



TANZANIA HEADS OF ISLAMIC SCHOOLS COUNCIL
FORM SIX INTER ISLAMIC MOCK EXAMINATION

ADVANCED MATHEMATICS 2

142/2

(For both School and Private Candidates)

TIME: 3 HOURS

Monday, 11th March 2024 p.m.

Instructions

1. This paper consists of sections A and B with a total of eight (8) questions.
2. Answer all questions in section A and two (2) questions from section B.
3. All work done in answering each question must be shown clearly.
4. Mathematical tables and non-programmable calculators may be used.
5. Cellular phone and any unauthorized materials are not allowed in the examination room.
6. Write your Examination Number on every page of your answer booklet(s).

1. (a) A team of 4 players is chosen at random from 5 girls and 6 boys. In how many ways can the team be chosen if:
 - (i) There are no restrictions
 - (ii) There must be more boys than girls.
 - (iii) Find the probability that the team contains only one boy.
- (b) (i) A factory packs bolts in boxes of 500. The probability that a bolt is defective is 0.002. Find the probability that a box contains 2 defective bolts.
- (ii) Students of a certain class were given an aptitude test. Their marks were found to be normally distributed with mean 60 and standard deviation 5. What percentage of students scored more than 60 marks?
- (c) A continuous random variable X has p.d.f $f(x)$ defined as:

$$f(x) = \begin{cases} k(x+2)^2 & : -2 \leq x < 0 \\ 4k & : 0 \leq x \leq 1\frac{1}{3} \\ 0 & : \text{else where} \end{cases}$$

- (i) Find the value of the constant k.
 - (ii) Find $p(|x| \leq 1)$
 - (iii) The variance of the random variable x.
2. (a) Assume p is false, q is true and r is false. What is the truth value of the statements?
 - (i) $p \wedge (q \vee 2r)$
 - (ii) $(2p \wedge q) \vee (2q)$
 - (b) (i) Write the statement “You can go to the movies when your work is done” in the form of “If, then”.
 - (ii) Using the “If ..., then” form. Write the converse, inverse and the contrapositive of the statement in 2(b)(i).
 - (c) Draw a simplified network that can perform same functions as the network if figure 1.

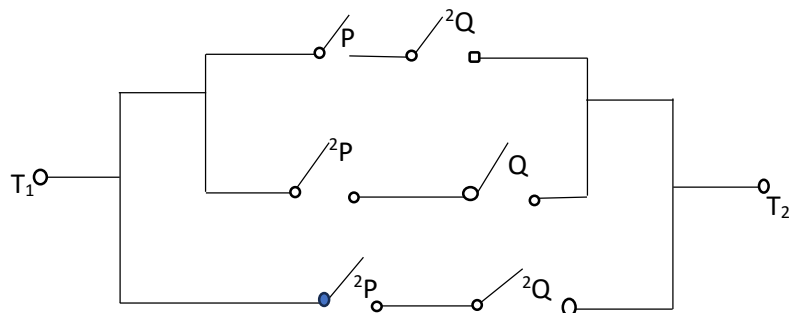
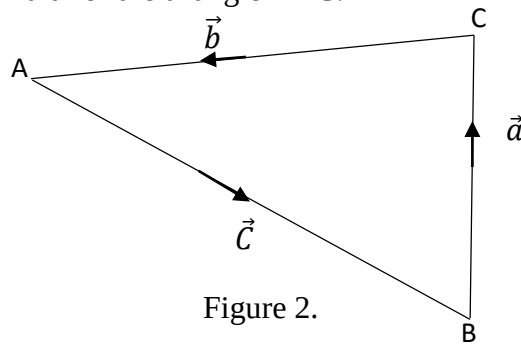


Figure 1

- (d) Use the truth table to test the validity of the Argument; I will either get A in this course or I will not graduate. If I don't graduate, I will go to my army. I got an A. Thus I will not go to the army.
3. (a) Find the direction ratios, direction Cosines and direction angles of the vector that is perpendicular to both $2\mathbf{i} + \mathbf{j} - \mathbf{k}$ and $3\mathbf{i} - \mathbf{j} - 2\mathbf{k}$.
- (b) Referring to figure 2, use the idea of cross product of vectors to derive the sine rule formula for the triangle ABC.



- (c) If $\vec{a}$ and $\vec{b}$ are unit vectors inclined at an angle of θ to each other, show that
- $$|\vec{a} - \vec{b}| = 2 \sin\left(\frac{\theta}{2}\right).$$
- (d) If the velocity of the body at time t is given by $\mathbf{v} = 3t^2 - 2t\mathbf{j} + 4\mathbf{k}$, find the expression for the displacement, "s" of the body at time t , given that $t=1$, $\mathbf{s}=3\mathbf{i}-\mathbf{j}+2\mathbf{k}$.
4. (a) Given that $2 - i$ and $1 + 3i$ are roots of the equation $ax^4 + bx^3 + cx^2 + dx + e = 0$, where a, b, c, d and e are real number coefficient, find;
- The other roots.
 - The sum of four roots of the equation.
 - The product of the four roots of the equation.
 - By using these four roots, form the quadratic equation.
- (b) Prove that $(1 + \cos\theta + i\sin\theta)^n = 2^n \cos^n \frac{\theta}{2} \left(\cos \frac{n\theta}{2} + i\sin \frac{n\theta}{2} \right)$.
- (c) If $z = x + yi$ and $w = u + Vi$ represent points $p(x, y)$ and $q(u, v)$ and $w = \frac{z+1}{2-1}$ Find the locus of Q when P describes the circle $x^2 + y^2 = 4$ hence, determine the centre and radius of the locus Q.

SECTION B (40 Marks)

Answer any two (2) questions from this section

5. (a) Solve for x if $\tan^{-1}(x + 1) = 3\tan^{-1}(x - 1)$.
- (b) If $\sin\theta = \frac{4}{5}$, find the value of
$$\frac{5\cos\theta + 4\operatorname{cosec}\theta + 3\tan\theta}{4\cot\theta + 3\sec\theta + 5\sin\theta}$$
- (c) Given that $10\sin\frac{\pi x}{3} + 24\cos\frac{\pi x}{3} = 13$. By using R – expression of Cosine , show that
$$x = (6n \pm 1) + \frac{3}{\pi} \tan^{-1}\left(\frac{5}{12}\right).$$
- (d) (i) Prove that $\frac{\sin\theta + \sin 2\theta}{1 + \cos\theta + \cos 2\theta} = \tan\theta$.
- (ii) If $x = \sin^2 2\theta$ and $y = \cos\theta$, find the equation connecting X and Y.
6. (a) Show that $2x^2 + 6x + 9$ is always positive. Hence solve the inequality
$$\frac{(x+1)(x+3)}{x} > \frac{x+6}{3}.$$
- (b) Use the standard result for $\sum_{k=1}^n k^3$ to show that
$$\sum_{k=n+1}^{2n} k^3 = \frac{n^2}{4}(3n+1)(5n+3)$$
- (c) The quadratic equation $2x^2 + 8x + 1 = 0$ has roots α and β . Find a quadratic equation with integar coefficients, which has roots $2\alpha^4 + \frac{1}{\beta^2}$ and $2\beta^4 + \frac{1}{\alpha^2}$.
- (d) (i) Use synthetic division to evaluate
$$\frac{6x^4 - 4x^3 - 2x^3 + 27x + 7}{3x - 4}$$
- (ii) Write down the constant term in the expansion for
$$\left(2x^2 - \frac{1}{2x}\right)^6$$
7. (a) Show that $y = A \cos nx + B \sin nx$ is a solution of the differential equation
$$\frac{d^2y}{dx^2} + n^2y = 0$$
, for all values of A and B. Find the solution when $n=2$, given that when $x = 0$, $y=5$ and $\frac{dy}{dx} = 8$.
- (b) State the type of differential equation for the equation $x dy - y dx = \sqrt{x^2 + y^2} dx$ and solve it.
- (c) Solve the differential equation

$$y \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = y \frac{dy}{dx}$$

- (d) A tank contains 40 litres of solution containing 2g of substance per litre. Salt water containing 3g of this substance per litre runs in at the rate of 4 litre/min and the well stirred mixture runs out at the same rate. If $A(t)$ is the amount of substance in the tank at any time t :
- Write down the differential equation for $A(t)$ and its associated initial conditions.
 - Solve the equation in 7(d) (i) for $A(t)$ at any time t .
 - Find the amount of substance in the tank after 15 minutes.
8. (a) Find the vertex, axis, focus, directrix, latusrectum of the parabola $4y^2+12x-20y+67=0$.
- (b) Find the equation of the tangent and normal to the ellipse $5x^2+3y^2=137$ at the point in the first quadrant whose ordinate is 2.
- (c) Prove that the line $y = mx \pm a\sqrt{(m^2 - 1)}$ are tangents to the hyperbola $x^2-y^2=a^2$ for all values of m .
- (d) (i) If a polar equation of a curve is $r^2=4\sin 2\theta$. Find the corresponding Cartesian equation.
- (ii) Sketch the graph of the curve $r = 1 - 2\cos\theta$.

Wabillah Taufiq