



TANZANIA HEADS OF ISLAMIC SCHOOLS COUNCIL
FORM SIX INTER ISLAMIC MOCK EXAMINATION

ADVANCED MATHEMATICS 1

142/1

(For both School and Private Candidates)

TIME: 3 HOURS

Tuesday, 5th March 2024 a.m.

Instructions

1. This paper consists of ten (10) compulsory questions.
2. All work done in answering each question must be shown clearly.
3. Mathematical tables and non-programmable calculators may be used.
4. Cellular phone and any unauthorized materials are not allowed in the examination room.
5. Write your Examination Number on every page of your answer booklet(s).

1. (a) The rate of spread of a COVID-19 virus is modeled mathematically by the equation $y = \frac{45000}{1+224e^{-0.899616t}}$. By using non programmable scientific calculator, evaluate:
 - (i) y when t = 5 correct to 6 significant figures.
 - (ii) t when y = 43785.
- (b) Use a calculator to find the value of $\hat{C}$ using formula $\tan(\hat{C} - \hat{B}) = \left(\frac{c-b}{c+b}\right) \cot \frac{1}{2}\hat{A}$. Where $b = 10.67\text{m}$, $c = 21.7\text{m}$, $\hat{A} = 44^\circ 46'$ and $\hat{B} = 28^\circ$.
2. (a) Given that $\sinh x = \frac{5}{12}$, find
 - (i) $\cosh 2x$
 - (ii) $\sinh 2x$
- (b) (i) Find the value of x for which $2\tanh^{-1}\left(\frac{x-2}{x+1}\right) = \ln 2$.
- (ii) If $y = \tanh^{-1}(\sin x)$, show that $\frac{dy}{dx} = \sec x$.
- (c) If θ is the angle of inclination to the x-axis of the tangent at a point of the curve $y = \frac{1}{2}(e^x + e^{-x})$. Show that
 - (i) $y = \sec \theta$
 - (ii) $x = \log_e (\tan \theta + \sec \theta)$
3. (a) A manufacturer of jams has 720kg of strawberry syrup and 800kg of mango syrup for making two types, each type is made by mixing – strawberry and mango as follows. Grade A: 60% strawberry and 40% mango. Grade B: 30% strawberry and 70% mango. The jam is sold in 400gram jars, the selling prices are as follows: Grade A is 48/= per jar while Grade B is 30/= per jar. Find how many of each type the manufacturer makes to maximize profit.
- (b) A wheat flour company has factories of A and B which supplies warehouses of C and D. The weekly factory capacities are 160 and 140 tons respectively and warehouses requirements are 70 and 120 tons respectively. The cost of transportation of one ton of wheat flour from A to C is Tsh 160 and cost from A to D is Tshs 240, similarly the transport cost from B to C is Tshs 200 and from B to D is Tsh 260. Formulate a mathematical model of the following:
 - (i) The objective function to be minimized by the company so as to supply tons of wheat flour to each warehouse.
 - (ii) The inequalities associated to the transportation problem.
 - (iii) Does the problem balance or not? Why?

4. (a) The mean and variance of a group of 100 items are 15 and 9. A second group containing 150 items is mixed with it and the mean and variance of the composite group are 15.6 and 13.44. Find the mean and the standard deviation of the second group.

- (b) The table below shows the height of 100 students.

Height (cm)	$150 \leq x < 155$	$155 \leq x < 160$	$160 \leq x < 165$	$165 \leq x < 170$	$170 \leq x < 175$
No. of students	30	20	18	17	15

- (i) Calculate the mean deviation about the median.
- (ii) By using coding method, calculate the standard deviation approximate to three decimal places.
5. (a) (i) What is a power set?
- (ii) Given that $A = \{1, 2, 3, 4\}$ and $B = \{1, 4, 5\}$. Find the power of $A \Delta B$.
- (b) Use the laws of algebra of sets to simplify $(A' \cap B' \cap C) \cup (B \cap C) \cup (A \cap C)$
- (c) Out of 60 students in a class at a certain school, all students do at least one of the 3 subjects; Physics, Chemistry or Mathematics and anyone who has chosen to do Mathematics elects to do Physics as well. But no student does mathematics and chemistry. 50 students do physics while 30 students do mathematics. If 16 students do Physics and Chemistry. Present this information on the Venn diagram and use it to find the number of students who do:
- (i) Physics only
- (ii) Chemistry only
6. (a) The function $f: x \rightarrow \frac{a}{x} + b$ is such that $f(-1) = 1\frac{1}{2}$ and $f(2) = 9$.
- (i) State the value of x for which f is defined.
- (ii) Find the value of a and b.
- (iii) Determine algebraically whether the function f is one to-one.
- (b) A function f(x) is defined as $f(x) = \frac{3x-9}{x^2-x-2}$. Determine:
- (i) The turning points of the function
- (ii) The asymptotes of f(x).
- (iv) Sketch the graph of f(x).
7. (a) A physical quantity P is related to four observables a, b, c and d by the equation $P = \frac{a^3 b^2}{\sqrt{cd}}$. The percentage errors of measurements in a, b, c and d are 1%, 3%, 4% and 2% respectively. What is the percentage error in the quantity P?
- (b) Use the Newton – Raphson method to derive the secant formula and comment why would you want to use it instead of the Newton-Raphson.

- (c) Find $\int_0^1 \frac{1}{1+x^2} dx$
- Trapezoidal rule with 5 ordinates.
 - By Simpson's rule with sub-intervals.
 - Find the approximate value of π referring to your results from the two methods above.
8. (a) Without using distance formula, show that the points (-2, -1), (4,0), (3,3), (3,2) are the vertices of a parallelogram.
- (b) Show that the area of the triangle formed by the lines $y = m_1x + c_1$, $y = m_2x + c_2$ and $x=0$ is $\frac{(c_1 - c_2)^2}{2|m_1 - m_2|}$
- (c) Find the equation of the circle whose centre is at the point of intersection of the two straight lines $x + y = 6$ and $2x - y - 3 = 0$ and passes through the centre of the circle $x^2 + y^2 - 2x + 2y - 10 = 0$.
9. (a) Evaluate the indefinite integral $\int \frac{\cos x}{1 + \cos x} dx$.
- (b) If $\int_0^1 \frac{x-a}{(x+1)(3x+1)} dx = 0$, find the value of a .
- (c) The curve has an equation $9y^2 = 16x^3$. Determine the length of the arc of the curve between the points $(1, \frac{4}{3})$ and $(4, \frac{32}{3})$
10. (a) Use definition to find $f'(x)$ given that $f(x) = x + \sin x$.
- (b) Prove that $\frac{d}{dx} \left[\frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \left(\frac{x}{a} \right) \right] = \sqrt{a^2 - x^2}$
- (c) Find the least area of the metal required to make a closed cylindrical container from a thin sheet of metal in order that it might have a capacity of $2000\pi \text{ cm}^3$.

Wabillah Taufiq