f(x) = e^{x^2},$$

$$h = \frac{1-0}{4} = 0.25.$$ ---- 00½

x	0.00	0.25	0.50	0.75	1.00
$f(x)$	1.0000	1.0645	1.2840	1.7551	2.7183

---- 01

16½

c) from

$$\int_a^b f(x)\,dx = \frac{h}{2}\left[\text{sum of extreme ordinate} + 2\left(\text{sum of middle ordinate}\right)\right]$$

$$\int_0^1 e^{x^2}\,dx = \frac{0.25}{2}\left[3.7183 + 2(4.1036)\right] \text{ ----- 01}$$

$$= 1.4907$$

---- 00½

$$\therefore \int_0^1 e^{x^2}\,dx \approx 1.4907$$

8 a) Given $A(1,2)$ and $B(4,6)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{6-2}{4-1} = 4/3 \text{ . ----- 00½}$$

~~Its eqn from~~

It's eqn will be

$$y = \frac{4}{3}(x-1) + 2$$

$$3y = 4x - 4 + 6 \text{ --- 00½}$$

17/21

8 a)

$$3y = 4x + 2$$

$$4x - 3y + 2 = 0 \quad - - - - \quad 00\tfrac{1}{2}$$

Distance from $(2,3)$ to the line $4x-3y+2=0$

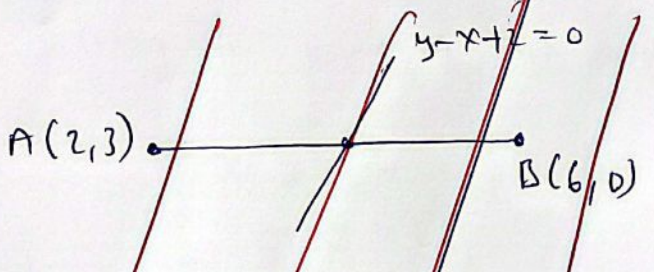
$$d = \left| \frac{4(2) - 3(3) + 2}{\sqrt{4^2+3^2}} \right| \quad - - - \quad 00\tfrac{1}{2}$$

$$d = \left| \frac{-1+2}{\sqrt{25}} \right| \quad - - - \quad 00\tfrac{1}{2}$$

$$d = \frac{1}{5} \text{ units.} \quad \text{———} \quad 00\tfrac{1}{2}$$

$\therefore$ The perpendicular distance is $\frac{1}{5}$ units.

b)



Slope of $\overrightarrow{AB}$, $m = \frac{0-3}{6-2}$

$$m = -\frac{3}{4}. \quad - - - - \quad \cancel{00\tfrac{1}{2}}$$

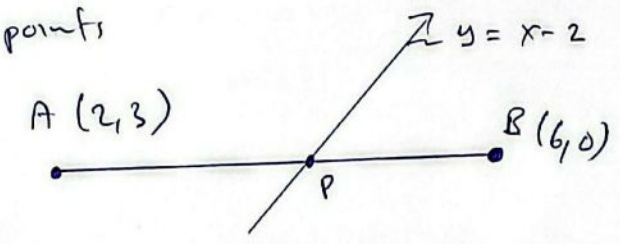
eqn of AB

$$y = -\frac{3}{4}(x-2) + 0$$

$$4y = -3x + 6.$$

15/31

8 b) Given points



Slope of AB, $m = \dfrac{0-3}{6-2}$

$m = -3/4$ - - - - 00½

Equation of line AB will be

$y = -\frac{3}{4}(x-2)+3$ - - - 00½

$4y = -3x + 6 + 12$

$4y = -3x + 18.$

$3x + 4y = 18.$ - - - - 00½

Solving for point of intersection P.

$$\begin{cases} x - y = 2 \\ 3x + 4y = 18 \end{cases}$$

So $(x, y) = \left(\frac{26}{7}, \frac{12}{7}\right)$ - - - 00½

Since P divide AB internally then

19/3

8 b) $P(x, y) = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$.

$\left(\frac{26}{7}, \frac{12}{7}\right) = \left(\frac{6m+2n}{m+n}, \frac{0m+3m}{m+n}\right)$ - - - $00\frac{1}{2}$

$\left(\frac{26}{7}, \frac{12}{7}\right) = \left(\frac{6m+2n}{m+n}, \frac{3m}{m+n}\right)$.

$\frac{26}{7} = \frac{6m+2n}{m+n}$ or $\frac{12}{7} = \frac{3m}{m+n}$. - - - $00\frac{1}{2}$

$12(m+n) = 21m$ - - - - - $00\frac{1}{2}$

$12m + 12n = 21m$

$12n = 21m - 12m$

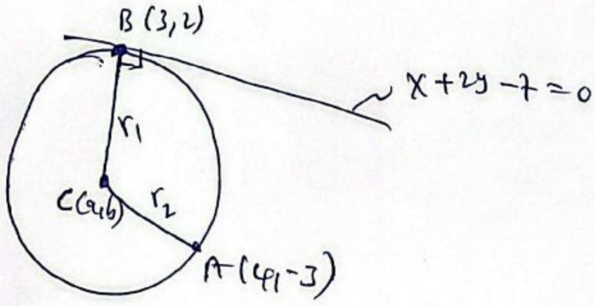
$12n = 9m$

$m/n = 9/12 = 3/4$.

$\therefore m : n = 3 : 4$. - - - - $00\frac{1}{2}$

$2\frac{1}{2}/3\frac{1}{2}$

8 c) consider



Slope of tangent line $x+2y-7=0$ is $-\frac{1}{2}$.

Since $\overline{BC}$ is $\perp$ to tangent line

Slope of $\overline{BC} = 2$. ------ 00½

equ of $\overline{BC}$

$$y = 2(x-3)+2$$

$$y = 2x-6+2$$

$$y = 2x-4$$

at (a,b) — centre,

$$2a-b = 4 \quad \text{--- (1)} \quad \text{------ 00½}$$

from

$$r_1 = r_2$$

$$(a-3)^2 + (b-2)^2 = (a-4)^2 + (b+3)^2$$

$$a^2-6a+9+b^2-4b+4 = a^2-8a+16+b^2+6b+9$$

$$2a-10b = 12 \quad \text{--- (11)} \quad \text{------ 00½}$$

2½/3

(b) Solve (i) and (ii)

$2a - b = 4$

$2a - 10b = 12$

$a = 14/9$, $b = -8/9$. ---------- 00½

from

$r^2 = (a-3)^2 + (b-2)^2$

$r^2 = \left(\frac{14}{9} - 3\right)^2 + \left(\frac{-8}{9} - 2\right)^2$

$r^2 = \frac{845}{81}$ ---------- 00½

from

equation of circle

$(x-a)^2 + (y-b)^2 = r^2$

$\left(x - \frac{14}{9}\right)^2 + \left(y + \frac{8}{9}\right)^2 = \frac{845}{81}$

$9x^2 + 9y^2 - 28x + 16y - 65 = 0$

---------- 00½

$\therefore$ The eqn of circle is

$\underline{9x^2 + 9y^2 - 28x + 16y - 65 = 0}$

22/3

9\. (a) $\int \frac{1}{x^3 - 3x^2 + 2x} = \int \frac{1}{x(x-2)(x-1)}$

$$\frac{1}{x(x-2)(x-1)} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x-1} \quad \text{--- } 00\tfrac{1}{2}$$

$$= \frac{A(x-2)(x-1) + Bx(x-1) + Cx(x-2)}{x(x-2)(x-1)}$$

$$1 = Ax^2 - 3Ax + 2A + Bx^2 - Bx + Cx^2 - 2Cx$$

$$= (A+B+C)x^2 + (-3A - B - 2C)x + 2A$$

$2A = 1, \quad A = \frac{1}{2}$ --- $00\frac{1}{2}$

$A + B + C = 0$

$-3A - B - 2C = 0$

$B + C - \frac{1}{2} = 0$

$B + C = \frac{1}{2}$ — (i)

$-\cancel{2A} - B - 2C = \frac{3}{2}$ — (ii)

(i) + (ii)

$B = \frac{1}{2}, \quad C = -1$ --- $00\frac{1}{2}$

$\frac{23}{31}$

$$\int \left(\frac{1}{2}\frac{A}{x} + \frac{1}{2}\left(\frac{B}{x-2}\right) + \frac{C}{x-1} \right) dx$$

$$= \int \left(\frac{2}{x} + \frac{2}{x-2} + \frac{1}{x-1} \right) dx \quad ---- 01$$

$$= \frac{1}{2}\ln x + \frac{1}{2}\ln(x-2) - \ln(x-1) + C \quad --- 0\tfrac{1}{2}$$

b) $\displaystyle\int_0^{\pi/4} \frac{1}{1+\cos^2 x}\, dx$

$$= \cos^2 x = \frac{1+\cos 2x}{2}$$

$$= \int_0^{\pi/4} \frac{2}{2+1+\cos 2x}\, dx = \int_0^{\pi/4} \frac{2}{3+\cos 2x}\, dx \quad --- 01$$

Let $t = \tan x$, $\cos 2x = \dfrac{1-t^2}{1+t^2}$

$$dt = \sec^2 x\, dx$$

$$3 + \cos 2x = 3 + \frac{1-t^3}{1+t^2} = \frac{4+2t^2}{1+t^2} \quad ---- 0\tfrac{1}{2}$$

24/34

b) $\frac{1}{2}\int_0^{\pi/4} \frac{1}{2+t^2} dt$

$t = \sqrt{2}\tan\theta$

$dt = \sqrt{2}\sec^2\theta \, d\theta$ ---- 00½

$\frac{1}{2}\int_0^{\pi/4} \frac{\sqrt{2}\sec^2\theta}{2(1+\tan^2\theta)} d\theta = \frac{\sqrt{2}}{4}\int_0^{\pi/4} d\theta = \frac{\sqrt{2}}{4}\theta\Big]_0^{\frac{\pi}{4}}$

$\theta = \tan^{-1}\frac{t}{\sqrt{2}}$ $\quad \theta = \tan^{-1}\left(\frac{\tan x}{\sqrt{2}}\right)$ ---01

$= \left[\frac{\sqrt{2}}{4}\tan^{-1}\left(\frac{\tan x}{\sqrt{2}}\right)\right]_0^{\frac{\pi}{4}}$

$= \frac{\sqrt{2}}{4}\left(\tan^{-1}\left(\frac{\tan \pi/4}{\sqrt{2}}\right)\right) - 0$

$= 0.4352099875$ ----01

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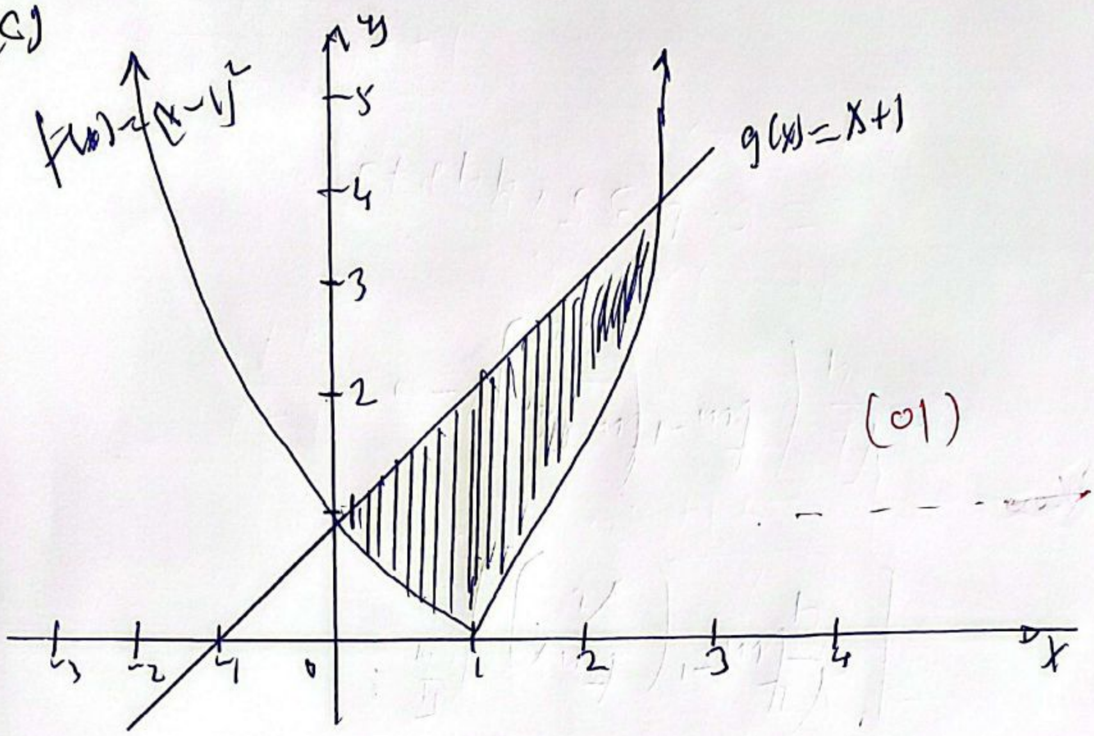
(c)



(01)

$$A = \int_0^3 (x+1) - (x-1)^2 \, dx$$

$$\int_0^3 (x+1 - x^2 + 2x - 1) \, dx \quad \text{---- } 01$$

$$\int_0^3 (3x - x^2) \, dx$$

$$\left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_0^3$$

$$\frac{3(3)^2}{2} - \frac{3^3}{3} - 0 = \frac{9}{2} \text{ sq. unit} \quad \text{---- } 01$$

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(a) $x = t^2 + 2t, \quad y = t^3 - t$

$$\frac{dx}{dt} = 2t + 2, \quad \frac{dy}{dt} = 3t^2 - 1 \quad \text{------ 01}$$

$$\frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt} = \frac{3t^2 - 1}{2t + 2}$$

$$\frac{dy}{dx} = \frac{3t^2 - 1}{2t + 2} \quad \text{------ 01/2}$$

(b) $y = \frac{1}{2}\sin 2x$

(ii) $\text{slope} = \left(\frac{dy}{dx}\right)_{t=1} = \frac{3(1)^2 - 1}{2(1) + 2} = \frac{2}{4}$

$$m = \frac{1}{2} \quad \text{------ 01/2}$$

The point $x = t^2 + 2t, \; y = t^3 - 1$

$x = 3, \quad y = 0$

$(3, 0)$ ------ 01

$\frac{2}{3}$

$m = \frac{1}{2}, \quad (3, 0)$

$y = \frac{1}{2}(x-3) + 0$

$y = \frac{1}{2}x - \frac{3}{2}$ ------ 01

(b) $y = \frac{1}{2}\sin 2x$

$f(x+h) = \frac{1}{2}\sin 2(x+h)$ --- 00½

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{\frac{1}{2}\sin 2(x+h) - \frac{1}{2}\sin 2x}{h}$ --- 00½

$= \frac{1}{2}\lim_{h \to 0} \frac{\sin[2x+2h] - \frac{1}{2}\sin 2x}{h}$

$\frac{1}{2}\lim_{h \to 0} \frac{\sin 2x\cos h + \sin h\cos 2x - \sin 2x}{h}$

$\frac{1}{2}\lim_{h \to 0} \frac{\sin 2x\cos h - \sin 2x + \sin h\cos 2x}{h}$ --- 01

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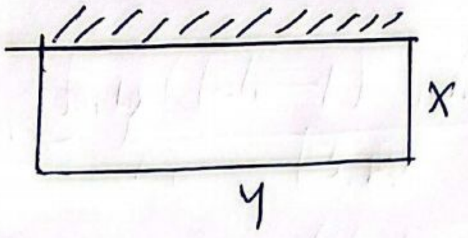
$$= \frac{1}{2} \lim_{h \to 0} \frac{\sin 2x(\cos h - 1) + \sin h \cos 2x}{h}$$

$$= \frac{1}{2} \lim_{h \to 0} \sin 2x \left(\frac{\cos h - 1}{h} \right) + \cos 2x \cdot \frac{\sin h}{h}$$

$$\frac{1}{2} \cos 2x \cdot 2 = \cos 2x$$

$$= \cos 2x \quad \text{- - - - ②}$$

0 (c)



total length = $2x+y$

$2x+y = 120$

$y = 120-2x$ ------ 00½

$A = xy$

$A = x(120-2x)$ ------ 00½

$A = 120x - 2x^2$

$\frac{dA}{dx} = 120 - 4x$ ------ 00½

$120 - 4x = 0$

$4x = 120$

$x = 30$ ------ 00½

$y = 120 - 2 \times 30$

$y = 60$ ------ 00½

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Maximum area = 30x60

Area of the field = 1800m² ---- 00 1/2

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CamScanner