TANZANIA HEAD OF ISLAMIC

COUNCIL.

FORM SIX INTER ISLAMIC MOCK

EXAMINATION

142/2 ADVANCED MATHETIC 2.

MARKING GUIDE.

1 a) i) There are 11 people, from whom 4 are choosen

∴ Number of ways of choosing the team = $^{11}C_4$

= 330 ways. (01½ marks)

ii) Number of ways to have more boys than girls;

$= {}^6C_3 \times {}^5C_1 + {}^6C_4 \times {}^5C_0 = 115$ ways. --(01½ marks)

iii) $n(S) = 330$

~~Let E be th~~ Let E be the event that only one boy is choosen

$n(E) = {}^6C_1 \times {}^5C_4 = 60$. ---- (01 marks)

So $P(E) = \frac{n(E)}{n(S)} = \frac{60}{330}$

$= 2/11$. --- (01 marks)

b) i) Let X be the number of defective bolts in a box.

This is a binomial; $X \sim Bin(500, 0.002)$.

$n = 500, \quad p = 0.002$.

from

$P(X=x) = {}^nC_x p^x q^{n-x}, \quad n = 0, 1, 2, \dots, n.$

$q = 0.998$. ---- (01)

$P(X=x) = {}^{500}C_x (0.002)^x (0.998)^{500-x}$

$P(X=2) = {}^{500}C_2 (0.002)^2 (0.998)^{500-2}$ -- (01)

$$= 0.184 .$$

$$\therefore P(X=2) = 0.184 \cdots\cdots (01)$$

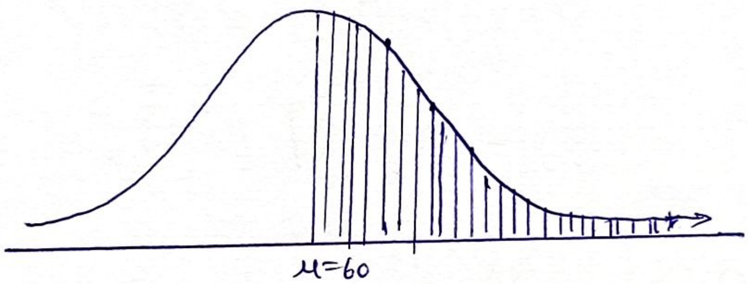
ii) Given

$$X = 60, \quad \bar{X} = 60, \quad \sigma = 5 .$$

$$Z = \frac{X-\bar{X}}{\sigma} = \frac{60-60}{5} = 0 .$$

--- (01 marks)

If $X > 60$, then $Z > 0$.



$$P(Z>0) = 0.5 .$$

$\therefore$ The percentage of students getting more than 60 marks $= 50\%$. --- (01 marks)

c) $$\int_{-2}^{0} k(x+2)^2 dx + \int_{0}^{4/3} 4k\,dx = 1$$ --- (01/2 marks)

$$k\left[\frac{1}{3}(x+2)^3\right]_{-2}^{0} + 4k\left[x\right]_{0}^{4/3} = 1$$

$$\frac{8}{3}k + \frac{16}{3}k = 1$$

$$\frac{24}{3}k = 1$$

$$k = \frac{1}{8} .$$ ---- (01/2 marks)

c) ii) $P(|X| \leq 1) = P(-1 \leq X \leq 1)$.

$= P(-1 \leq X < 0) + P(0 \leq X < 1)$ -- (00 1/2 Marks)

$= \int_{-1}^{0} \frac{1}{8}(x+2)^2 dx + \int_{0}^{1} \frac{1}{2} dx$.

$= \left(\frac{1}{3} - \frac{1}{24}\right) + \left(\frac{1}{2}\right)$

$= \frac{19}{24}$. ---- (00 1/2 Marks)

iii) From

$Var(X) = E(X^2) - (E(X))^2$ --- (00 1/2 Marks)

$E(X) = \int_{-\infty}^{\infty} x f(x) dx$

$= \int_{-2}^{0} \frac{1}{8} x (x+2)^2 dx + \int_{0}^{4/3} \frac{1}{2} x dx$. -- (00 1/2 Marks)

$= -\frac{1}{6} + \frac{4}{9}$

$= \frac{5}{18}$ --- (00 1/2 marks).

$E(X^2) = \int_{-\infty}^{\infty} x^2 f(x) dx$

$= \int_{-2}^{0} \frac{1}{8} x^2 (x+2)^2 dx + \int_{0}^{4/3} \frac{1}{2} x^2 dx$.

$= \int_{-2}^{0} \frac{1}{8} x^2 (x+2)^2 dx + \int_{0}^{4/3} \frac{1}{2} x^2 dx$ -- (00 1/2 Marks)

$= \frac{2}{5} + \frac{32}{81} = \frac{214}{405}$ -- (00 1/2 Marks)

$$Var(X) = \frac{214}{405} - \left(\frac{5}{18}\right)^2 = \frac{731}{1620}$$

$$\therefore Var(X) = \frac{731}{1620} \text{ ...... (00 1/2 Marks)}$$

2 a) i) $p \wedge (q \vee \sim r)$

$F \wedge (T \vee \sim F)$

$F \wedge (T \vee T)$ (01 marks)

$F \wedge T$

F

$\therefore$ The statement is false. - - (00 1/2)

ii) $(\sim p \wedge q) \vee \sim q$

$(\sim F \wedge T) \vee \sim T$

$(T \wedge T) \vee F$ (01 marks)

$T \vee F$

T

$\therefore$ The statement is true. (00 1/2 marks).

b) i) If your work is done, then you can go to the movies. (01 marks)

ii) Converse

If you go to the movies then your work is done (01 marks)

Inverse

If your work is done, then you can not go to the movies. (01 Marks) contrapositive;

If you cannot go to the movies, then your work is not done. --- (01 marks)

c) The statement is;

$[(p \wedge \sim q) \vee (\sim p \wedge q)] \vee (\sim p \wedge \sim q)$ --- (01 Marks)

$(p \wedge \sim q) \vee [(\sim p \wedge q) \vee (\sim p \wedge \sim q)]$ .. Associative law

$(p \wedge \sim q) \vee [\sim p \wedge (q \vee \sim q)]$ -- Distributive law

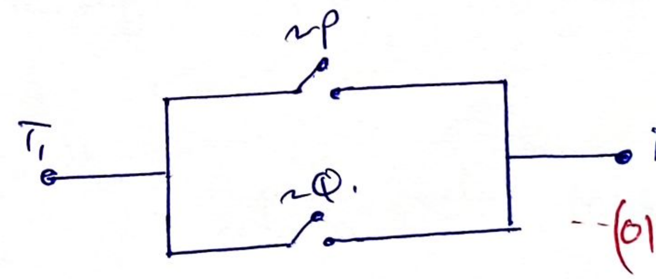
$(p \wedge \sim q) \vee [\sim p \wedge T]$ ..... Negation law

$(p \wedge \sim q) \vee \sim p$ --- Identity law

$(p \vee \sim p) \wedge (\sim q \vee \sim p)$ --- Distributive law (02 Marks)

$T \wedge (\sim q \vee \sim p)$ -- Negation law

$\sim p \vee \sim q$ --- Identity and commutative laws.



--(01 Marks)

d) let

$p \equiv$ I will get an A

$q \equiv$ I will graduate

$r \equiv$ I will go to my army

The argument is;

$(p \vee \sim q), (\sim p \rightarrow r), p \vdash \sim r.$

$$[(p \vee \sim q) \wedge (\sim q \rightarrow r) \wedge p] \rightarrow \sim r \quad \text{------} (00\tfrac{1}{2})$$

Check validity by using truth table.

p	q	r	~q	~r	A $p \vee \sim q$	B $\sim q \rightarrow r$	C $A \wedge B$	D $C \wedge p$	$D \rightarrow \sim r$
T	T	T	F	F	T	T	T	T	F
T	T	F	F	T	T	T	T	T	T
T	F	T	T	F	T	T	T	T	F
T	F	F	T	T	T	F	F	F	T
F	T	T	F	F	F	T	F	F	T
F	T	F	F	T	F	T	F	F	T
F	F	T	T	F	T	T	T	F	T
F	F	F	T	T	T	F	F	F	T
					$(00\tfrac{1}{2})$	$(00\tfrac{1}{2})$	$(00\tfrac{1}{2})$	$(00\tfrac{1}{2})$	$(00\tfrac{1}{2})$

Since it is not tautology then the argument is not valid. (01 marks).

3 a) let

$$\underline{a} = 2\underline{i} + \underline{j} - \underline{k}$$

$$\underline{b} = 3\underline{i} - \underline{j} - 2\underline{k}$$

Suppose that vector $\underline{c}$ is perpendicular to both $\underline{a}$ and $\underline{b}$. then

$$\underline{c} = \underline{a} \times \underline{b} = \begin{vmatrix} i & j & k \\ 2 & 1 & -1 \\ 3 & -1 & -2 \end{vmatrix}$$

$$\underline{c} = -3\underline{i} + \underline{j} - 5\underline{k}. \quad \text{——} (00\tfrac{1}{2} \text{ Marks})$$

$$|\underline{c}| = \sqrt{35}$$

The direction ratios are $-3 : 1 : -5$ --(co½ Marks)

The direction cosine are $\frac{-3}{\sqrt{35}}, \frac{1}{\sqrt{35}}, \frac{-5}{\sqrt{35}}$.

@ co½ = 1½ Marks.

The direction angles are;

$$\cos^{-1}\left(\frac{-3}{\sqrt{35}}\right) = 120.5^\circ$$ ---(co½ Marks)

$$\cos^{-1}\left(\frac{1}{\sqrt{35}}\right) = 80.3^\circ$$ --- (co½ Marks)

$$\cos^{-1}\left(\frac{-5}{\sqrt{35}}\right) = 147^\circ.7^\circ$$ --- (co½ Marks.)

b) From the given triangle,

$$\vec{a} + \vec{b} + \vec{c} = 0 \quad \text{---} (1)$$

Cross by $\vec{a}$ both side.

$$\vec{a}(\vec{a} + \vec{b} + \vec{c}) = \vec{a} \times 0$$

$$\vec{a} \times \vec{a} + \vec{a} \times \vec{b} + \vec{a} \times \vec{c} = 0$$

$$\vec{a} \times \vec{b} = -\vec{a} \times \vec{c}$$

$$|\vec{a} \times \vec{b}| = |\vec{a} \times \vec{c}| \quad \text{---} (ii)$$ - (0 marks)

Also

$$\vec{b} \times (\vec{a} + \vec{b} + \vec{c}) = \vec{b} \times 0$$

$$\vec{b} \times \vec{a} + \vec{b} \times \vec{b} + \vec{b} \times \vec{c} = 0$$

$$\vec{b} \times \vec{a} = -\vec{b} \times \vec{c}$$

$$|\vec{b} \times \vec{a}| = |\vec{b} \times \vec{c}|.$$

$$|\vec{a} \times \vec{b}| = |\vec{b} \times \vec{c}|; \quad \text{---} (iii)$$ --- (0 marks)

Comparing (ii) and (iii)

$$|\vec{a}\times\vec{b}| = |\vec{a}\times\vec{c}| = |\vec{b}\times\vec{c}|.$$

$$|\vec{a}||\vec{b}|\sin C = |\vec{a}||\vec{c}|\sin B = |\vec{b}||\vec{c}|\sin A.$$

Dividing by $|\vec{a}||\vec{b}||\vec{c}|$ both side,

$$\therefore \frac{\sin C}{|\vec{c}|} = \frac{\sin B}{|\vec{b}|} = \frac{\sin A}{|\vec{a}|}. \quad \text{--- (01 marks)}$$

c) $$|\vec{a}-\vec{b}|^2 = (\vec{a}-\vec{b})\cdot(\vec{a}-\vec{b}).$$

$$= \vec{a}^2 - 2\vec{a}\vec{b} + \vec{b}^2 \quad \text{--- (01 Marks)}$$

$$= |\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}|\cos\theta$$

$$= 1+1-2\cos\theta \quad \text{---- (01 marks)}$$

$$= 2-2\cos\theta$$

$$= 2(1-\cos\theta)$$

$$= 2(2\sin^2\tfrac{\theta}{2}) \quad \text{---- (01 marks)}$$

$$|\vec{a}-\vec{b}|^2 = 4\sin^2\theta/2$$

$$\therefore |\vec{a}-\vec{b}| = 2\sin\frac{\theta}{2}. \quad \text{---- (01 Marks)}$$

d) Given.

$$V = 3t^2\underline{i} - \underline{j} + \underline{k}.$$

$$\frac{ds}{dt} = 3t^2\underline{i} - 2t\underline{j} + 4\underline{k}.$$

$$\int ds = \int (3t^2\underline{i} - 2t\underline{j} + 4\underline{k})\,dt.$$

$$\underline{s} = t^3\underline{i} - t^2\underline{j} + 4t\underline{k} + \underline{c} \quad \text{-- (01)}$$

when $t=1$ $\underline{s} = 3\underline{i} - \underline{j} + 2\underline{k}$.

$$3\underline{i} - \underline{j} + 2\underline{k} = \underline{i} - \underline{j} + 4\underline{k} + \underline{c}$$

$$\underline{c} = 2\underline{i} - 2\underline{k}. \quad \text{--- (02 marks)}$$

$$\underline{s} = t^3\underline{i} - t^2\underline{j} + 4t\underline{k} + 2\underline{i} - 2\underline{k}.$$

$$\underline{s} = (t^3+2)\underline{i} - t^2\underline{j} + (4t-2)\underline{k}. \quad \text{--- (01 marks)}$$

4 a) i) The other roots are $2+i$ and $1-3i$ (01 marks).

ii) The sum of the four roots of the equation

$$= (2-i) + (2+i) + (1-3i) + (1-3i) = 6. \quad \text{(01 marks)}$$

iii) The product of four roots of the equation

$$= (2-i)(2+i)(1+3i)(1-3i) = 50. \quad \text{(01 marks)}$$

(iv) Consider $(2-i)$ and $(2+i)$

$$(x-(2-i))(x-(2+i)) = 0$$

$$x^2 - 4x + 5 = 0 \quad \text{---- } 01/2$$

also

$$(x-(1+3i))(x-(1-3i)) = 0$$

$$x^2 - 2x + 10 = 0 \quad \text{---- } 01/2$$

Then,

$$(x^2-4x+5)(x-2x+10) = 0$$

$$x^4 - 2x^3 + 10x^2 - 4x^3 + 8x^2 - 40x + 5x^2 - 10x + 50 = 0$$

$$x^4 - 6x^3 + 23x^2 - 50x + 50 = 0 \quad \text{---- } 01$$

b) $(1+\cos\theta+i\sin\theta)^n = 2^n \cos^n \frac{1}{2}\theta \left(\cos\frac{n\theta}{2}+i\sin\frac{n\theta}{2}\right)$.

Consider L.H.S.

$$L.H.S = (1+\cos\theta+i\sin\theta)^n.$$

let

$$z = 1+\cos\theta+i\sin\theta$$

$$|z| = \sqrt{(1+\cos\theta)^2+\sin^2\theta}$$

$$|z| = \sqrt{1+2\cos\theta+\cos^2\theta+\sin^2\theta} \quad \text{---(01 marks)}$$

$$|z| = \sqrt{1+2\cos\theta+1}$$

$$= \sqrt{2+2\cos\theta}$$

$$= \sqrt{2(1+\cos\theta)}$$

$$= \sqrt{2(2\cos^2\theta/2)}$$

$$\therefore r = 2\cos\tfrac{1}{2}\theta. \quad \text{--- 01 marks}$$

$$\operatorname{Arg}(1+\cos\theta+i\sin\theta) = \tan^{-1}\left(\frac{\sin\theta}{1+\cos\theta}\right)$$

$$= \tan^{-1}\left(\frac{2\sin\theta/2\cos\theta/2}{2\cos^2\theta/2}\right).$$

$$= \tan^{-1}(\tan\theta/2)$$

$$= \frac{\theta}{2}. \quad \text{--- 01 Marks}$$

Recall that

$$z = r(\cos\theta+i\sin\theta)$$

$$z = 2\cos\tfrac{1}{2}\theta\left(\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}\right) \quad \text{--- 01 Marks}$$

$$z^n = 2^n\cos^n\tfrac{1}{2}\theta\left[\cos\tfrac{\theta}{2} + i\sin\frac{\theta}{2}\right]^n.$$

$$= 2^n\cos^n\tfrac{1}{2}\theta\left(\cos\frac{n\theta}{2} + i\sin\frac{n\theta}{2}\right) \quad \text{--- 01 marks.}$$

$\therefore$ L.H.S = R.H.S. Proved.

c) Given

$$z = x + iy, \quad x^2 + y^2 = 4$$

$$|z| = 2.$$

$$w = \frac{z+1}{z-1}$$

$$z = \frac{w+1}{w-1} \quad \text{--- 00 1/2}$$

$$|z| = 2$$

$$\left|\frac{w+1}{w-1}\right| = 2.$$

$$|w+1| = 2|w-1| \quad \text{--- 00 1/2}$$

$$|u+1+vi| = 2|u-1+vi|.$$

$$(u+1)^2 + v^2 = 4\left((u-1)^2 + v^2\right) \quad \text{--- 01}$$

$$3u^2 + 3v^2 - 10u + 3 = 0$$

$\therefore$ $x^2+y^2=4$ maps on another circle with equation $3u^2+3v^2-10u+3=0$. --- 01

We have to find centre and radius.

$$u^2+v^2-\frac{10}{3}u+1=0$$

Centre will be $C\left(\frac{5}{3}, 0\right)$. ---- 01 marks

and radius, $r=\frac{4}{3}$. ------ 01 Marks

5 (a) $\tan^{-1}(x+1)=3\tan^{-1}(x-1)$.

let $A=\tan^{-1}(x+1) \Rightarrow x+1=\tan A$

$B=\tan^{-1}(x-1) \Rightarrow x-1=\tan B$. (01 marks)

Then

$$A=3B$$

Apply tan both side.

$$\tan A=\tan 3B.$$

$$\tan A=\frac{3\tan B-\tan^3 B}{1-3\tan^2 B}.$$ ---- (01 Marks)

$$x+1=\frac{3(x-1)-(x-1)^3}{1-3(x-1)^2}$$

$$=\frac{3x-3-x^3+3x^2-3x+1}{1-3x^2+6x-3}$$ ---- (01 Marks)

$$x+1=\frac{-x^3+3x^2-2}{-3x^2+6x-2}$$

$(x+1)(-3x^2+6x-2)=-x^3+3x^2-2$. ---- (01 Marks)

$-3x^3+6x^2-2x-3x^2+6x-2=-x^3+3x^2-2$

$$-3x^3+3x^2+4x-2=-x^3+3x^2-2.$$

$$2x^3+4x-2=0$$

$$x=0 \text{ and } x=\sqrt{2}. \text{ ---- } (0|\text{marks}).$$

b) We have

$$\sin\theta = 4/5.$$

$$\cos\theta = \sqrt{1-\sin^2\theta}$$

$$= \sqrt{1-\left(\tfrac{4}{5}\right)^2} = 3/5 \text{ --- } 00\tfrac{1}{2}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{4/5}{3/5} = 4/3 \text{ --- } 00\tfrac{1}{2}$$

$$\cot\theta = 1/\tan\theta = 3/4. \text{ --- } 00\tfrac{1}{2}$$

$$\mathrm{cosec}\,\theta = 1/\sin\theta = 5/4. \text{ --- } 00\tfrac{1}{2}$$

$$\sec\theta = 1/\cos\theta = 5/3. \text{ --- } 00\tfrac{1}{2}$$

$$\therefore \frac{5\cos\theta+4\,\mathrm{cosec}\,\theta+3\tan\theta}{4\cot\theta+3\sec\theta+5\sin\theta} = \frac{5(3/5)+4(5/4)+3(4/3)}{4(3/4)+3(5/3)+5(4/5)} \text{ - } (0\tfrac{1}{2}\text{ Mark})$$

$$= \frac{3+5+4}{3+5+4} = 1.$$

$$\therefore \frac{5\cos\theta+4\,\mathrm{cosec}\,\theta+3\tan\theta}{4\cot\theta+3\sec\theta+5\sin\theta} = 1. \text{ -- } (0|\text{Marks})$$

c) Given

$$10\sin\frac{\pi x}{3} + 24\cos\frac{\pi x}{3} = 13.$$

$$10\sin\frac{\pi x}{3} + 24\cos\frac{\pi x}{3} = R\cos\left(\frac{\pi x}{3} - \alpha\right). \quad \text{--- } 00\tfrac{1}{2}$$

$$10\sin\frac{\pi}{3}x + 24\cos\frac{\pi}{3}x = R\cos\frac{\pi x}{3}\cos\alpha + R\sin\frac{\pi x}{3}\sin\alpha. \quad \text{--- } 00\tfrac{1}{2}$$

By comparing

$$10\sin\frac{\pi x}{3} = R\sin\frac{\pi x}{3}\sin\alpha \Rightarrow \sin\alpha = \frac{10}{R} \text{ --- (I) } \quad 00\tfrac{1}{2}$$

$$24\cos\frac{\pi x}{3} = R\cos\frac{\pi x}{3}\cos\alpha \Rightarrow \cos\alpha = \frac{24}{R} \text{ --- (II) } \quad 00\tfrac{1}{2}$$

From

$$\cos^2\alpha + \sin^2\alpha = 1$$

$$\left(\frac{24}{R}\right)^2 + \left(\frac{10}{R}\right)^2 = 1$$

$$R = 26. \text{ ------ } 00\tfrac{1}{2}$$

Also

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{10}{24} = \frac{5}{12}.$$

$$\alpha = \tan^{-1}\left(\frac{5}{12}\right) \text{ --- } 00\tfrac{1}{2}$$

Recall

$$R\cos\left(\frac{\pi x}{3} - \alpha\right) = 13.$$

$$26\cos\left(\frac{\pi x}{3} - \alpha\right) = 13$$

$$\cos\left(\frac{\pi x}{3} - \alpha\right) = \frac{1}{2}. \text{ --- } 00\tfrac{1}{2}$$

$$\frac{\pi x}{3} - \alpha = 2\pi n \pm \cos^{-1}(1/2).$$

$$= 2\pi n \pm \pi/3 \quad \text{---- } 00 1/2$$

$$\frac{\pi x}{3} = 2\pi n \pm \pi/3 + \alpha$$

but $\alpha = \tan^{-1}(5/12)$.

$$\frac{\pi x}{3} = 2\pi n \pm \pi/3 + \tan^{-1}(5/12) \quad \text{---- } 00 1/2$$

$$x = \frac{3}{\pi}\left(2\pi n \pm \frac{\pi}{3} + \tan^{-1}(5/12)\right).$$

$$\therefore x = (6n \pm 1) + \frac{3}{\pi}\tan^{-1}(5/12) \quad \text{---- } 00 1/2$$

d) i) Given

$$\frac{\sin\theta + \sin 2\theta}{1 + \cos\theta + \cos 2\theta} = \tan\theta.$$

Consider L.H.S

$$\frac{\sin\theta + \sin 2\theta}{1 + \cos\theta + \cos 2\theta} = \frac{\sin\theta + 2\sin\theta\cos\theta}{1 + \cos\theta + \cos^2\theta - \sin^2\theta} \quad \text{-- 01 marks}$$

$$= \frac{\sin\theta(1 + 2\cos\theta)}{\cos\theta + \cos^2\theta + 1 - \sin^2\theta}$$

$$= \frac{\sin\theta(1 + 2\cos\theta)}{\cos\theta + \cos^2\theta + \cos^2\theta} \quad \text{---- 01 marks}$$

$$= \frac{\sin\theta(1 + 2\cos\theta)}{\cos\theta(1 + 2\cos\theta)}$$

$$= \frac{\sin\theta}{\cos\theta} = \tan\theta \quad \text{--- 01 marks}$$

ii) $X = \sin^2 2\theta$

$X = (\sin 2\theta)^2$

$X = (2\sin\theta\cos\theta)^2$ - - 01/2

$X = 4\sin^2\theta\cos^2\theta$

$X = 4(1-\cos^2\theta)\cos^2\theta$ - - 01/2

$X = 4(1-y^2)y^2$

$\therefore X = 4y^2(1-y^2)$ - - - - 01

6 a) $2X^2+6X+9 = 2(x^2+3x)+9$

$= 2(x^2+3x+\frac{9}{4})+9-\frac{9}{2}$

$= 2(x+\frac{3}{2})^2+\frac{9}{2}$ - - 01 Marks.

Now

$(x+\frac{3}{2})^2 \geq 0$ - - 01/2

$2(x+\frac{3}{2})+\frac{9}{2} > 0$ - - - 01/2

so $2X^2+6X+9$ is always positive, - - - 01/2

$\frac{(X+1)(x+3)}{X} > \frac{X+6}{3}$

$\frac{(X+1)(x+3)}{X} - \frac{X+6}{3} > 0$ - - - - - 01/2

$\frac{3X^2+12X+9-X^2-6X}{3X} > 0$

$\frac{2X^2+6X+9}{3X} > 0$ —— 01/2

01

but $2X^2+6X+9 > 0$

$3x > 0$

$x > 0$ ------ (01 marks)

b) Req

$$\sum_{k=n+1}^{2n} k^3 = \frac{n^2}{4}(3n+1)(5n+3).$$

From

$$\sum_{k=1}^{n} k^3 = \frac{n^2}{4}(n+1)^2. \text{------ 01}$$

$$\sum_{k=n+1}^{2n} k^3 = \sum_{k=1}^{2n} k^3 - \sum_{k=1}^{n} k^3 \text{ ---- 01}$$

$$= \frac{(2n)^2}{4}(2n+1)^2 - \frac{n^2}{4}(n+1)^2.$$

$$= \frac{4n^2}{4}(4n^2+4n+1) - \frac{n^2}{4}(n^2+2n+1). \text{ ---01}$$

$$= \frac{n^2}{4}(16n^2+16n+4-n^2-2n-1).$$

$$= \frac{n^2}{4}(15n^2+14n+3) \text{ ---- 01}$$

$$= \frac{n^2}{4}(3n+1)(5n+3).$$

$$\therefore \sum_{k=n+1}^{2n} k^3 = \frac{n^4}{4}(3n+1)(5n+3). \text{ --- 01}$$

c) $2x^2+8x+1=0$

$\Rightarrow \quad x^2+4x+\frac{1}{2}=0$.

$\Rightarrow \quad \alpha+\beta=-4 \quad$ and $\alpha\beta=\frac{1}{2}$ --- (01 Marks)

New equation:

Sum of roots $= 2\alpha^4+\frac{1}{\beta^2}+2\beta^4+\frac{1}{\alpha^2}$

$= 2(\alpha^4+\beta^4)+\frac{1}{\beta^2}+\frac{1}{\alpha^2}$

$= 2\left((\alpha^2+\beta^2)^2-2\alpha^2\beta^2\right)+\frac{\alpha^2+\beta^2}{\alpha^2\beta^2}$ --- (01 Marks)

$= 2\left(((\alpha+\beta)^2-2\alpha\beta)^2-2(\alpha\beta)^2\right)+\frac{(\alpha+\beta)^2-2\alpha\beta}{(\alpha\beta)^2}$

$= 2\left(((-4)^2-2(\frac{1}{2}))^2-2(\frac{1}{2})^2\right)+\frac{(-4)^2-2(\frac{1}{2})}{(\frac{1}{2})^2}$

$= 2\left(225-\frac{1}{2}\right)+15\times 4.$

$= 2\left(\frac{449}{2}\right)+60$

$= 509.$ ---- (00½ Marks)

Product of roots $= \left(2\alpha^4+\frac{1}{\beta^2}\right)\left(2\beta^4+\frac{1}{\alpha^2}\right)$.

$= 4\alpha^4\beta^4+2\alpha^2+2\beta^2+\frac{1}{\alpha^2\beta^2}$ - (01 Marks)

$\cancel{=4(\alpha\beta)^4+2[(-4)^2-2(}$

$$= 4(\alpha\beta)^4 + 2\left[(\alpha+\beta)^2 - 2\alpha\beta\right] + \frac{1}{(\alpha\beta)^2}$$

$$= 4\left(\tfrac{1}{2}\right)^4 + 2\left[(-4)^2 - 2\left(\tfrac{1}{2}\right)\right] + \frac{1}{\left(\tfrac{1}{2}\right)^2}$$

$$= \frac{137}{4}$$ ------- (01/2 marks)

The equation is $x^2 - 509x + \frac{137}{4} = 0$. (01 Marks)

d) $$\frac{6x^4 - 4x^3 - 2x^2 + 27x + 7}{3x - 4}$$

$$= \frac{6x^4 - 4x^3 - 2x^2 + 27x + 7}{3(x - 4/3)}.$$ --- (01/2 Marks)

Consider

$$\frac{6x^4 - 4x^3 - 2x^2 + 27x + 7}{x - 4/3}$$

$$x - 4/3 = 0 \Rightarrow x = 4/3.$$

$$\begin{array}{c|ccccc}
4/3 & 6 & -4 & -2 & 27 & 7 \\
+ & 0 & 8 & 16/3 & \frac{40}{9} & \frac{1132}{27} \\
\hline
 & 6 & 4 & 10/3 & \frac{283}{9} & \frac{1321}{7}
\end{array}$$ -- (01 Marks)

$$\Rightarrow \frac{6x^4 - 4x^3 - 2x^2 + 27x + 7}{x - 4/3} = 6x^3 + 4x^2 + \frac{10}{3}x + \frac{283}{9} + \frac{\frac{1321}{27}}{x - 4/3}$$ ·01/2

$$\Rightarrow \frac{6x^4-4x^3-2x^2+27x+7}{3(x-4/3)} = \frac{6x^3+4x^2+16/3x+\frac{283}{9}}{3} + \frac{\frac{1321}{27}}{3(x-4/3)}$$

$$\Rightarrow \frac{6x^4-4x^3-2x^2+27x+7}{3x-4} = 2x^3+\frac{4}{3}x^2+\frac{16}{9}x+\frac{283}{27} + \frac{\frac{1321}{27}}{3x-4}.$$ -- (01 Marks)

ii) $\left(2x^2-\frac{1}{2x}\right)^6$.

from $T_{r+1} = {}^nC_r\, a^{n-r}\, b^r$.

$T_{r+1} = {}^6C_r\,(2x^2)^{6-r}\left(-\frac{1}{2x}\right)^r$ ---- 00½

$= {}^6C_r\, 2^{6-2r} \times (-1)^r \times x^{12-3r}$ --- 00½

for constant term the coefficient of x is zero.

$12-3r=0$

$r=4.$ ---- 00½

Constant term $= {}^6C_4\, 2^{-2} \times (-1)^4$

$= 15/4.$ --- 00½

7 a) let $y = A\cos nx + B\sin nx$. --- ①

$\frac{dy}{dx} = -An\sin nx + Bn\cos nx$ --- (02) --- 00½

$\frac{d^2y}{dx^2} = -An^2\cos nx - Bn^2\sin nx$

$\frac{d^2y}{dx^2} = -n^2(A\cos nx + B\sin nx).$ --- 00½

$$\frac{d^2y}{dx^2} = -n^2y.$$

$$\frac{d^2y}{dx^2} + n^2y = 0 \text{ ---- } 00\tfrac{1}{2}$$

Required the solution when $n = 2$.

$$\frac{d^2y}{dx^2} + 4y = 0$$

Its A.Q.E

$$m^2 + 4 = 0$$

$$m = \pm 2i \text{ ------ } 01$$

From

$$y = e^{ax}(A\cos\beta x + B\sin\beta x).$$

Given $y = A\cos 2x + B\sin 2x$. --- $00\tfrac{1}{2}$

when

$x = 0$, $y = 5$ and $\frac{dy}{dx} = 8$.

$$5 = A\cos 2(0) + B\sin(2(0)).$$

$$A = 5. \text{ ---- } 00\tfrac{1}{2}$$

$$\frac{dy}{dx} = -2A\sin 2x + 2B\cos 2x. \text{ --- } 00\tfrac{1}{2}$$

$$8 = -2A\sin 0 + 2B\cos 0.$$

$$B = 4. \text{ ---- } 00\tfrac{1}{2}$$

$$\therefore y = 5\cos 2x + 4\sin 2x. \text{ --- } 00\tfrac{1}{2}$$

b) $x\,dy - y\,dx = \sqrt{x^2-y^2}\,dx.$

$$\frac{dy}{dx} = \frac{\sqrt{x^2+y^2}+y}{x}.$$

let $x = L,\ y = L.$

$$\frac{dy}{dx} = \frac{\sqrt{L^2+L^2}+L}{L} = \frac{\sqrt{2}L+L}{L}. \quad \text{--- } 01$$

$\therefore$ The given differential is homogeneous $\left(01/2\right)$

then

Let

$$y = ux.$$

$$\frac{dy}{dx} = u + x\frac{du}{dx}. \quad \text{--- } 01/2$$

$$u + x\frac{du}{dx} = \frac{\sqrt{\cancel{x^2}+u^2x^2}+ux}{x}$$

$$u + x\frac{du}{dx} = \sqrt{1+u^2}+u.$$

$$x\frac{du}{dx} = \sqrt{1+u^2} \quad \text{--- } 01$$

$$\frac{du}{\sqrt{1+u^2}} = \frac{dx}{x}$$

$$\int\frac{du}{\sqrt{1+u^2}} = \int\frac{dx}{x}.$$

Use substitution $\cos^2 A = 1 + \sinh^2 A.$

$$\ln\left(u+\sqrt{1+u^2}\right) = \ln x + \ln c \quad \text{--- } 01$$

$$\ln\left(u+\sqrt{1+u^2}\right)=\ln(cx).$$

$$\ln\left(\frac{y}{x}+\sqrt{1+\frac{y^2}{x^2}}\right)=\ln(cx).$$

$$\frac{y}{x}+\frac{\sqrt{x^2+y^2}}{x}=cx.$$

$$\therefore\ y+\sqrt{x^2+y^2}=cx^2. \quad --- 01$$

c) Given.

$$y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=y\frac{dy}{dx}$$

let

$$p=\frac{dy}{dx} \quad -- \text{(I)} \quad --- 00\tfrac{1}{2}$$

$$\frac{dp}{dx}=\frac{d^2y}{dx^2} \quad -- \text{(II)} \quad --- 00\tfrac{1}{2}$$

Substitute (I) and (ii) into the given d.e

$$y\frac{dp}{dx}+p^2=yp. \quad ---- 00\tfrac{1}{2}$$

but

$$\frac{dp}{dx}=\frac{dy}{dx}\times\frac{dp}{dy} \Rightarrow \frac{dp}{dx}=p\frac{dp}{dy} \quad --- 00\tfrac{1}{2}$$

$$\Rightarrow\ yp\frac{dp}{dy}+p^2=yp.$$

$$y\frac{dp}{dy}+p=y. \quad ----- 01$$

$$\frac{d}{dy}(Py) = y.$$

$$\int d(Py) = \int y\,dy$$

$$Py = \frac{1}{2}y^2 + C_1 \quad \text{-----} \; 01$$

$$P = \frac{y^2 + 2C_1}{2y}$$

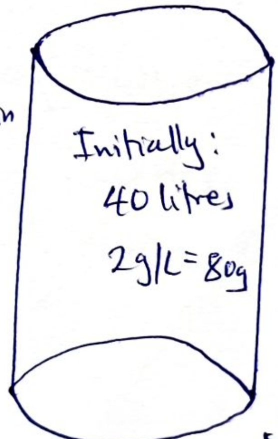
$$\frac{dy}{dx} = \frac{y^2 + 2C_1}{2y}$$

$$\int \frac{2y}{y^2 + 2C_1}\,dy = \int dx.$$

$$\therefore \ln(y^2 + 2C_1) = x + C_2. \quad \text{-----} \; 01$$

d) Conc = 3g / Litre

Rate in = 4 liter/min



Rate out = 4L/min.

1) $$\frac{dA}{dt} = 3 \times 4 - \left(\frac{A}{40 + 4t - 4t}\right) \times 4. \quad \text{-----} \; 00\tfrac{1}{2}$$

$$\frac{dA}{dt} = 12 - \frac{A}{10}.$$

$\therefore$ The d.e is $\frac{dA}{dt} + \frac{1}{10}A = 12.$ ----- $00\tfrac{1}{2}$

Initial conditions: $A = 80g$ when $t = 0$. (01/2 Marks)

ii) $$\frac{dA}{dt} + \frac{1}{10}A = 12$$

$$I.F = e^{\int \frac{1}{10} dt} = e^{\frac{1}{10}t} \quad \text{---- } 01/2$$

$$e^{\frac{1}{10}t}\frac{dA}{dt} + \frac{1}{10}Ae^{\frac{1}{10}t} = 12e^{\frac{1}{10}t} \quad \text{--- } 01/2$$

$$\int \frac{d(Ae^{\frac{1}{10}t})}{dt} = \int 12e^{\frac{1}{10}t}\,dt.$$

$$Ae^{\frac{1}{10}t} = 120e^{\frac{1}{10}t} + c$$

$$A(t) = 120 + ce^{-\frac{1}{10}t} \quad \text{--- } 01/2$$

but $A = 80g$ when $t = 0$.

$$80 = 120 + c$$

$$c = -40 \quad \text{--- } 01/2$$

Therefore

$$A(t) = 120 - 40e^{-\frac{1}{10}t} \quad \text{--- } 01$$

iii) when $t = 15$ min.

$$A = 120 - 40e^{-\frac{1}{10}\times 15}$$

$$A = 111.075\,kg. \quad \text{--- } 01$$

a) Given

$$4y^2 + 12x - 20y + 67 = 0$$

$$y^2 + 3x - 5y + \frac{67}{4} = 0.$$

$$y^2 - 5y = -3x - \frac{67}{4}.$$

$$\left(y - \frac{5}{2}\right)^2 = -3x - \frac{21}{2}.$$

$$\left(y - \frac{5}{2}\right)^2 = -3\left(x + \frac{7}{2}\right) \quad \text{--- } 0\tfrac{1}{2}$$

Translation factor, $= \left(-\frac{7}{2}, \frac{5}{2}\right)$ --- $0\frac{1}{2}$

Vertex $= \left(-\frac{7}{2}, \frac{5}{2}\right)$ ---- $0\frac{1}{2}$

Axis of symmetry; $y = \frac{5}{2}$ ---- $0\frac{1}{2}$

Focus $= \left(-\frac{3}{4}, 0\right) + \left(-\frac{7}{2}, \frac{5}{2}\right)$

Focus $= \left(-\frac{17}{4}, \frac{5}{2}\right)$ ---- 01

Directrix; $x = -\frac{11}{4}$. ---- 01

Latus rectum $= 4\left(\frac{3}{4}\right)$

$= 3$ units. ---- 01

b) We have $5x^2+3y^2=137$ and $y=2$.

$$5(x^2)+3(2)^2=137.$$

$$5x^2=125.$$

$$x=\pm 5.$$

in the first quadrant the point is $(5,2)$
(01 marks)

From

$$5x^2+3y^2=137$$

$$10x+6y\frac{dy}{dx}=0 \quad \text{--- (01 marks)}.$$

$$\frac{dy}{dx}=\frac{-10x}{6y}$$

$$\frac{dy}{dx}=\frac{-10(5)}{6(2)}.$$

$$\text{Slope}=-\frac{25}{6} \quad \text{----- (01 marks)}$$

It's equation is

$$y=-\frac{25}{6}(x-x_0)+y_0$$

$$y=-\frac{25}{6}(x-5)+2.$$

$$\therefore 25x+6y-137=0 \quad \text{--- (01 marks)}$$

Normal equation;

from

$$M_1M_2=-1.$$

$$\text{Slope of normal}=\frac{6}{25}$$

The equation will be.

$$y = \frac{6}{25}(x-5)+2.$$

$\therefore$ $6x-25y+20=0$. --- (01 Marks)

c) We have

$$x^2-y^2=a^2$$

let the equation of line be $y=mx+c$

Hence

$$x^2-(mx+c)^2=a^2$$

$$x^2-m^2x^2-2mcx-c^2=a^2 \text{ --- (01 marks)}$$

$$(1-m^2)x-2mcx-(c^2+a^2)=0.$$

For tangency.

$$b^2=4ac$$

$$(-2mc)^2=4(1-m^2)(-(a^2+c^2)) \text{ --- (01 Marks)}$$

$$4m^2c^2=-4(a^2+c^2-a^2m^2-c^2m^2).$$

$$m^2c^2=-a^2-c^2+a^2m^2+c^2m^2.$$

$$0=-a^2-c^2+a^2m^2 \text{ --- (01 Marks)}$$

$$c^2=a^2m^2-a^2.$$

$$c^2=a^2(m^2-1).$$

$$c^2=\pm a\sqrt{(m^2-1)} \text{ --- (01 marks)}$$

From

$$y=mx+c.$$

$$\therefore y=mx\pm a\sqrt{m^2-1}. \text{ --- (01 Marks)}$$

d) i) We have.

$$r^2 = 4\sin 2\theta$$
$$= 4(2\sin\theta\cos\theta).$$
$$= 8\sin\theta\cos\theta. \quad \text{--- (00½ mark)}$$

but

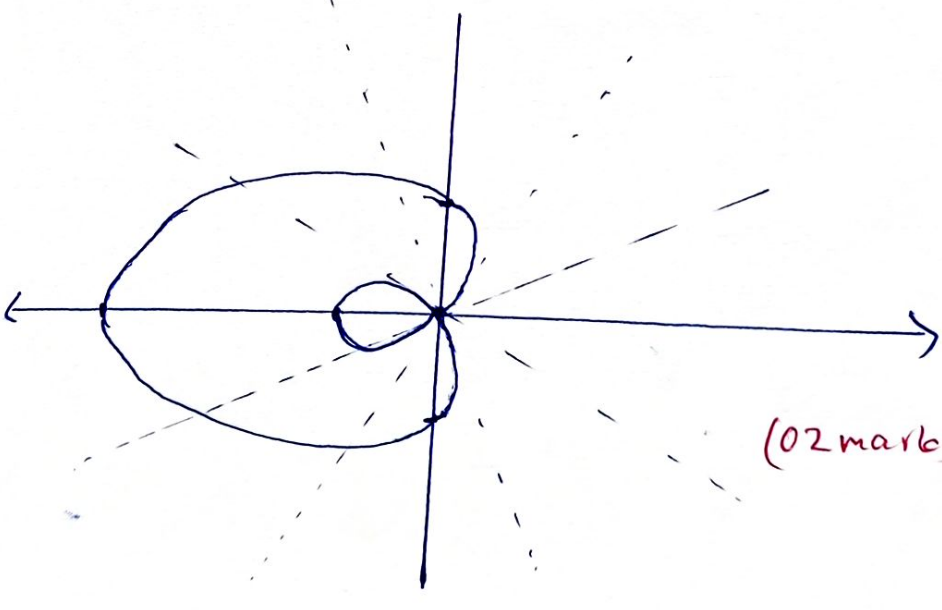
$\sin\theta = y/r$, $\cos\theta = x/r$ and $r^2 = x^2+y^2$

$$r^2 = 8\left(\frac{y}{r}\right)\left(\frac{x}{r}\right) \quad \text{--- (00½)}$$
$$r^4 = 8xy$$
$$(r^2)^2 = 8xy.$$
$$(x^2+y^2)^2 = 8xy. \quad \text{--- (01 marks)}$$

iii) $r = 1 - 2\cos\theta.$

θ	$\pi/4$	$\pi/2$	$\frac{3}{4}\pi$	π	$5/4\pi$	$3/2\pi$	$7/4\pi$	2π
r	-0.41	1	2.41	3	2.41	1	-0.41	-1.

(01 mark)



(02 mark)