FORM THREE JOINT EXAMINATION
BASIC MATHEMATICS
MARKING SCHEME

1 (a) Let X be the award distributed to teachers

Thus, $23\% + 15\% + 27\% + X = 960,000/= 100\%$

$65\% + X = 100\%$

$X = 100 - 65$

$X = 35\%$

Thus $\frac{35}{100} \times 960,000 = 336,000/=$

Therefore, the amount distributed to teachers $= 336,000/=$

(b)

$$\begin{array}{r|ccc} 2 & 8, & 10, & 12 \\ \hline 2 & 4 & 5 & 6 \\ \hline 2 & 2 & 5 & 3 \\ \hline 3 & 1 & 5 & 3 \\ \hline 5 & 1 & 5 & 1 \\ \hline & 1 & 1 & 1 \end{array}$$

$2 \times 2 \times 2 \times 3 \times 5 = 120$ seconds

But $1 \min = 60 \sec$

$X = 120 \sec$

$\frac{60X}{60} = \frac{120}{60}$

$X = 2 \min$

The interval to which they will toll together $= \frac{50}{2} = 25$

Thus, they will toll together at interval of 25

2 (a) $(3^{x+1}) \cdot (2^{2y+9}) = 69984$ (02) Pg.

$3^{x+1} \cdot 2^{2y+9} = 3^7 \cdot 2^5$

but $3^{x+1} = 3^7$ and $2^{2y+9} = 2^5$

$x+1 = 7$

$x = 7-1$

$x = 6$

$2y+9 = 5$

$2y = 5-9$

$\frac{2y}{2} = \frac{-4}{2}$

$y = -2$

(03 marks)

from $x+y = 6+-2$

$x+y = 6-2$

$x+y = 4$

thus the value of $x+y = 4$

(b) from $\log_4 5x - \log_4(x+2) - \log_4 3 = 0$

$\log_4 5x - \log_4(x+2) = \log_4 3$

$\log_4\left(\frac{5x}{x+2}\right) = \log_4 3$

$\frac{5x}{x+2} = 3$

(03 marks)

$5x = 3(x+2)$

$5x = 3x+6$

$5x - 3x = 6$

$\frac{2x}{2} = \frac{6}{2}$

$x = 3$

thus

$x = 3$

3 ⓐ Given that

$P = \{\text{multiple of } 5\}$.

Then $P = \{5, 10, 15, 20, 25, 30 \cdots\}$.

$Q = \{\text{Even numbers b/n } 10 \text{ and } 22\}$.

Then $Q = \{12, 14, 16, 18, 20\}$.

$P \cap Q = \{20\}$

Then $P \cap Q = \{\ \}\ \{20\}$.

Therefore; The subsets of $P \cap Q = \{\ \}\ \{20\}$.

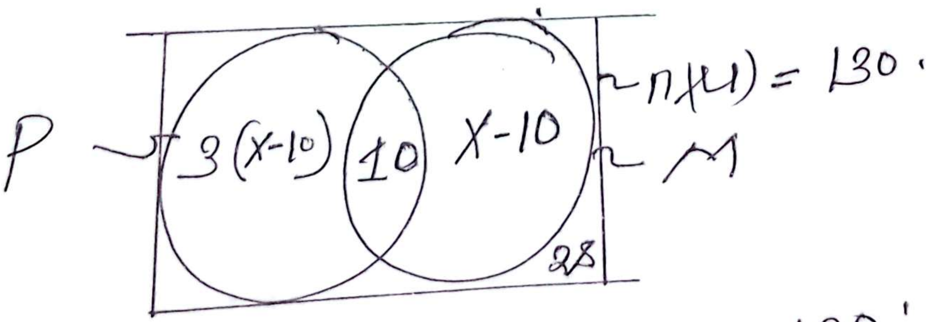
(b) $n(U) = 130$

$n(P \cup M)' = 28$

let X be the students who study maths

Then, ~~p~~ mathematics $= X$

→ Consider a venn-diagram below.



$3(X-10) + 10 + X - 10 + 28 = 130$

$3X - 30 + 10 + X - 10 + 28 = 130$

$3X + X - 2 = 130$

$4X - 2 = 130$

$4X = 130 + 2$

$4X = 132$

$X = 33$

Mathematics $= X - 10$

$= 33 - 10$

$= 23$

Thus, The number of students who study mathematics only are 23 | (04) Pg

4 | If Tsh 28256 $\longrightarrow$ 100 Kronor
x $\longrightarrow$ 100,000 Kronor

(a)
$$\frac{100x}{100} = \frac{28256 \times 100{,}000}{100}$$
$$x = 28256 \times 1000$$
$$x = 28256{,}000/=$$
(03 marks)

Thus, he received 28256000/= Tsh

(b)
$$x \propto y^2/z$$
$$x = ky^2/z$$
$$x \cdot z = ky^2$$
$$k = \frac{x \cdot z}{y^2}$$
$$k = \frac{16 \times 2}{4^2}$$
$$k = \frac{16 \times 2}{16}$$
$$k = 2$$

from
$$k = \frac{x \cdot z}{y^2}$$
$$2 = \frac{x \cdot 4}{3^2}$$
$$\frac{2 \times 9}{4} = \frac{4x}{4}$$
$$x = 4.5$$
(03 marks)

Thus, the value of $x = 4.5$

05

Pg

5 (a) from $1m = 100cm$

$37.5m = ?$

$3750 cm$

Then $\frac{3750 cm}{75 cm} = 50$ pieces (03 marks)

Therefore 50 pieces were obtained.

(b) Let x be the total number of girls and boys in the school

Then $7 + 12 = 19$

$\frac{12}{19} \times x = 1380$

$\frac{12x}{19} = 1380$ (03 marks)

$\frac{12x}{12} = \frac{1380 \times 19}{12}$

$x = 2185$

Then, the total number of girls and boys in the school $= 2185$

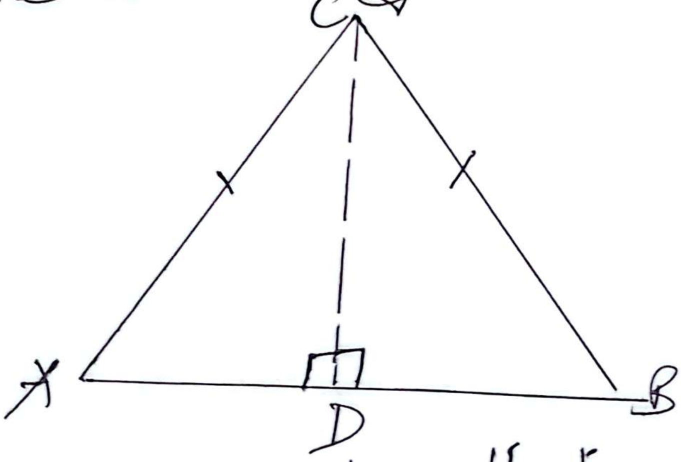
from girls $= \frac{7}{19} \times 2185$

$= 7 \times 115$

$= 805$

Then, the number of girls in the school $= 805$

6 (a) Consider a triangle below



Required to show that
$m(A\hat{D}C) = m(B\hat{D}C)$.

In $\triangle ACD$ and In $\triangle BCD$

$\overline{AC} = \overline{BC}$ (Given).

$\overline{AD} = \overline{BD}$ ($\overline{CD}$ bisect $\overline{AB}$).

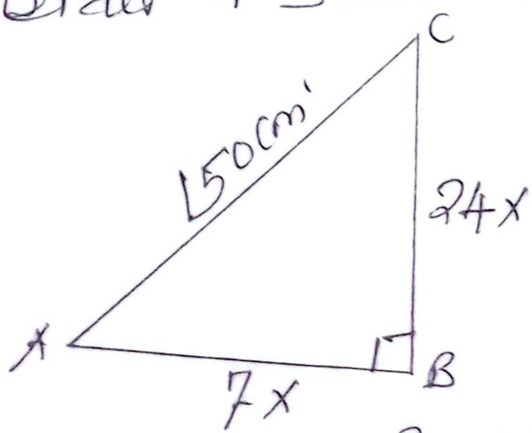
$\overline{CD} = \overline{CD}$ (Common side)

Thus $\triangle ACD \cong \triangle BCD$. (SSS)

Since $\triangle ACD \cong \triangle BCD$ hence

$m(A\hat{D}C) = m(B\hat{D}C)$ (definition of Congruence).

(b) Consider a sketch below



(i) from $(\overline{AC})^2 = (\overline{AB})^2 + (\overline{BC})^2$

$(150)^2 = (7x)^2 + (24x)^2$

$22500 = 49x^2 + 576x^2$

6 (b) from $22500 = 625x^2$

(i) $x^2 = \frac{22500}{625}$

$x^2 = 36$

$x = \sqrt{36}$

$x = 6$

Thus, the value of $x = 6$

(ii) from $\overline{AB} = 7x$

Thus $\overline{AB} = 7 \times 6 \Rightarrow 42$ cm

$\overline{BC} = 24x$

but $x = 6$

$\overline{BC} = 24 \times 6$

$\overline{BC} = 144$ cm

from

$\text{Area} = \frac{1}{2} \cdot \overline{AB} \cdot \overline{BC}$

$= \frac{1}{2} \times 42 \times 144$

$= 3024$

Thus, the Area of a triangle

$(A) = 3024 \text{ cm}^2$

(08) pg'

7 (a)' From $\% \text{ loss} = \frac{B \cdot P - S \cdot P}{B \cdot P}$

$$\frac{25}{100} = \frac{40{,}000 - S \cdot P}{40{,}000}$$

$$\frac{100(40{,}000 - S \cdot P)}{100} = \frac{40{,}000 \times 25}{100}$$

$$40{,}000 - S \cdot P = 400 \times 25$$

(03 marks)

$$40{,}000 - S \cdot P = 10{,}000$$

$$-S \cdot P = 10{,}000 - 40{,}000$$

$$+S \cdot P = +30{,}000$$

Thus' the selling price = 30,000/=

(b) discount $= \frac{15}{100} \times 140{,}000/=$

discount = 21,000

(03 marks)

From 140,000 − 21,000

119,000/=

Thus' he paid Tsh 119,000/=

8 (a) $T_3 = 18$

$T_1 r^2 = 18$ ——— (i)

$T_6 = 486$

$T_1 r^5 = 486$ ——— (ii)

take eqn (ii) then divide by eqn (i).

$$\frac{T_1 r^5}{T_1 r^2} = \frac{486}{18}$$

$$r^{5-2} = 27$$

$$r^3 = 27$$

$$\sqrt[3]{r^3} = \sqrt[3]{27}$$

$$\underline{r = 3}$$

(04 marks)

from eqn (i)

$$T_1 r^2 = 18$$

$$T_1 \cdot (3)^2 = 18$$

$$T_1 \cdot 9 = 18$$

$$T_1 = 18/9$$

$$T_1 = 2$$

thus, the first term $(T_1) = 2$

7. (a) $S_{10} = ?$

from $S_n = \frac{a(r^n - 1)}{r-1}$

$S_{10} = \frac{2(3^{10}-1)}{3-1}$

$S_{10} = \frac{2 \times 59048}{2}$

$S_{10} = 59048$

Then, the sum of the first ten terms are 59048

(b) $x_1 = 2$

$d = 5$

$S_n = 245$

from $S_n = \frac{n}{2}(2x_1 + (n-1)d)$

$245 = \frac{n}{2}((2 \cdot 2) + (n-1)5)$

$245 = \frac{n}{2}(4 + 5n - 5)$

$245 = \frac{n}{2}(5n - 1)$

$245 \times 2 = n(5n - 1)$

$490 = 5n^2 - n$

$5n^2 - n - 490 = 0$

(02 marks)

$n = 10$

8 ⓐ $S_{10} = ?$

From $S_n = \frac{a_1(r^n - 1)}{r-1}$

$S_{10} = \frac{2(3^{10}-1)}{3-1}$

$S_{10} = \frac{2 \times 59048}{2}$

$S_{10} = 59048$

Thus, the sum of the first ten terms are 59048

ⓑ $A_1 = 2$

$d = 5$

$S_n = 245$

From $S_n = \frac{n}{2}(2A_1 + (n-1)d)$

$245 = \frac{n}{2}((2 \cdot 2) + (n-1)5)$

$245 = \frac{n}{2}(4 + 5n - 5)$

$245 = \frac{n}{2}(5n - 1)$

$245 \times 2 = n(5n-1)$

$490 = 5n^2 - n$

$5n^2 - n - 490 = 0$

$n = 10$

(02 marks)

Thus, There are ten

10 pg

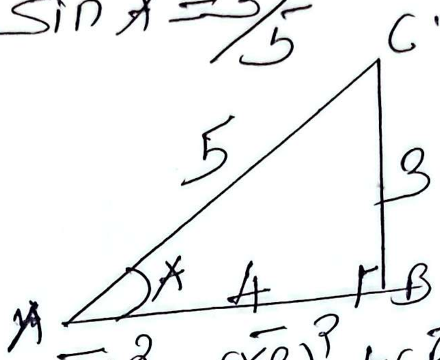
9 (a) If

$5 \sin A = 3$

$\sin A = 3/5$

(11) B

from



from $(\overline{AC})^2 = (\overline{AB})^2 + (\overline{BC})^2$

$5^2 = (\overline{AB})^2 + 3^2$

$25 = (\overline{AB})^2 + 9$

$25 - 9 = (\overline{AB})^2$

$(\overline{AB})^2 = 16$

$\overline{AB} = \sqrt{16}$

$\overline{AB} = 4$

then $\sin A = 3/5$

$\cos A = 4/5$

$\tan A = 3/4$

from (i) then

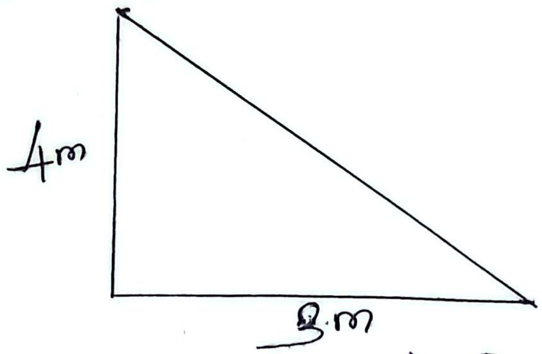
$\cos A = 4/5$

from $\dfrac{\tan A - \sin A}{1 + \cos A} = \dfrac{3/4 - 3/5}{1 + 4/5} = \dfrac{3/20}{9/5}$

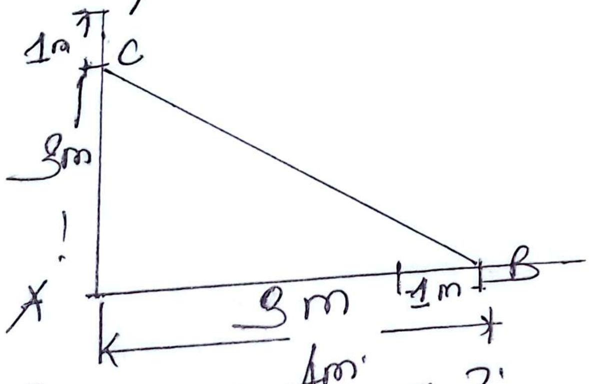
$= 3/20 \times 5/9 = 1/12$

(ii) then $\dfrac{\tan A - \sin A}{1 + \cos A} = 1/12$

9 (b) consider a sketch before sliding down (12) pg



when the lower end is pulled away by 1m and the upper end slide down by 1m.



from $(\overline{BC})^2 = (\overline{AB})^2 + (\overline{AC})^2$

$(\overline{BC})^2 = 4^2 + 3^2$

$(\overline{BC})^2 = 16 + 9$

$(\overline{BC})^2 = 25$

$\overline{BC} = \sqrt{25}$

thus $\overline{BC} = 5m$,

thus, the length of a ladder $= 5m$

10 (a) Given that (13) pg

$$\frac{3x^2}{3} - \frac{7x}{3} - \frac{6}{3} = \frac{0}{3}$$

$$x^2 - \frac{7}{3}x - 2 = 0$$

$$x^2 - \frac{7}{3}x = 2 \text{ ——— } (1)$$

from $C = \left(\frac{b}{2a}\right)^2$

$$C = \left(\frac{-7/3}{2\times1}\right)^2$$

$$C = \left(-7/6\right)^2$$

Then add "C" to both sides of the equation (1)

$$x^2 - \frac{7}{3}x + \left(-7/6\right)^2 = 2 + 49/36.$$

$$\left(x - 7/6\right)^2 = \frac{121}{36}$$

$$\sqrt{\left(x - 7/6\right)^2} = \pm\sqrt{\frac{121}{36}}$$

$$x - 7/6 = \pm 11/6$$

$$x = 7/6 \pm 11/6$$

$$x = 7/6 + 11/6 \text{ or } 7/6 - 11/6.$$

$$x = 18/6 \text{ or } -4/6$$

Then the value of $x = 3$ or $-2/3$

10 (b) Factorize a^2-b^2 (14) PG

thus $a^2-b^2=(a+b)(a-b)$

from $(1005)^2-995\times 994$

$(1005)^2-(994+1)\times 994$

$(1005)^2-(994^2+994)$

$(1005)^2-994^2-994$

$(1005^2-994^2)-994$

let $a=1005$ and $b=994$

from $a^2-b^2=(a+b)(a-b)-994$

$(1005)^2-(994)^2=(1005+994)(1005-994)-994$

$=(1999\times 11)-994$

$=21989-994$

$=20995$

thus $(1005)^2-995\times 994=20995$

11 (a) Consider a table below

15 Pg

$\Sigma f = 40$

Class interval (KG)	Class mark (X)	(f)	$d = X - A$	fd	CF
40 - 44	42	7	-10	-70	7
45 - 49	47	8	-5	-40	15
50 - 54	52	x = 11	0	0	26
55 - 59	57	10	5	50	36
60 - 64	62	4	10	40	40

Let x be the lower limit (L.L)
y be the upper limit (U.L)

then $\frac{x+y}{2} = 42$

$x + y = 84$ —— (i)

Let U.R.L be $y + 0.5$
L.R.L be $x - 0.5$
class size $= 47 - 42$
class size $= 5$

then from
U.R.L - L.R.L = class size
$y + 0.5 - (x - 0.5) = 5$
$y + 0.5 - x + 0.5 = 5$
$y + 1 - x = 5$
$y - x = 5 - 1$
$y - x = 4$

then $-x + y = 4$ —— (ii)

... eqn (i) and eqn (ii) simultan...

11

$x+y = 84$ — (i)

$-x+y = 4$ — (ii)

$x = 40$ and $y = 44$

$\therefore$ $x = 40$ and $y = 44$

But $\bar{x} = 52$ and $d = x - \bar{x}$; $\sum fd = -70$

From $fd = f(x-\bar{x})$ $\qquad$ $fd = -70$

$fd = f(42-52)$ $\qquad$ $d = -10$

$-70 = f(42-52)$ or from $-10f = -70$

$f = 7$

$-70 = -10f$ $\qquad$ $\frac{-5f}{-5} = \frac{-40}{-5}$

$f = \frac{-70}{-10}$ $\qquad$ $f = 8$

$f = 7$

But $\sum f = 7 + 8 + x + 10 + 4$

But $\sum f = 40$

$40 = 7 + 8 + x + 10 + 4$

$40 = 29 + x$

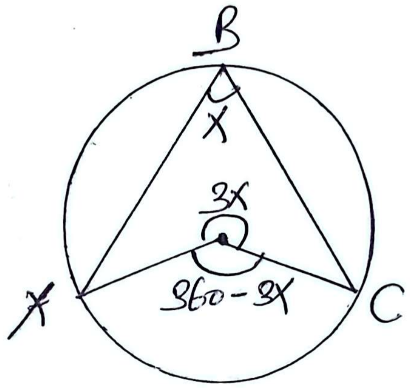
$40 - 29 = x$

$x = 11$

16

15

12 (a) Consider a sketch below:



From $m(A\hat{B}C) = \frac{1}{2}$ arc AC (Angle inscribed in a circle)

but $m(A\hat{B}C) = x$

arc $AC = 360 - 3x$ (central angle)

Thus $m(A\hat{B}C) = \frac{1}{2}$ arc AC

$$x = \frac{1}{2}(360 - 3x)$$

$$2x = 360 - 3x$$

$$2x + 3x = 360$$

$$5x = 360$$

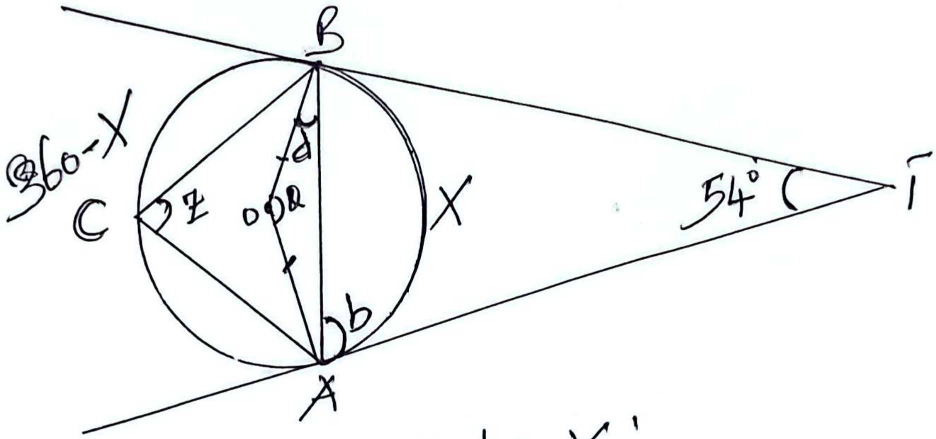
$$x = 72$$

Thus, the value of $x = 72^\circ$

12 (b) Consider a sketch below'

(18)

19



Let arc AB be X'

arc ACB = 360 - X'

from $m(A\hat{T}B) = \frac{1}{2}$ (arc ACB - arc AB) (Angle formed outside the circle)

but $m(A\hat{T}B) = 54'$

arc AB = X

arc ACB = 360 - X.

from $m(A\hat{T}B) = \frac{1}{2}$ (arc ACB - arc AB)

$54° = \frac{1}{2}(360 - X - X)$

$54 \times 2 = 360 - 2X$

$108 = 360 - 2X$

$108 - 360 = -2X$

$-252 = -2X$

$X = 126°$

but $m(A\hat{O}B)$ = arc AB (Central angle)

but $m(A\hat{O}B) = Q$

arc AB = 126'

Then from $m(A\hat{O}B)$ = arc AB

$Q = 126°$

12B Thus angle $Q = 126°$ (19) pg

Also; $m(X\hat{C}B) = \frac{1}{2}$ arc XB (Angle inscribed in a circle)

but $m(X\hat{C}B) = Z$ and arc $AB = 126°$

From $m(X\hat{C}B) = \frac{1}{2}$ arc XB

$$Z = \frac{1}{2} \times 126$$

$$Z = 63°$$

From Isosceles triangle:

$$126° + d + d = 180°$$

$$126 + 2d = 180°$$

$$2d = 180 - 126$$

$$\frac{2d}{2} = \frac{54}{2}$$

$$d = 27°$$

but $27 + b = 90°$

$$b = 90 - 27$$

$$b = 63°$$

Thus

$$Q = 126°$$

$$Z = 63°$$

13 (a) Given that

$f(x) = -2x^2 + 12x - 16$

where $a = -2$; $b = 12$ and $c = -16$

(i) from $x = -b/2a$

$x = -12/2 \times -2$

$x = -12/-4$

$x = 3$

thus; the axis of symmetry $(x) = 3$

(ii) since the coefficient of x^2 is negative, thus, the graph is maximum

from $y = \frac{4ac - b^2}{4a}$

$y = \frac{4(-2)(-16) - 12^2}{4(-2)}$

$y = 2$

thus; the maximum value $(y) = 2$

(iii) from $(x,y) = \left(-b/2a, \frac{4ac - b^2}{4a}\right)$

$(x,y) = -12/-4, \frac{4(-2)(-16) - 12^2}{4(-2)}$

$(x,y) = 3, 2$

thus; the turning point $(x,y) = (3, 2)$

13 (b) Given that

$$f(x) = \sqrt{x-2}$$

$$y = \sqrt{x-2}$$

Interchange the variables

from $y = \sqrt{x-2}$

$$x = \sqrt{y-2}$$

Make y as the subject of the formula.

$$x^2 = \left(\sqrt{y-2}\right)^2$$

$$x^2 = y-2$$

$$x^2+2 = y$$

$$y = x^2+2$$

Then $f^{-1}(x) = x^2+2$

when $x = \left(\frac{5}{2}\right)$

$$f^{-1}\left(\frac{5}{2}\right) = \left(\frac{5}{2}\right)^2+2$$

$$= \frac{25}{4}+2$$

$$= \frac{25}{4}+2 = \frac{33}{4}$$

Then $f^{-1}\left(\frac{5}{2}\right) = \frac{33}{4}$ or 8.25

(21) Pg

13 (b) from $f^{-1}(x) = x^2+2$
when $x = \sqrt{3}$

$$f^{-1}(\sqrt{3}) = (\sqrt{3})^2+2$$

$$f^{-1}(\sqrt{3}) = 3+2$$

$$f^{-1}(\sqrt{3}) = 5$$

thus

$$f^{-1}(\sqrt{3}) = 5$$

23
Pg

14 Given that

$$R = \{(x,y) : y = x^2\}$$

x	-3	-2	-1	0	1	2	3
y	9	4	1	0	1	4	9

from $y = x^2$
Interchange the variables

$$y = x^2$$

$$x = y^2$$

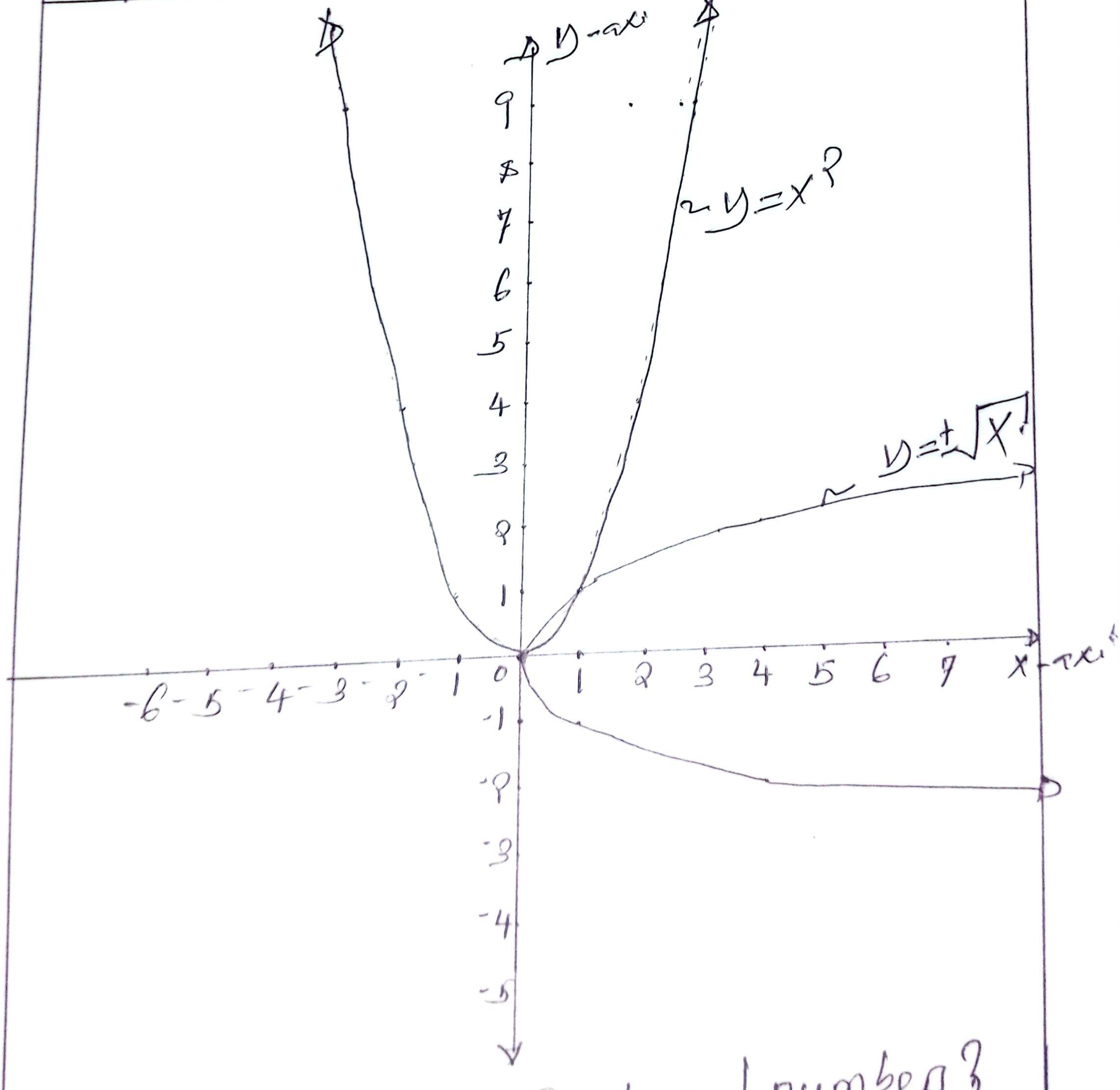
make y as the subject of the formular

$$x = y^2$$

$$y = \pm\sqrt{x}$$

$y = \pm\sqrt{x}$

x	0	1	4	9	16	25	
y	0	± 1	± 2	± 3	± 4	± 5	



Domain of $R = \{$All real numbers$\}$

of $R = \{y : y \geq 0\}$.