TANZANIA HEAD OF ISLAMIC SCHOOL COUNCIL
FORM SIX INTER ISLAMIC MOCK EXAMINATION

142/1 ADVANCED MATHEMATICS 1

MARKING GUIDE.

01. (a) (i) $y = 12882.2$ (03 Marks)

(ii) $t = 10.00003698$ (03 Marks)

(b) $\hat{C} = 107^\circ 14'$. (04 Marks).

02 (a)(i) Given $\sinh x = 5/12$

from

$\cosh 2x = 1 + 2\sinh^2 x$ ———— $0\frac{1}{2}$

$= 1 + 2\left(\frac{5}{12}\right)^2$ ———— $0\frac{1}{2}$

$= \frac{97}{12}$ ———— $0\frac{1}{2}$

ii) $\sinh 2x = 2\sinh x \cosh x$ ———— $0\frac{1}{2}$

$= 2 \times \frac{5}{12} \times \frac{13}{12}$ ———— $0\frac{1}{2}$

$= \frac{65}{72}$ ———— $0\frac{1}{2}$

b) $2\tanh^{-1}\left(\frac{x-2}{x+1}\right) = \ln 2$

$\tanh^{-1}\left(\frac{x-2}{x+1}\right) = \frac{1}{2}\ln 2$ ———— $0\frac{1}{2}$

$\frac{1}{2}\ln\left(\frac{1+\frac{x-2}{x+1}}{1-\frac{x-2}{x+1}}\right) = \frac{1}{2}\ln 2$

$\ln\left(\frac{2x-1}{3}\right) = \ln 2$ ———— $0\frac{1}{2}$

$\frac{2x-1}{3} = 2$

$x = 7/2$. 01/2

ii) Given

$y = \tanh^{-1}(\sin x)$

$\frac{dy}{dx} = \frac{1}{1-\sin^2 x} \cdot \frac{d(\sin x)}{dx}$ 01/2

$= \frac{\cos x}{\cos^2 x}$. 01/2

$= \frac{1}{\cos x}$.

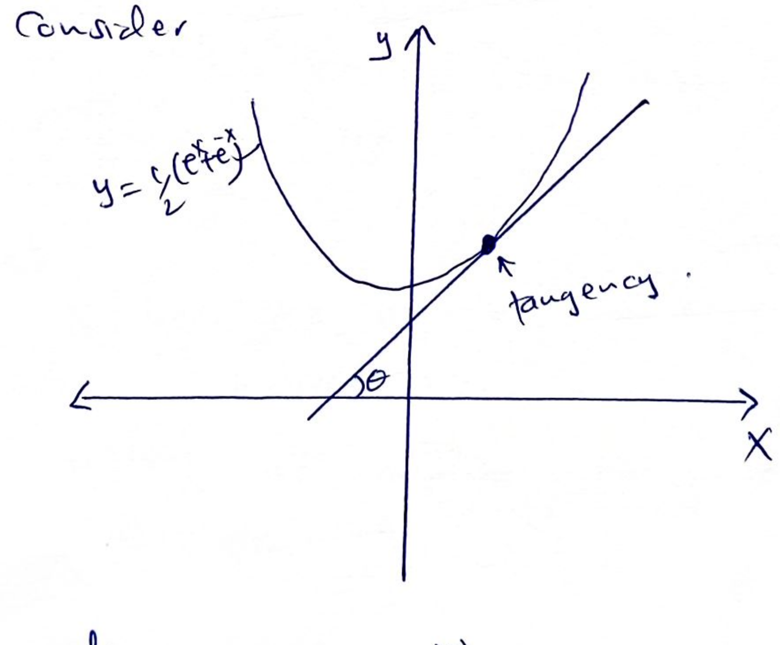
$= \sec x$.

$\therefore \frac{dy}{dx} = \sec x$. 01/2

c) i) Given

$y = \frac{1}{2}(e^x + e^{-x})$.

Consider



$\frac{dy}{dx} = \frac{1}{2}(e^x - e^{-x})$.

but $\frac{dy}{dx} = \tan\theta$ 01/2

$\therefore \tan\theta = \sinh x$. ——— $\frac{1}{2}$

from

$y = \frac{1}{2}(e^x + e^{-x})$

$= \cos x$.

$= \sqrt{1+\sinh^2 x}$ ——— $\frac{1}{2}$

but

$\sinh x = \tan\theta$.

$y = \sqrt{1+\tan^2\theta}$ ——— $\frac{1}{2}$

$y = \sec\theta$. ——— $\frac{1}{2}$

ii) from

$x = \sinh^{-1}(\tan\theta)$

Recall

$\sinh^{-1} x = \log_e(x + \sqrt{x^2+1})$ ——— $\frac{1}{2}$

$x = \log_e(\tan\theta + \sqrt{\tan^2\theta + 1})$. ——— $\frac{1}{2}$

$= \log_e(\tan\theta + \sec\theta)$.

$\therefore x = \log_e(\tan\theta + \sec\theta)$. ——— $\frac{1}{2}$

03 a) let

x be the number of grade A jar produced

y be the number of grade B jar produced

Mass of a jar $400g = 0.4kg$.

In grade A: 60% strawberry and 40% mango

60% strawberry $= \frac{60}{100} \times 0.4 = 0.24 kg$

40% mango $= \frac{40}{100} \times 0.4 = 0.16$ kg.

In grade B: 30% strawberry and 70% mango

30% strawberry $= \frac{30}{100} \times 0.4 = 0.12$ kg

70% mango $= \frac{70}{100} \times 0.4 = 0.28$ kg.

	Grade A	Grade B	Total maximum
Strawberry	0.24	0.12	720
Mango	0.16	0.28	800
Price.	48	30	

Constraints:

$0.24x + 0.12y \leq 720$ $0\frac{1}{2}$

$0.16x + 0.28y \leq 800$ $0\frac{1}{2}$

$x, y \geq 0$. $0\frac{1}{2}$

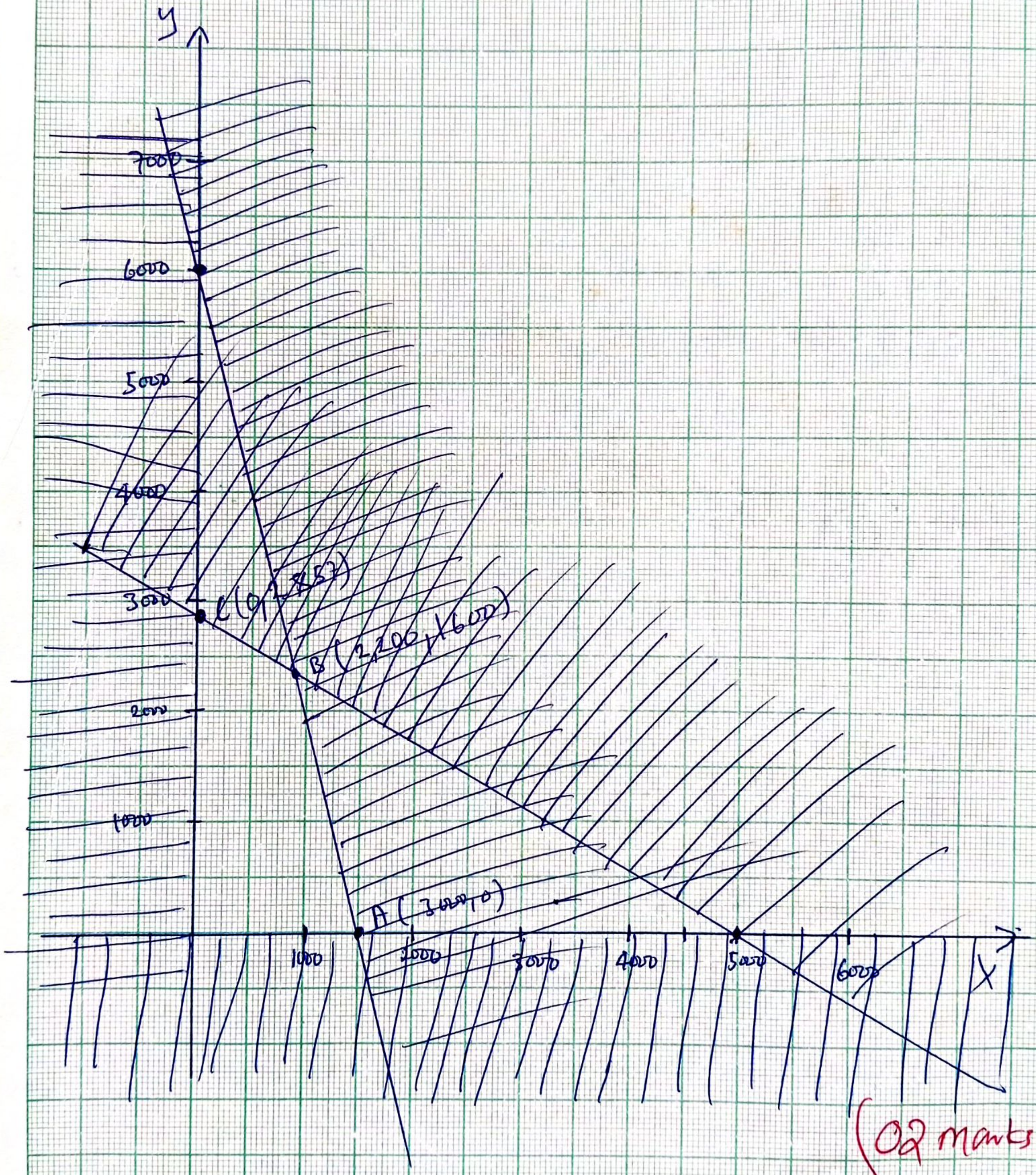
Maximize

$f(x,y) = 48x + 30y$. $0\frac{1}{2}$

Intercepts:

$0.24x + 0.12y = 720$ $(3000, 0)$ and $(0, 6000)$

$0.16x + 0.28y = 800$ $(5000, 0)$ and $(0, 2857)$



(02 marks)

(1 cm x 1 cm)squares of (10mm x 10mm)

Corner point	$f(x,y) = 48x + 30y$.
$A(3000, 0)$	144,000
$B(2200, 1600)$	153,600
$C(0, 2857)$	85,710

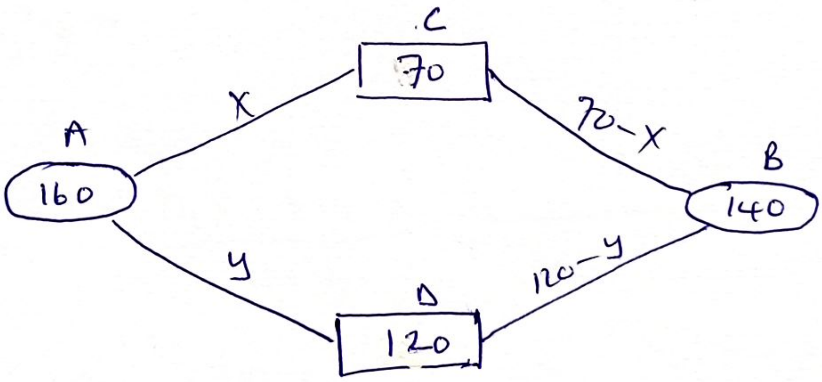
01

The maximum profit is 153,600 when $x = 2200$ and $y = 1600$. 01

b) Let

x be the number of units from A to C

y be the number of units from A to D



(i) Minimize

$$f(x,y) = 160x + 240y + 200(70-x) + 260(120-y)$$

$$f(x,y) = -40x - 20y + 45,200.$$ 01/2

(ii) $x + y \leq 60$ 01/2

$x + y \geq 50$ 01/2

$x \leq 70$ 01/2

$y \leq 120$ 01/2

$x \geq 0, y \geq 0$. 01/2

iii) Total demand $= 70 + 120 = 190$

Total supply $= 160 + 140 = 300$

Since total supply $\neq$ total demand, then the problem is not balanced. ——— 01

04 a) Given

$n_1 = 100$

$\bar{X}_1 = 15$

$\delta_1^2 = 9$

$n_2 = 150$

$\bar{X}_2 = ?$

$\delta_2^2 = ?$

From

$$\bar{X} = \frac{n_1\bar{X}_1 + n_2\bar{X}_2}{n_1 + n_2}$$ ——— 01/2

$$15.6 = \frac{100 \times 15 + 150\bar{X}_2}{100 + 150}$$ ——— 01/2

$$\bar{X}_2 = \frac{3900 - 1500}{150}$$

$$= 16.$$ ——— 01/2

Now;

$d_1 = \bar{X} - \bar{X}_1$ and $d_2 = \bar{X} - \bar{X}_2$

$= 15.6 - 15$ $= 15.6 - 16.$

$= 0.6$ $= -0.4.$

$d_1 = 0.6$ $d_2 = -0.4$

from

$$\sigma^2 = \frac{n_1(\sigma_1^2+d_1^2)+n_2(\sigma_2^2+d_2^2)}{n_1+n_2}$$ — $\frac{1}{2}$

$$13.44 = \frac{100(9+(0.6)^2)+150(\sigma_2^2+(-0.4)^2)}{250}$$

$$13.44 = \frac{936+150\sigma_2^2+24}{250}$$

$$150\sigma_2^2 = 2400$$

$$\sigma^2 = 16.$$

$$\sigma_2 = 4.$$ — $\frac{1}{2}$

$\therefore$ The mean and standard deviation of the second group are 16 and 4 respectively. — $\frac{1}{2}$

b) Consider

class interval	x	f	c.f	$\lvert x-m\rvert$	$f\lvert x-m\rvert$
150 – 155	152.5	30	30	7.5	225
155 – 160	157.5	20	50	2.5	50
160 – 165	162.5	18	68	2.5	45
165 – 170	167.5	17	85	7.5	127.5
170 – 175	172.5	15	100	15.5	232.5

0 mark   0 mark

$$\sum f\lvert x-m\rvert = 680$$

i) The median.

$$\text{Median} = L + \left[\frac{N/2 - n_b}{n_w}\right] c$$

$$= 155 + \left[\frac{50-30}{20}\right] 5$$

$$= 160$$

Median = 160 cm. ——— 01/2

Mean deviation about median.

$$M.D = \frac{\Sigma f|x-M|}{\Sigma f}$$ ——— 01/2

$$= \frac{680}{100}$$

$$= 6.8$$

M.D = 6.8.

ii) Assumed mean, = 162.5 ——— 01/2

Class interval (cm)	f	x	u	fu	fu²
150 – 155	30	152.5	-2	-60	120
155 – 160	20	157.5	-1	-20	20
160 – 165	18	162.5	0	0	0
165 – 170	17	167.5	1	17	17
170 – 175.	15	172.5	2	30	60

$\Sigma f = 100$ $\Sigma fu = -33$ $\Sigma fu^2 = 217$ 02

From

$$S.D = c\sqrt{\frac{\Sigma fu^2}{\Sigma f} - \left(\frac{\Sigma fu}{\Sigma f}\right)^2}$$ ——— 01/2

$$= 5\sqrt{\frac{217}{100} - \left(\frac{-33}{100}\right)^2}$$

$= 7.178.$

Therefore, the standard deviation is 7.178. 01

5 a) i) A power set is a set that contains all subsets of the given set. 01

ii) $A \Delta B = (A-B) \cup (B-A)$ 01/2

$= \{2, 3, 5\}.$ 01/2

$P(A \Delta B) = \{\phi, \{2\}, \{3\}, \{5\}, \{2,3\}, \{2,5\}, \{3,5\}, \{2,3,5\}\}$ 01

b) $(A' \cap B' \cap C) \cup (B \cap C) \cup (A \cap C)$.

$((A' \cap B') \cap C) \cup (B \cap C) \cup (A \cap C)$ --- Given.

$(A' \cap B' \cap C) \cup [(B \cup A) \cap C]$ --- Distributive law 01/2

$[(A' \cap B') \cup (A \cup B)] \cap C$ --- Distributive and commutative laws. 01

$[(A \cup B)' \cup (A \cup B)] \cap C$ --- De Morgan's law 01/2

$\mu \cap C$ ---- Complement law 01/2

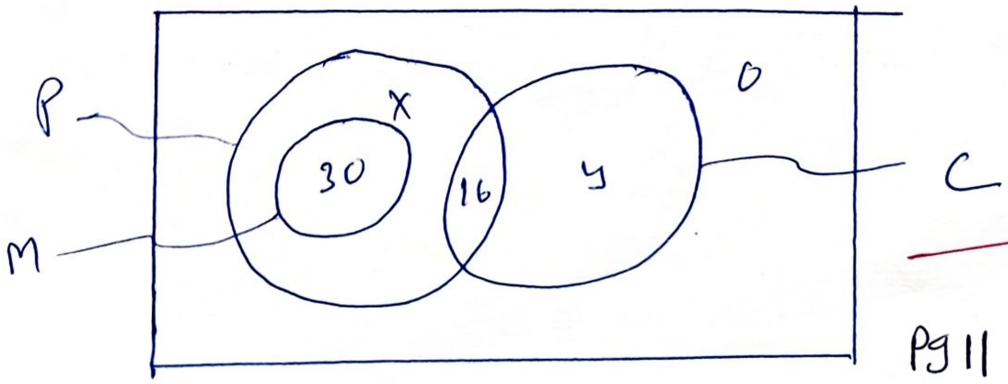
C ---- Identity law. 01/2

$\therefore (A' \cap B' \cap C) \cup (B \cap C) \cup (A \cap C) = C.$

c) ~~Given.~~ let

M = Mathematics, C = chemistry and P = physics.

$n(P) = 50$, $n(M) = 30$, $n(\mu) = 60$



01

For physics

$x + 30 + 16 = 50$

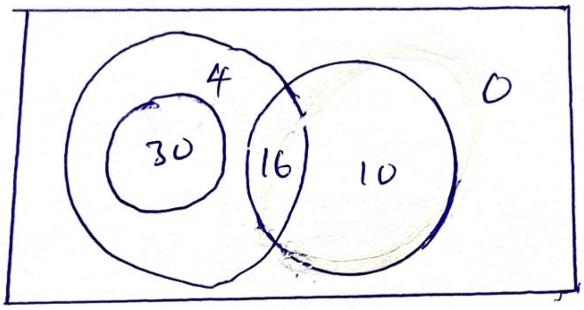
$x = 4$ —— $01/2$

Also

$x + 30 + 16 + y = 60$

$y = 10$ —— $01/2$

Now;



—— $01/2$

i) 4 students do physics only —— $01/2$

ii) 26 students do chemistry. —— $01/2$

06 a) i) The function f is defined for all values of x except $x = 0$ —— 01

ii) Given

$$f(x) = \frac{a}{x} + b.$$

$$f(-1) = -a + b = 3/2 \quad \text{---} (i)$$

$$f(2) = \frac{a}{2} + b = 9 \quad \text{---} (ii)$$

$a = 5$ and $b = 13/2$. —— $01/2$ @

iii) Given

$f(x) = a/x + b$

$f(x_1) = \frac{a}{x_1} + b$ and $f(x_2) = a/x_2 + b$.

Suppose $f(x_1) = f(x_2)$.

$\frac{a}{x_1} + b = a/x_2 + b$. — cw 1/2

$\frac{a}{x_1} = a/x_2$

$x_1 = x_2$,

$f(x_1) = f(x_2)$ implies $x_1 = x_2$ so the function f — cw 1/2

is one to one function.

b) $f(x) = \frac{3x-9}{x^2-x-2}$.

$y = \frac{3x-9}{x^2-x-2} = \frac{3(x-3)}{(x+1)(x-2)}$

$$\frac{dy}{dx} = \frac{(x^2-x-2)\frac{d}{dx}(3x-9) - (3x-9)\frac{d}{dx}(x^2-x-2)}{(x^2-x-2)^2}$$ — cw 1/2

$$\frac{dy}{dx} = \frac{(x^2-x-2)3 - (3x-9)(2x-1)}{(x^2-x-2)^2}$$

at turning point, $dy/dx = 0$.

$$0 = \frac{3(x^2-x-2) - (3x-9)(2x-1)}{(x^2-x-2)^2}$$ — cw 1/2

$$0 = 3x^2 - 3x - 6 - (6x^2 - 3x - 18x + 9)$$

$$-3x^2 + 18x - 15 = 0$$ a$\frac{1}{2}$

$$\therefore x = 1,\ x = 5$$ a$\frac{1}{2}$

If $x = 1$ then $y = \dfrac{3(1) - 9}{1^2 - 1 - 2} = 3$

$$\Rightarrow (1, 3)$$ a$\frac{1}{2}$

If $x = 5$ then $y = \dfrac{3(5) - 9}{5^2 - 5 - 2} = \frac{1}{3}$.

$$\Rightarrow (5, \tfrac{1}{3})$$ a$\frac{1}{2}$

$\therefore$ The turning points are $(1, 3)$ and $(5, \frac{1}{3})$.

ii) Vertical asymptotes

let $(x+1)(x-2) = 0$

$$x = -1 \text{ and } x = 2$$ a$\frac{1}{2}$

iii) Horizontal asymptotes

Since Degree of Numerator < Degree of denominator

then

$$y = 0$$ a$\frac{1}{2}$

x and y- intercepts.

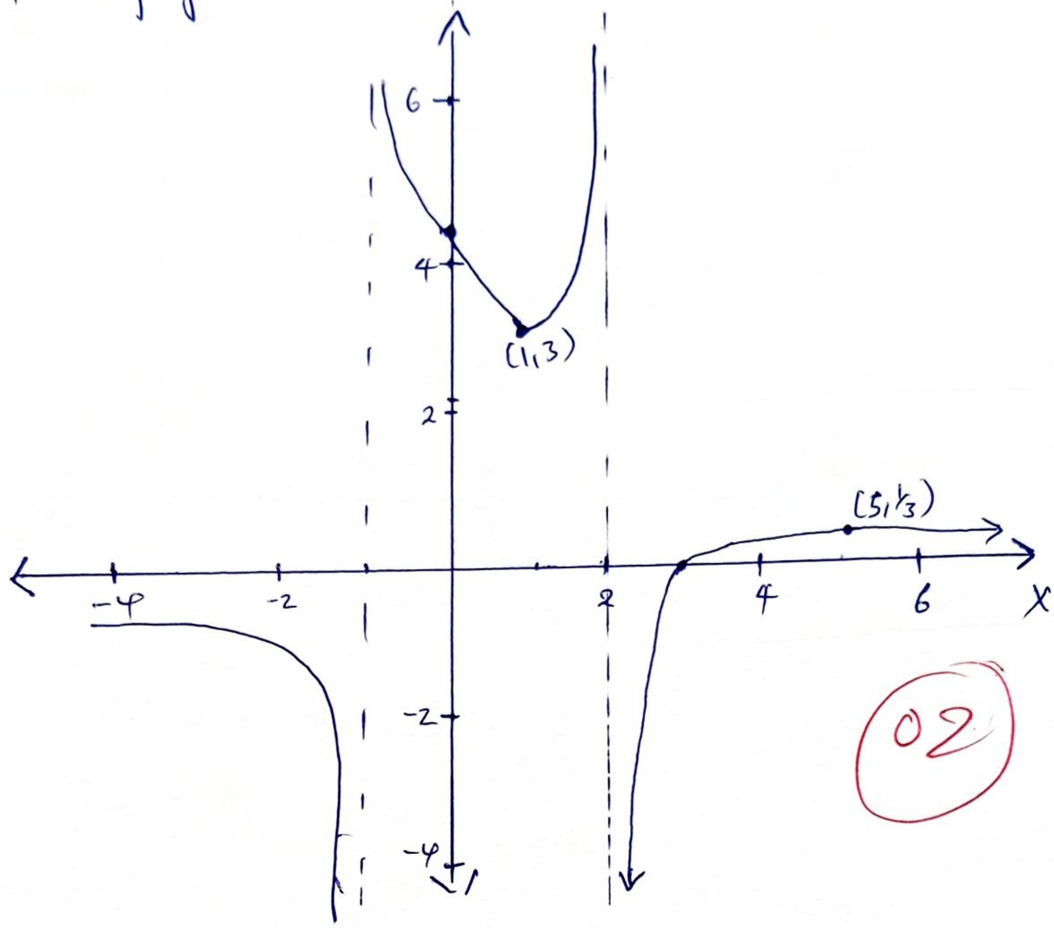
If $x = 0$, $y = \frac{9}{2}$

$$\Rightarrow (0, \tfrac{9}{2})$$ a$\frac{1}{2}$

If $y = 0$, $x = 3$

$$\Rightarrow (3, 0)$$ a$\frac{1}{2}$

It's graph.



07 a) Given

$$P = \frac{a^3 b^2}{\sqrt{c}\, d}$$

$$\ln P = \ln(a^3 b^2) - \ln(\sqrt{c}\, d).$$

$$\ln P = 3\ln a + 2\ln b - \tfrac{1}{2}\ln c - \ln d$$ $\frac{1}{2}$

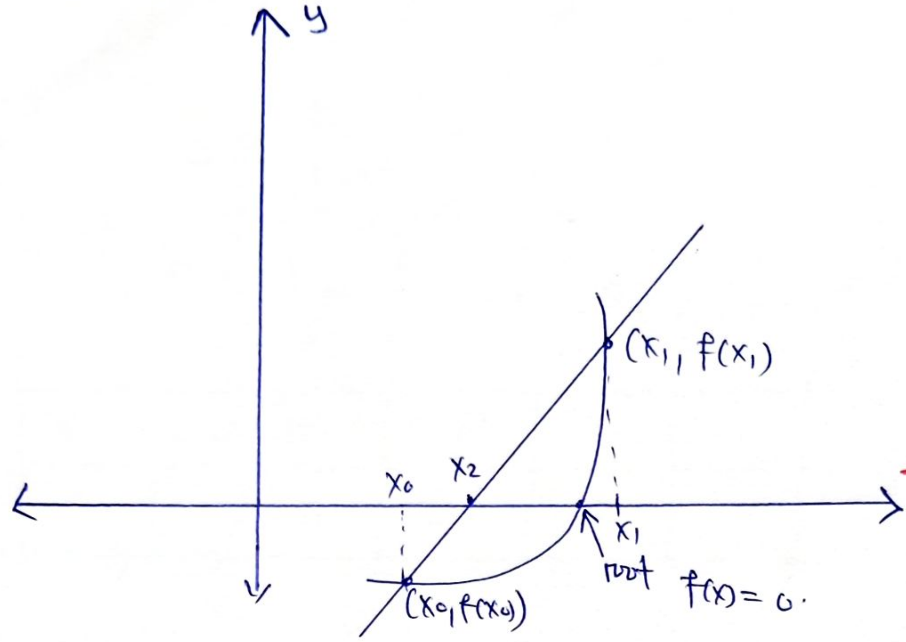
$$\frac{dP}{P} = 3\left(\frac{da}{a}\right) + 2\left(\frac{db}{b}\right) + \frac{1}{2}\left(\frac{dc}{c}\right) + \frac{dd}{d}.$$

$$\frac{dP}{P} = 3(0.01) + 2(0.03) + \tfrac{1}{2}(0.04) + 0.02.$$ $\frac{1}{2}$

$$\frac{dP}{P} = 0.13$$ $\frac{1}{2}$

∴ The percentage error in p is 13% $\frac{1}{2}$

b) Consider:



— 02

Newton - Raphson Method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \quad (f(x)=0) \text{ --- } (1)$$ — 01/2

By using the approximation from the graph above.

$$f'(x_n) \approx \frac{f(x_n) - f(x_{n-1})}{x_n - x_{n-1}} \text{ ---- } (2)$$ — 01/2

Substitute (2) into (1),

$$x_{n+1} = x_n - \frac{(x_n - x_{n-1}) f(x_n)}{f(x_n) - f(x_{n-1})} ; \; n = 1, 2, 3, \ldots$$ — 01/2

Secant formula for $f(x)=0$.

Comment:

The secant Method does not require the knowledge of $f'(x)$, where N-R method requires both $f(x)$ and $f'(x)$.

— 01/2

b) $\displaystyle\int_0^1 \frac{1}{1+x^2}\,dx$

let $y = \dfrac{1}{1+x^2}$

$h = \dfrac{1-0}{5-1} = 0.25$ — 01/2

x	0.00	0.25	0.50	0.75	1.00
y	1.0000	0.9412	0.8	0.64	0.5

— 01

i) By trapezoidal rule;

$$\int_0^1 \frac{dx}{1+x^2} = \frac{h}{2}\left[y_0+y_4+2\Sigma(y_1+y_2+y_3)\right]$$ — 01/2

$$= \frac{0.25}{2}\left(1.5+2(2.3812)\right)$$

$$= 0.7828$$ — 01/2

ii) By Simpson's rule.

$$\int_0^1 \frac{dx}{1+x^2} = \frac{h}{3}\left[(y_0+y_4)+4(y_1+y_3)+2(y_2)\right]$$

$$= \frac{0.25}{3}\left(1.5+4(1.5812)+2(2.8)\right)$$

$$= 0.7854$$ — 01

iii) $\displaystyle\int_0^1 \frac{dx}{1+x^2} = \tan^{-1}x\Big|_0^1 = \pi/4.$

- Find the exact value ~~of~~ by using calculator.

$$\int_0^1 \frac{dx}{1+x^2} = 0.7854.$$ — 01/2

By Simpson's rule. numerical value.

$\pi/4 = 0.7854 \Rightarrow \pi = 3.1416.$ — 01

a) Given points are $A(-2,-1)$, $B(4,0)$, $C(3,3)$, $D(-3,2)$

Slope of $AB = \frac{1}{4+2} = \frac{1}{6}$. ——— 001/2

Slope of $DC = \frac{2-1}{3+3} = \frac{1}{6}$. ——— a1/2

Slope of AB = Slope of DC thus $AB \parallel DC$

Slope of $AD = \frac{2+1}{-3+2} = \frac{+3}{-1} = -3$. ——— a1/2

Slope of $\overline{BC} = \frac{3-0}{3-4} = \frac{3}{-1} = -3$ ——— a1/2

Slope of AD = Slope of BC $\Rightarrow$ $AD \parallel BC$ ——— 001/2

Hence $ABCD$ is a parallelogram. ——— a1/2

b) Given $y = m_1x + c_1$, $y_2 = m_2x + c_2$ and $x = 0$

- Point of intersection btn $y = m_1x + c_1$ and $x = 0$

$\Rightarrow (0, c_1)$. ——— 001/2

- Point of intersection of $y = m_2x + c_2$ and $x = 0$

$\Rightarrow (0, c_2)$ ——— a1/2

- Point of intersection of $y = m_1x + c_1$ and $y = m_2x + c_2$;

$m_1x + c_1 = m_2x + c_2$.

$x = \frac{c_2 - c_1}{m_1 - m_2}$. ——— 001/2

from

$y = m_1x + c_1$

$$y = m_1\left(\frac{c_2-c_1}{m_1-m_2}\right)+c_1. \qquad \frac{1}{2}$$

thus
The triangle will be formed by the points $(0, c_1)$ $(0, c_2)$ and $\left(\frac{c_2-c_1}{m_1-m_2},\ m_1\left(\frac{c_2-c_1}{m_1-m_2}\right)+c_1\right)$.

Its area.

$$A = \frac{1}{2}\begin{vmatrix} 0 & c_1 & 1 \\ 0 & c_2 & 1 \\ \frac{c_2-c_1}{m_1-m_2} & m_1\left(\frac{c_2-c_1}{m_1-m_2}\right)+c_1 & 1 \end{vmatrix} \qquad 1$$

$$A = \frac{1}{2}\left[+c_1\left[\frac{c_2-c_1}{m_1-m_2}\right]+(-c_2)\left[\frac{c_2-c_1}{m_1-m_2}\right]\right| \qquad \frac{1}{2}$$

$$= \frac{1}{2}\,\frac{(c_2-c_1)^2}{|m_1-m_2|}.$$

$$\therefore \text{Area of triangle} = \frac{(c_1-c_2)^2}{2|m_1-m_2|}. \qquad \frac{1}{2}$$

c) $x + y = 6$ and $2x - y - 3 = 0$

upon solving

$$(x, y) = (3, 3).$$

Centre of $x^2 + y^2 - 2x + 2y - 10 = 0$

$$\Rightarrow C(1, -1).$$ 01/2

radius, $r = d = \sqrt{(3-1)^2 + (3+1)^2}$

$$= \sqrt{20}$$ 01/2

from,

$$(x-a)^2 + (y-b)^2 = r^2$$

$$(x-3)^2 + (y-3)^2 = (\sqrt{20})^2$$ 01

$$x^2 + y^2 - 6x - 6y - 2 = 0.$$ 01

9 a)

$$\int \frac{\cos x}{1+\cos x}\,dx = \int \frac{(1+\cos x) - 1}{1+\cos x}\,dx$$ 01/2

$$= \int dx - \int \frac{dx}{1+\cos x}.$$ 01/2

$$= x - \int \frac{1}{2\cos^2 x/2}\,dx + c$$ 01/2

$$= x - \frac{1}{2}\int \sec^2 \frac{x}{2} + c$$ 01/2

$$= x - \frac{2}{2}\tan\frac{x}{2} + c$$

$$\therefore \int \frac{\cos x}{1+\cos x}\,dx = x - \tan\frac{x}{2} + c$$ 01.

b) $\displaystyle\int_0^1 \frac{x-a}{(x+1)(3x+1)}\,dx = 0.$

$$\frac{x-a}{(x+1)(3x+1)} = \frac{A}{x+1} + \frac{B}{3x+1} \qquad a0\tfrac{1}{2}$$

$$x-a = A(3x+1) + B(x+1).$$

when $x = 1$; $A = \dfrac{a+1}{2}$ $\qquad a\tfrac{1}{2}$

when $x = -\frac{1}{3}$, $B = \dfrac{-3a-p}{2}$. $\qquad a\tfrac{1}{2}$

$$\frac{x-a}{(x+1)(3x+1)} = \frac{\frac{a+1}{2}}{x+1} - \frac{\frac{3x+1}{2}}{3x+1} \qquad a\tfrac{1}{2}$$

$$\int_0^1 \frac{x-a}{(x+1)(3x+1)} = \frac{a+1}{2}\int_0^1 \frac{1}{x+1}\,dx - \frac{3a+1}{2}\int_0^1 \frac{1}{3x+1}\,dx \qquad a\tfrac{1}{2}$$

$$= \frac{a+1}{2}\ln(x+1)\Big|_0^1 - \frac{3a+1}{6}\ln(3x+1)\Big|_0^1 \qquad a\tfrac{1}{2}$$

$$= \frac{a+1}{2}\ln 2 - \frac{3a+1}{3}\ln 2. \qquad a\tfrac{1}{2}$$

$$\frac{a+1}{2}\ln 2 - \frac{3a+1}{3}\ln 2 = 0.$$

$$\frac{a+1}{2} = \frac{3a+1}{3} \qquad a\tfrac{1}{2}$$

$$a = \frac{1}{3}.$$

c) Given

$$9y^2 = 16x^3 \Rightarrow y = \tfrac{4}{3} x^{3/2}.$$

$$\frac{dy}{dx} = 2x^{1/2}$$

Length, $L = \displaystyle\int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx.$

$$= \int_0^4 \sqrt{1 + (2x^{1/2})^2}\, dx$$

$$= \int_1^4 \sqrt{1+4x}\, dx$$

$$= \left[\frac{1}{6}\left(\sqrt{1+4x}\right)^3\right]_1^3$$

$$= \frac{1}{6}(\sqrt{17})^3 - \frac{1}{6}(\sqrt{5})^3$$

$$= 9.8187 \text{ units}.$$

10 a) $f(x) = x + \sin x.$

$f(x+h) = (x+h) + \sin(x+h).$

from

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

$$= \lim_{h \to 0} \frac{x+h+\sin(x+h) - x - \sin(x+h)}{h}.$$

$$= \lim_{h \to 0} \frac{h + \sin(x+h) - \sin x}{h}.$$

$$= \lim_{h \to 0} \frac{h}{h} + \lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h}.$$

$$= \lim_{h \to 0} 1 + \lim_{h \to 0} \frac{2\cos(x+h/2)\sin h/2}{h}.$$ 2

$$= 1 + \lim_{h \to 0} \cos(x+h/2) \times \lim_{h \to 0} \frac{\sin h/2}{h/2}.$$

$$= 1 + \cos(x+0/2) \times 1.$$

$$= 1 + \cos x.$$

$\therefore f'(x) = 1 + \cos x.$ 1

b) Given

$$\frac{d}{dx}\left[\frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right] = \sqrt{a^2-x^2}$$

Consider L.H.S.

$$\frac{d}{dx}\left[\frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(x/a\right)\right] = \frac{1}{2}\frac{d}{dx}\left[x\sqrt{a^2-x^2}\right] + \frac{a^2}{2}\frac{d}{dx}\left(\sin^{-1}\frac{x}{a}\right)$$ 1

$$= \frac{1}{2}\left[x\frac{d}{dx}\left(\sqrt{a^2-x^2}\right) + \sqrt{a^2-x^2}\frac{d}{dx}(x)\right] + \frac{a^2}{2}\frac{1}{\sqrt{1+x^2/a^2}}\left(\frac{1}{a}\right)$$ 1

$$= \frac{1}{2}\left[\frac{x}{2}\frac{(-2x)}{\sqrt{a^2-x^2}} + \sqrt{a^2-x^2}\right] + \frac{a^2}{2}\frac{1}{\sqrt{a^2-x^2}}$$

$$= \frac{1}{2}\left[\frac{-x^2+(a^2-x^2)+a^2}{\sqrt{a^2-x^2}}\right]$$ 1

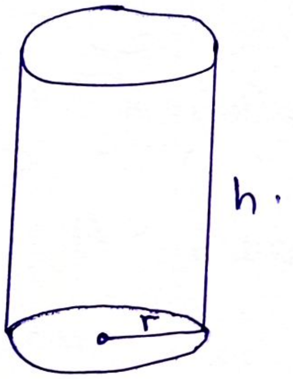
PJ22

$$= \frac{2(a^2-x^2)}{2\sqrt{a^2-x^2}} = \frac{a^2-x^2}{\sqrt{a^2-x^2}}$$

$$= \sqrt{a^2-x^2}.$$

$$\therefore \frac{d}{dx}\left[\frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right] = \sqrt{a^2-x^2}$$

c)



Volume, $V = \pi r^2 h.$

$$\pi r^2 h = 2000\pi$$

$$r^2 h = 2000$$

$$h = \frac{2000}{r^2} \quad \text{---} \; (1)$$

Area, $A = 2\pi r^2 + 2\pi r h$ --- (ii)

Substitute (1) into (ii)

$$A = 2\pi r^2 + 2\pi r\left(\frac{2000}{r^2}\right).$$

$$A = 2\pi r^2 + \frac{4000\pi}{r}$$

$$\frac{dA}{dr} = 4\pi r - \frac{4000\pi}{r^2}.$$

For least area, $\frac{dA}{dr} = 0$. — $a\frac{1}{2}$

$$4\pi r - \frac{4000\pi}{r^2} = 0$$

$$r = 10$$

from

$$A = 2\pi r^2 + \frac{4000\pi}{r}.$$ — $a\frac{1}{2}$

$$A = 2\pi(10)^2 + \frac{4000\pi}{10}$$

$$A = 600\pi.$$

$\therefore$ The least area of metal is $600\pi\ cm^2$. — $a\frac{1}{2}$