TANZANIA HEADS OF ISLAMIC SCHOOL COUNCIL
FORM SIX INTER ISLAMIC MOCK EXAMINATION

141 BASIC APPLIED MATHEMATICS

MARKING GUIDE

1 (a) 1.4333 ——— 02

(b) $\begin{pmatrix} 21 & 18 & 18 \\ 29 & 37 & 7 \\ 28 & 26 & 29 \end{pmatrix}$ ——— 02

(c) $\bar{X} = 66.5784$ ——— 02

$S.D = 1.8849$ ——— 02

$\sum X^2 = 2799269$ ——— 02

2 (a) Here we have $f(x) = 3 + 4X - X^2$

$\Rightarrow a = -1, \; b = 4, \; c = 3.$

$h = \frac{-b}{2a}$ ——— $00\frac{1}{2}$

$= \frac{-4}{2 \times -1} = 2.$

$h = 2.$ ——— $00\frac{1}{2}$

$K = \frac{4ac - b^2}{4a}$ ——— $00\frac{1}{2}$

$= \frac{4(-1)(3) - (4)^2}{4(-1)}$

$= \frac{-28}{-4}$

$K = 7.$ ——— $00\frac{1}{2}$

∴ (i) The turning points (h, k) is $(2, 7)$ ——— 01 (a)(ii) The axis of symmetry is a line $X=2$. — 001

(iii) Since the coefficient of x^2 is negative then there is a maximum turning point which is $(2,7)$ — 00 $\frac{1}{2}$

$\therefore$ The maximum value of the function is $y=7$. — 00 $\frac{1}{2}$

(b) we have $f(x)=\frac{x-1}{x-4}$. Then,

Vertical asymptote — 00 $\frac{1}{2}$

$x-4=0$

$x=4$

$\therefore$ Vertical asymptote is a line $x=4$. — 00 $\frac{1}{2}$

Horizontal asymptote

Since the degree of numerator is equal to that of the denominator then

Horizontal asymptote is $\dfrac{\frac{x}{x}-\frac{1}{x}}{\frac{x}{x}-\frac{4}{x}}$ — 00 $\frac{1}{2}$.

as $x\to\infty$

$=\frac{1}{1}$

$=1$

$\therefore$ The horizontal asymptote is a line $y=1$. — 00 $\frac{1}{2}$

b) X intercept $(y=0)$.

$$0 = \frac{x-1}{x-4}$$

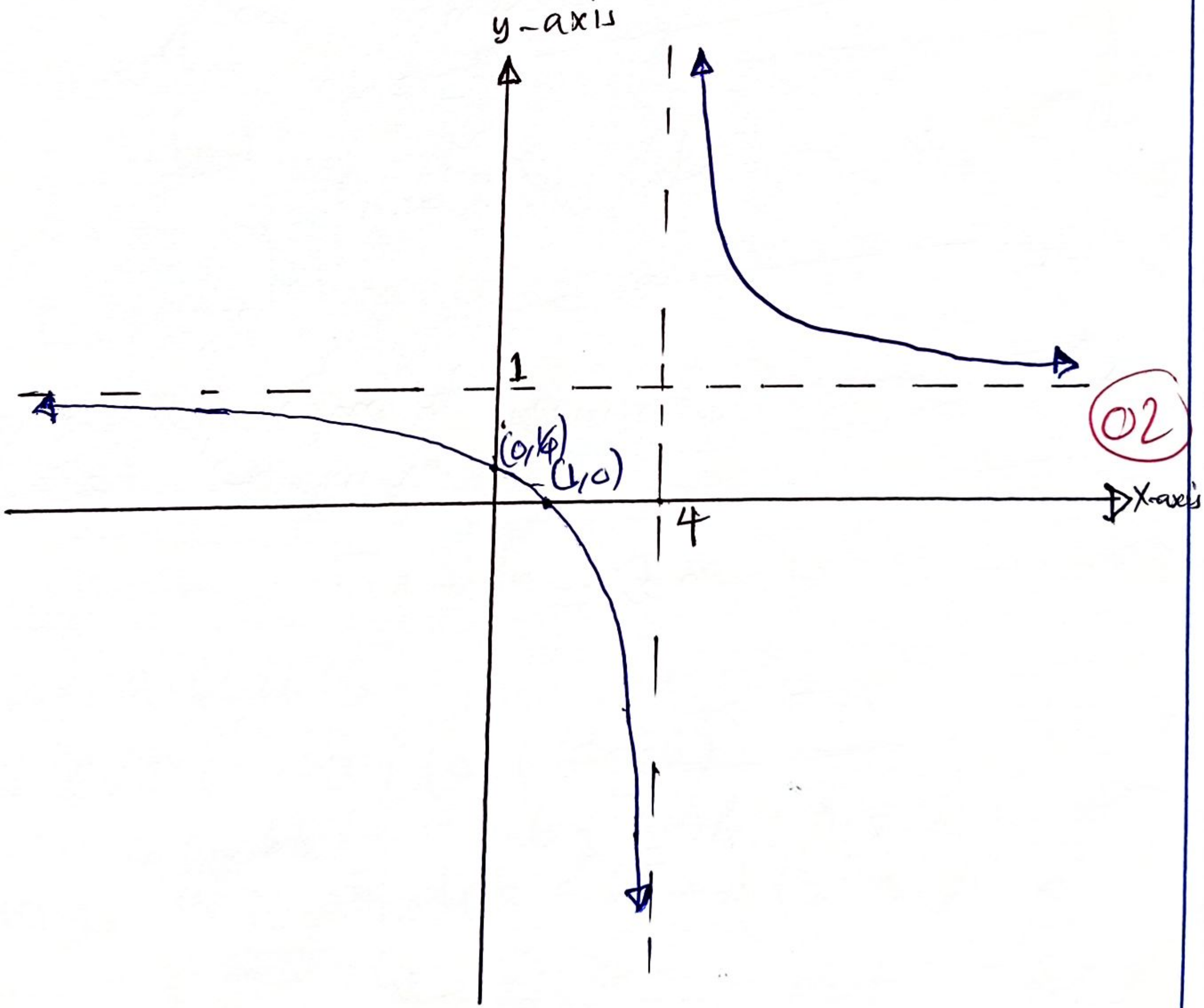
$x = 1$

$\therefore$ x-intercept is at point $(1, 0)$. — $a\frac{1}{2}$

y-intercept $(x=0)$.

$\therefore$ y-intercept is at point $(0, \frac{1}{4})$ — $a\frac{1}{2}$

The graph of $f(x) = \dfrac{x-1}{x-4}$.



wehave.

$4^x - 6(2^x) - 16 = 0$.

Then.

$(2^2)^x - 6(2^x) - 16 = 0$

$(2^x)^2 - 6(2^x) - 16 = 0$ ——— $01\frac{1}{2}$

Let. $2^x = a$ ——— $01\frac{1}{2}$

$a^2 - 6a - 16 = 0$.

Solving quadratic equation

$(a-8)(a+2) = 0$ ——— $01\frac{1}{2}$

$a = 8$ or $a = -2$ ——— $01\frac{1}{2}$

but

$a = 2^x$

taking $a = 8$

$2^x = 2^3$ ——— $01\frac{1}{2}$

$x = 3$.

$\therefore$ The value of x is 3. ——— $01\frac{1}{2}$

(b)(i) wehave. 2.666666 ---

$2.666666 = 2 + (0.666666)$ ——— $01\frac{1}{2}$

Consider $0.666666 = 0.6 + 0.06 + 0.006 + 0.0006 + \ldots$ ——— $01\frac{1}{2}$

This is G.P with $a_1 = 0.6$, and $r = 0.1$

From $S_\infty = \frac{a_1}{1-r} = \frac{0.6}{1-0.1} = \frac{0.6}{0.9} = 2/3$. ——— $01\frac{1}{2}$

Then the required fraction $= 2 + \frac{2}{3} = 8/3$. ——— $01\frac{1}{2}$ $01\frac{1}{2}$

(ii) wehave $a_1 = 1$, $r = 2$, $S_n = 255$.

(ii) from: $S_n = \frac{a(r^n - 1)}{r-1}$ ——— $01/2$

$255 = \frac{1(2^n - 1)}{2-1}$ ——— $01/2$

$255 = 2^n - 1$

$2^n = 256.$

$2^n = 2^8$ ——— $01/2$

$n = 8$

$\therefore$ There are 8 terms. ——— $01/2$

(c) We have:

$3x - y = 9.$ — (i)

$x^2 - xy + 2 = 0$ — (ii)

Using eqn (i) make y as a subject

$y = 3x - 9.$

$x^2 - x(3x-9) + 2 = 0$ ——— $01/2$

$x^2 - 3x^2 + 9x + 2 = 0$

$-2x^2 + 9x + 2 = 0$ ——— $01/2$

Solving quadratically

$x_1 = \frac{9+\sqrt{97}}{4}$, or $x_2 = \frac{9-\sqrt{97}}{4}$ — 01

$y_1 = \frac{27+3\sqrt{97} - 36}{4}$

$= \frac{-9+3\sqrt{97}}{4}$

$y_2 = \frac{27-3\sqrt{97} - 36}{4}$ — 01

$= \frac{-9-3\sqrt{97}}{4}$

$\therefore$

1. From: $f'(x) = \lim_{h \to 0} \left[\frac{f(x+h) - f(x)}{h} \right]$ — 01/2

where $f(x+h) = \sqrt{2x+2h+1}$

$f(x) = \sqrt{2x+1}$

Then $f'(x) = \lim_{h \to 0} \left(\frac{\sqrt{2x+2h+1} - \sqrt{2x+1}}{h} \right]$ — 01/2

By rationalizing the Numerator.

$= \lim_{h \to 0} \frac{\left(\sqrt{2x+2h+1} - \sqrt{2x+1}\right)\left(\sqrt{2x+2h+1} + \sqrt{2x+1}\right)}{h\left(\sqrt{2x+2h+1} + \sqrt{2x+1}\right)}$ — 01/2

$= \lim_{h \to 0} \left(\frac{(2x+2h+1) - (2x+1)}{h\left(\sqrt{2x+2h+1} + \sqrt{2x+1}\right)} \right)$

$= \lim_{h \to 0} \left(\frac{2h}{h\sqrt{2x+2h+1} + \sqrt{2x+1}} \right)$ — 01/2

$= \lim_{h \to 0} \left(\frac{2}{\sqrt{2x+2h+1} + \sqrt{2x+1}} \right)$

$= \frac{2}{\sqrt{2x+1} + \sqrt{2x+1}}$

$= \frac{1}{\sqrt{2x+1}}$ 01/2

4(i) we have

$8x^3 + xy^3 - 5y^2 = 0, \ (x, y) = (1, -1)$

$\frac{d}{dx}(8x^3 + xy^3 - 5y^2) = \frac{d}{dx}(0)$ ——— $\frac{1}{2}$

$24x^2 + 3xy^2\frac{dy}{dx} + y^3 - 10y\frac{dy}{dx} = 0$ ——— $\frac{1}{2}$

$(10y - 3xy^2)\frac{dy}{dx} = y^3 + 24x^2$

$\frac{dy}{dx} = \frac{y^3 + 24x^2}{10y - 3xy^2}$ ——— $\frac{1}{2}$

at $(1, -1)$

$\frac{dy}{dx} = \frac{(-1)^3 + 24(1)^2}{10(-1) - 3(1)(-1)^2}$ ——— $\frac{1}{2}$

$= \frac{-1 + 24}{-10 - 3} = \frac{-23}{13}.$

$\therefore$ slope of tangent is $\frac{-23}{13}$ ——— $\frac{1}{2}$

4(b)(ii) we have $f(x) = 2x^3 + 3x^2 - 12x - 5$

$\frac{dy}{dx} = 6x^2 + 6x - 12$ ——— $\frac{1}{2}$

at critical point $\frac{dy}{dx} = 0.$

$6x^2 + 6x - 12 = 0.$

$x = 1$ or $x = -2$ ——— $\frac{1}{2}$

$\frac{d^2y}{dx^2} = \frac{d}{dx}(6x^2 + 6x - 12)$

(ii) $\frac{d^2y}{dx^2} = 12x + 6$ when $x = 1$

$= 12(1) + 6$

$= 18$ —————— a$\frac{1}{2}$

Since the second derivative is positive then there is a minimum turning point at $x = 1$.

$y_{min} = 2(1)^3 + 3(1)^2 - 12(1) - 5$

$= 2 + 3 - 17$

$= -12$

minimum turning point is at $(1, -12)$. —— a$\frac{1}{2}$

when $x = -2$

$\frac{d^2y}{dx^2} = 12(-2) + 6$

$= -24 + 6$

$= -18$ —————— a$\frac{1}{2}$

Since the second derivative is negative then there is a maximum turning point at $x = -2$

$y_{max} = 2(-2)^3 + 3(-2)^2 - 12(-2) - 5$

$= 15$

$\therefore$ The maximum turning point is at $(-2, 15)$. a$\frac{1}{2}$

(c) We have. $E(t) = \frac{1}{27}(9t + 3t^2 - t^3)$

$$\frac{dE(t)}{dt} = \frac{1}{27}(9 + 6t - 3t^2)$$ — $a_1 \frac{1}{2}$

at $t = 3$

$$\frac{dE(t)}{dt} = \frac{1}{27}(9 + 6(3) - 3(3)^2)$$ — $a_1 \frac{1}{2}$

$$= \frac{1}{27}(27 - 27)$$

$$= 0$$ — a_1

5(a). $$\int_1^2 \frac{x^2+1}{x^3+3x}\,dx$$

Let $u = x^3 + 3x$

$$\frac{du}{dx} = 3x^2 + 3$$ — $a_1 \frac{1}{2}$

$$dx(x^2+1) = \frac{du}{3}.$$

$$\int_1^2 \frac{x^2+1}{x^3+3x}\,dx = \int \frac{du}{3u} = \frac{1}{3}\ln u$$ — $a_1 \frac{1}{2}$

$$= \frac{1}{3}\ln(x^3+3x)\Big|_1^2$$

$$= \frac{1}{3}\ln(14) - \frac{1}{3}\ln 4$$ — $a_1 \frac{1}{2}$

$$= \frac{1}{3}\ln\left(\frac{14}{4}\right)$$ — $a_1 \frac{1}{2}$

$$= \frac{1}{3}\ln\left(\frac{7}{2}\right) \text{ or } 0.4175876$$ — a_1

(b) $\int \frac{1}{2x-1}\,dx$

let

$u = 2x-1$ — $a_1 \frac{1}{2}$

$\frac{du}{dx} = 2$

$dx = \frac{du}{2}$ — $a_1 \frac{1}{2}$

$\int \frac{1}{2x-1}\,dx = \int \frac{du}{2u} = \frac{1}{2}\ln|u| + C$ — a_1

$= \frac{1}{2}\ln|2x-1| + C$ — $a_1 \frac{1}{2}$

$\therefore \int \frac{1}{2x-1}\,dx = \frac{1}{2}\ln|2x-1| + C$ — $a_1 \frac{1}{2}$

(c) We have.

$MC(x) = \frac{3}{7}e^{-2x} + 6x^2 + 11$

Then the total cost $C(x) = \int MC(x)dx$

$C(x) = \int\left[\frac{3}{7}e^{-2x} + 6x^2 + 11\right]dx$ — $a_1 \frac{1}{2}$

$= \frac{-3}{14}e^{-2x} + 12x + 11x + T$ — $a_1 \frac{1}{2}$

when $x = 3$, $C(x) = 10{,}000$,

Then

$10{,}000 = \frac{-3}{14}e^{0} + 0 + 0 + F$

$T = 10000 + \frac{3}{14}$

$= 10{,}000.2142$ ——— 01

Then

$C(3) = \frac{-3}{14}e^{-6} + 54 + 33 + 10{,}000.2142$

$= 10087.21375.$ ——— $01\frac{1}{2}$

$\therefore$ The total cost of production is

Tsh 10,087.2137 ——— $01\frac{1}{2}$

6 Consider the frequency distribution table below

A = 28.

Interval	Frequency	X	C.F	d = X−A	u = d/c	Fu	Fu²
1 − 5	8	3	8	−25	−5	−40	200
6 − 10	13	8	21	−20	−4	−52	208
11 − 15	14	13	35	−15	−3	−42	126
16 − 20	9	18	44	−10	−2	−18	36
21 − 25	16	23	60	−5	−1	−16	16
26 − 30	18	28	78	0	0	0	0
31 − 35	12	33	90	5	1	12	12
36 − 40	10	38	100	10	2	20	40

03

$\Sigma Fu = -136.$ $\Sigma Fu^2 = 638$

(a) Required Variance $= c^2\left[\frac{\sum_{i=1}^{n} fu^2}{N} - \left(\frac{\sum_{i=1}^{n} fu}{N}\right)^2\right]$ — a1 $\frac{1}{2}$

$= 5^2\left[\frac{638}{100} - \left(\frac{-136}{100}\right)^2\right]$ — a1 $\frac{1}{2}$

$= 25(6.8 - 1.8496)$

$\therefore Var(x) = 123.76$ — a1 1

(b) Range of the data:

Range $=$ (upper class boundary of the highest class interval) $-$ (lower class boundary of the lowest class interval) — a1 $\frac{1}{2}$

$= 39.5 - 0.5$ — a1 $\frac{1}{2}$

$= 39.$ — a1 $\frac{1}{2}$

(c). The upper quartile

$= L_Q + \left(\frac{\frac{3}{4}N - fb}{fw}\right)c$ — a1 $\frac{1}{2}$

but path position $\left(\frac{3}{4}N\right)^{th} = 75^{th}$ — a1 $\frac{1}{2}$

upper quartile $= 25.5 + \left(\frac{75 - 60}{18}\right)5$ — a1 $\frac{1}{2}$

$= 29.6667$ — a1 $\frac{1}{2}$

(d) The 80^{th} percentile

from $P_K = \left(\frac{K}{100}\right)N$

$P_{80} = \frac{80 \times 100}{100} = 80^{th}$ position

$P_{80} = 30.5 + \left(\frac{80-78}{12}\right) \times 5$ ———— $01\frac{1}{2}$

$= 31.3333.$

$\therefore 80^{th}$ percentile $= 31.333$ ———— 01.

7(a): $E = \{(4,6), (5,5), (6,4).\}$ ———— 01

$n(E) = 3.$

$n(S) = 6 \times 6$

$= 36.$

from $P(E) = \frac{n(E)}{n(S)}$ ———— $01\frac{1}{2}$

$= \frac{3}{36}$

$= \frac{1}{12}$ ———— 01

(b). Given: $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{4}.$

for mutual exclussive event

$P(A \cap B) = 0$ ———— $01\frac{1}{2}$

Then $P(A \cap B') = P(A) + P(A \cap B)$ ———— $01\frac{1}{2}$

(b) $P(A \cap B') = 0 + \frac{1}{2}$.

$P(A \cap B') = \frac{1}{2}$ — 01

7 (c) Given $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{4}$, $P(A \cap B) = \frac{1}{8}$

(i) $P(A/B) = \frac{P(A \cap B)}{P(B)}$ — 0½

$= \frac{1/8}{1/4}$

$= \frac{1}{8} \times 4$

$= \frac{1}{2}$. — 0½

(ii) $P(A/B^c) = \frac{P(A \cap B^c)}{P(B^c)}$ — 0½

Where

$P(A \cap B^c) = P(A) - P(A \cap B)$ — 0½

$= \frac{1}{3} - \frac{1}{8}$

$= \frac{8-3}{24} = \frac{5}{24}$ — 01

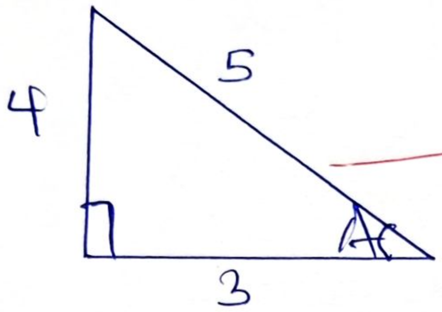
$P(B^c) = 1 - \frac{1}{4} = 3/4$ — 01

$P(A/B^c) = \frac{5/24}{3/4} = \frac{5}{24} \times \frac{4}{3} = \frac{5}{18}$ — 01½

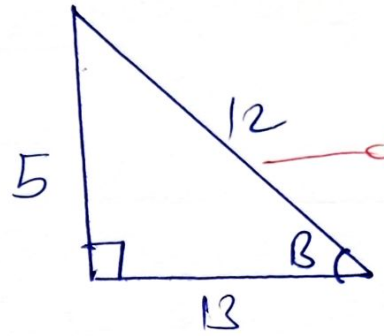
8-(a) Given $\sin A = 4/5$

$\cos B = 12/13$

Consider the figures below to find $\cos A$ and $\sin B$.



— $\frac{1}{2}$



— $\frac{1}{2}$

Then $\cos A = 3/4$ — $\frac{1}{2}$ $\sin B = 5/12$ — $\frac{1}{2}$

from

$\cos(A+B) = \cos A \cos B - \sin A \sin B$ — 1

$= 3/5 \times \frac{12}{13} - 4/5 \times 5/13$

$= \frac{36}{65} - \frac{20}{65} = \frac{16}{65}$ — $\frac{1}{2}$

$\therefore \cos(A+B) = 16/65$ — $\frac{1}{2}$

(b) Given $x = 1 + \cos 2\theta$ — (i)

$y = 4\sin\theta$ — (ii)

from eqn (ii) $\sin\theta = y/4$

Also using eqn(i)

(b) $X = 1 + \cos^2\alpha - \sin^2\alpha$ ——— $\alpha 1\frac{1}{2}$

$X = 1 + 1 - 2\sin^2\alpha$ ——— $\alpha 1\frac{1}{2}$

$X = 2 - 2(\sin\alpha)^2$ ——— $\alpha 1\frac{1}{2}$

$X = 2 - 2\left(\frac{y}{4}\right)^2$

$X = 2 - 2\left(\frac{y^2}{16}\right)$ ——— $\alpha 1\frac{1}{2}$

$8X = 16 - y^2$

$y^2 = 16 - 8x.$ ——— $\alpha 1$

(c) Consider the L.H.S

$\frac{2\tan\alpha}{1+\tan^2\alpha}$

$= \frac{2\sin\alpha/\cos\alpha}{\sec^2\alpha}$ ——— $\alpha 1$

$= \frac{2\sin\alpha/\cos\alpha}{1/\cos^2\alpha}$ ——— $\alpha 1$

$= \frac{2\sin\alpha}{\cos\alpha} \times \frac{\cos^2\alpha}{1}$

$= 2\sin\alpha\cos\alpha$ ——— $\alpha 1\frac{1}{2}$

$= \sin 2\alpha$ ——— $\alpha 1\frac{1}{2}$

Hence proved.

9 a) given

$a = \log_b c \;,\; b = \log_c a \quad c = \log_a b$

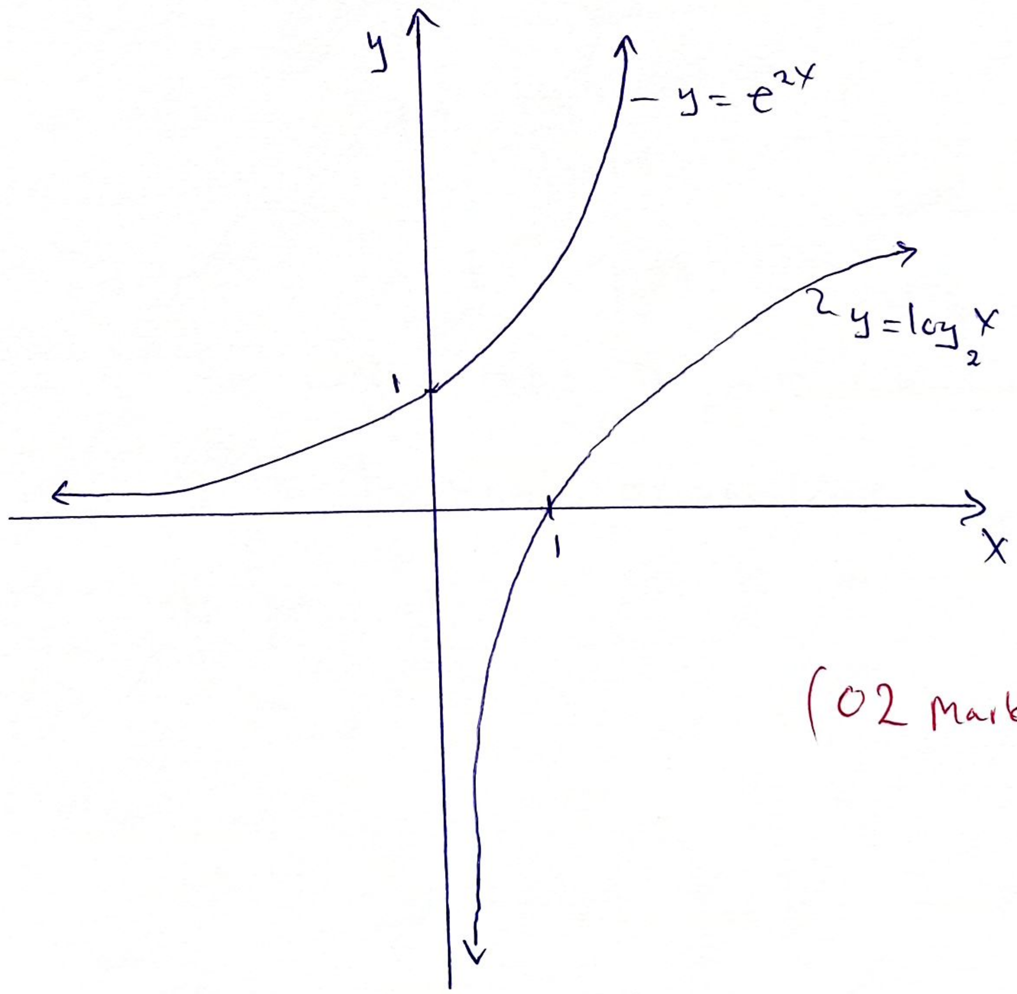
$abc = \log_b c \, \log_c a \cdot \log_a b$. - - - 01

$= \dfrac{\log c}{\log b} \cdot \dfrac{\log a}{\log c} \cdot \dfrac{\log b}{\log a}$ - - 01

$= 1$.

$\therefore abc = 1$. Proved. - - - 01

b)



(02 marks)

The graph of $y = \log_2 x$ is inverse of $y = 2^x$.

(01 mark)

c) Given

$r = 10$, P = principal

$A = 2P$, $n = 1$ and t = number of years.

$$A_n = P\left(1 + \frac{r}{100n}\right)^{nt} \quad \text{------ 01}$$

$$2P = P\left(1 + \frac{10}{100}\right)^{t}$$

$$2 = (1.1)^{t} \quad \text{------ 01}$$

$$\log 2 = t \log 1.1$$

$$t = \frac{\log 2}{\log 1.1} \quad \text{------ 01}$$

$$t = 7.273$$

$\therefore$ It will take 7 years and 4 month for mary to receive accumulated amount twice its principal. ------ 01

10 Let

x be the number of chairs per week -- (00½)

y be the number of table per week. -- (00½)

Maximize

$$f(x, y) = 1500x + 2500y \quad \text{- - - - 01}$$

Subject to;

$$3x + 3y \leq 36 \quad \text{- - - - - 00½}$$

$$5x + 2y \leq 50 \quad \text{- - - - 00½}$$

$$2x + 6y \leq 60 \quad \text{- - - - 00½}$$

$$x \geq 0, y \geq 0 \quad \text{- - - - - 00½}$$

Intercepts

$3x + 3y = 36$

$\Rightarrow (0, 12)$ and $(12, 0)$

$5x + 2y = 50$

$\Rightarrow (0, 25)$ and $(10, 0)$.

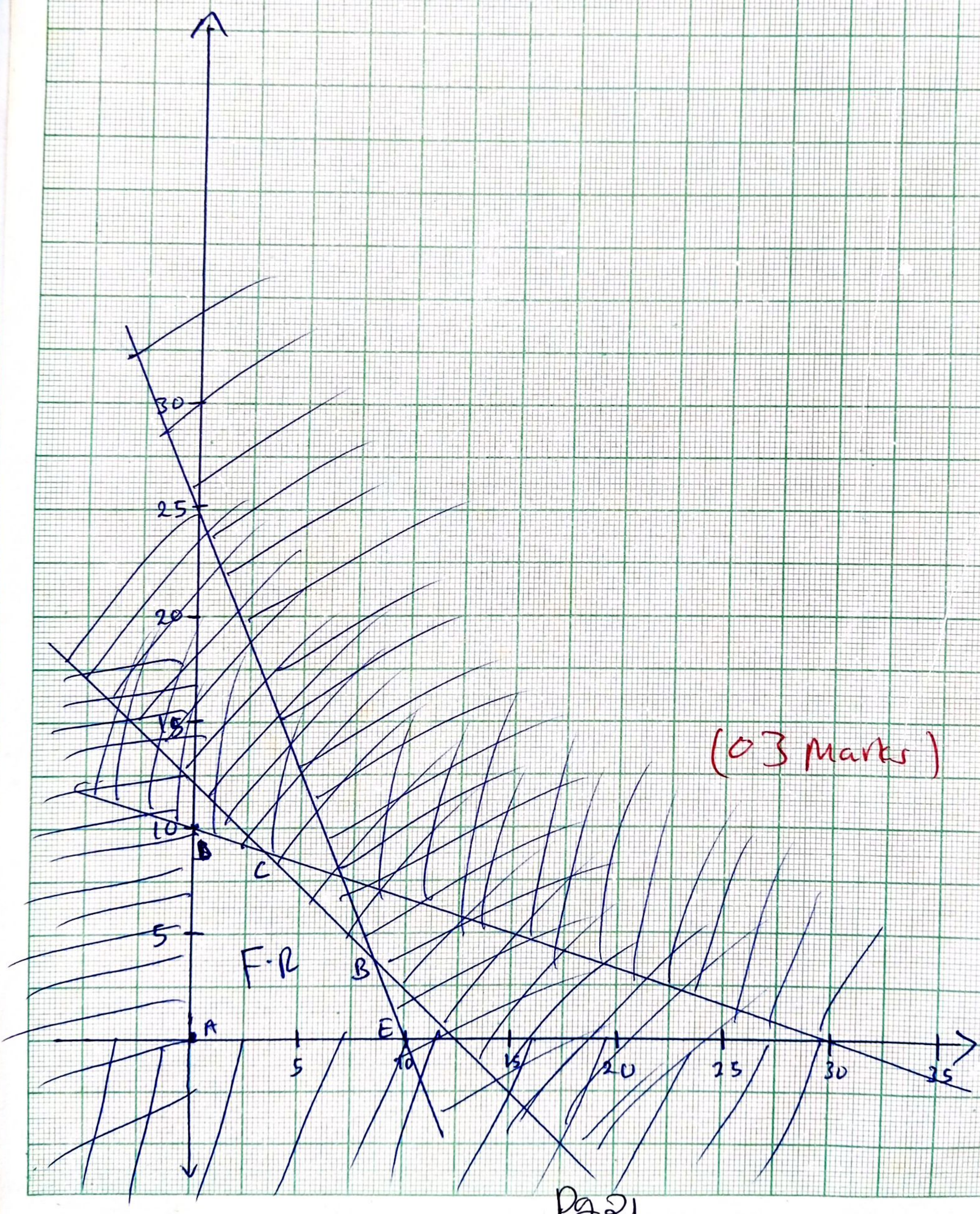
$2x + 6y = 60$

$\Rightarrow (0, 10)$ and $(30, 0)$.

Corner point	$f(x, y) = 1500x + 2500y$
A (0, 0)	0
B (8.67, 3.33)	21330
C (3, 9)	27000
D (0, 9)	22500
E (10, 0	15000

01

(10) The graph of question 10.



optimal point is (3, 9).
Maximum value = 27000

In order to maximize production the company should produce 3 chairs per week and 9 tables per week. (01 Marks)

iii) The maximum contribution is 27000/= per week. - - - - - 01