

Physics – Form 3

Classical notes for CSEE – 2024

Competency Based

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Preface

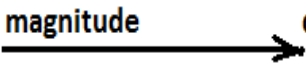
- Physics Classical Note for CSEE has undergone a great deal of improvement since its first publication (**2020**). It has become so popular among the CSEE students in such a way that most of the candidates go through this book before appearing for the Examination.
- You must have already gone through first edition of Physics Classical Note for CSEE (2020). We hope that you liked the book. We are happy to release the revised edition of Physics Classical Note (**2024**). This new edition is competency based on the latest revised syllabus prescribed by the ministry of education, science and technology
- Several new diagrams have been incorporated in this edition in order to facilitate the understanding of the subject matter. Many examples from daily life are given to make the topics relevant and interesting.
- In each chapter of this book, sufficient exercises (class Activities and Self Assessment) are given at the end of each sub – topic and at the end of each topic respectively.
- Finally I must express my thanks to teachers and students who have kindly read parts of the manuscript and sending their valuable suggestions. Further suggestions for more improvements to come of this book will be gratefully acknowledged

Chapter– 01: Application of Vectors

Differences between Scalar and Vector Quantities

Scalar quantities	Vector quantities
Are those physical quantities which have only magnitude	Are the physical quantities which have both magnitude and direction
They change if magnitude change	They change either if magnitude or direction or both changes
They are specified by a magnitude and a unit only	They are also specified by a number along with the direction and unit
They obey the ordinary laws of algebra	They do not obey the ordinary laws of algebra
Examples are mass temperature, energy, volume, density, distance, area and length	Examples are force, acceleration, gravitational force, velocity, momentum

Vector addition

- The addition of vector is done with the help of vector diagrams. The straight line is drawn to scale. The length of a line segment represents the **magnitude** of the vector and the arrow represents the **direction** ie 
- When two or more vectors are added the sum is known as **The resultant vector**
- In order to find the sum of two vectors, which are inclined to each other, **triangle law of vectors or parallelogram law of vectors**, can be used
- **Vectors can be added by** mathematical methods using mathematical formula such as Pythagoras' theorem, trigonometric Ratios etc
- **Also vectors can be added by graphical method**

Adding vectors by Graphical method

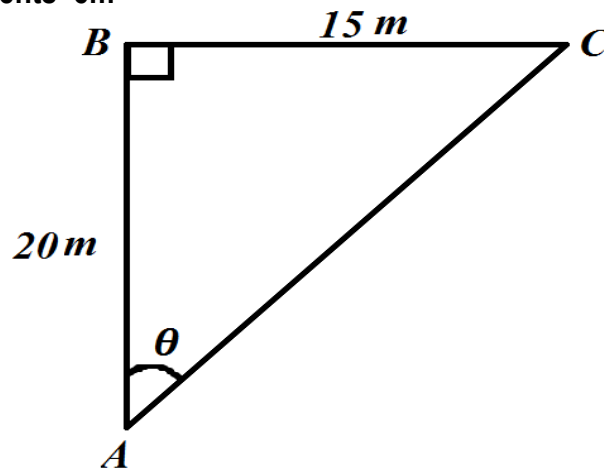
- Choose a suitable scale and write it down on a graph paper
- Pick a starting point and draw the first vector to scale direction stated (indicate the magnitude and direction)
- Starting from the head of the first vector, draw the second vector to scale in the started direction until all given vectors finished
- Draw the line to connect tail of the first drawn vector and the head of the last vector. **This is called resultant vector**
- Measure the length of the resultant vector and convert to actual unit
- Determine the direction of the resultant vector

Example

1. Suppose a man walks starting from point A, a distance of 20m due north and then walks 15m due east. Find his new position from A

Solution

Scale: 1cm represents 5m



∴ From the fig above, AC (R) is 25 m at an angle of $36^{\circ} 51'$ east of north

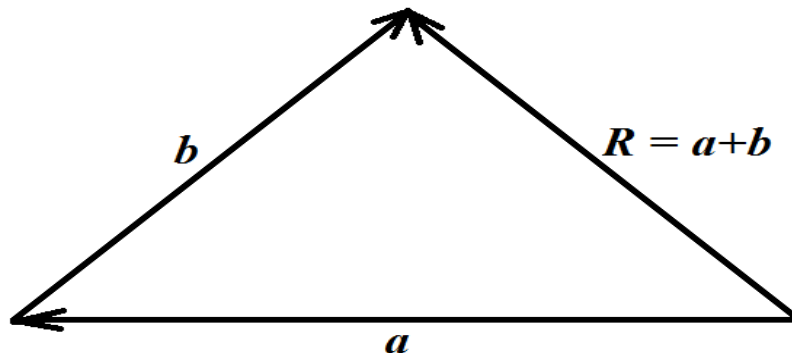
The Triangle Method (Triangle law of vector addition)

- States that
“If two vectors are represented by two sides of a triangle in sequence, then the third closing side of the triangle, drawn from the tail of the first vector to the head of the second vector, represents the resultant of the two vectors in both magnitude and direction “

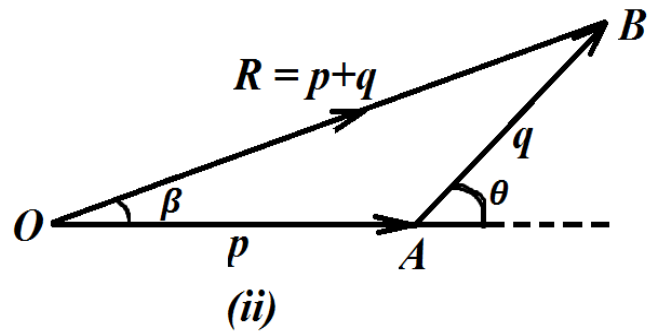
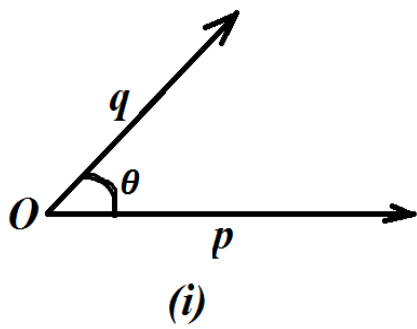
OR

“If three vectors are in equilibrium and that two of the vectors are represented in magnitude and direction by two sides of a triangle, then the third side of the triangle represents the resultant of the two forces”

See the diagram below



- In order to find the resultant of two vectors \mathbf{p} and \mathbf{q} which are acting at an angle θ , the following procedure is adopted. First draw \overline{OA} to represent vector \mathbf{p} . Then starting from the arrow head of \mathbf{p} , draw \overline{AB} to represent vector \mathbf{q} . Finally, draw \overline{OB} to represent magnitude \mathbf{R} from the tail of vector \mathbf{p} to the head of vector \mathbf{q} . (see figure (ii) below)
- $\overline{OB} = \mathbf{R}$ is the sum of the vectors \mathbf{p} and \mathbf{q} . Thus, $\mathbf{R} = \mathbf{p} + \mathbf{q}$.
- The magnitude of $\mathbf{p} + \mathbf{q}$ is determined by measuring the length of \mathbf{R} and direction by measuring the angle between \mathbf{p} and \mathbf{R} .



triangle law of vector addition

Alternatively;

- The magnitude and direction of R , can be obtained by using the sine rule and cosine rule of triangles. Let β be the angle made by the resultant Vector R with Vector p .

- The magnitude of R is,

$$R^2 = p^2 + q^2 - 2pq\cos(180 - \theta)$$

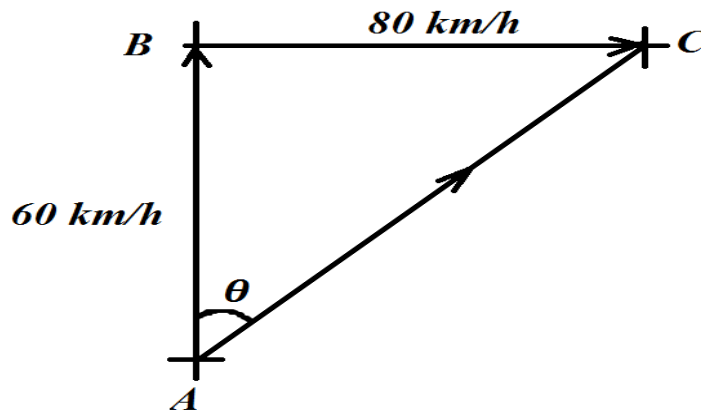
$$\therefore R = \sqrt{p^2 + q^2 - 2pq\cos(180 - \theta)}$$

- The direction is also given by; $\frac{q}{\sin\beta} = \frac{R}{\sin(180-\theta)}$

Worked Examples:

1. A car is travelling due north at 60 km/hr .It turns and then travels due east at 80 km/hr .Find the magnitude and direction of the resultant velocity of the car

Soln: By calculation method:



From: Pythagoras theorem: $a^2 + b^2 = c^2$

$$a^2 + b^2 = c^2 \rightarrow R^2 = 60^2 + 80^2 = 3400 + 6400 = 10000 \rightarrow R = \sqrt{10000} = 100$$

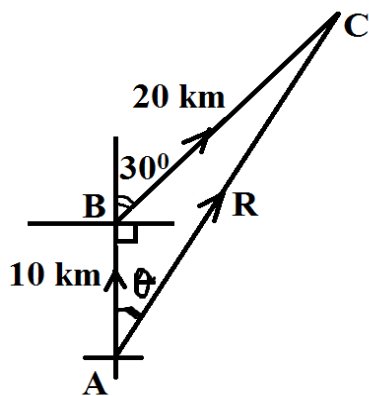
$$\text{Now consider } \tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{80}{60} = 1.3333$$

$$\rightarrow \theta = \tan^{-1}(1.333) = 53.1^\circ$$

\therefore The resultant , R is 100 km/h to the direction of N 53.1° E

2. A moving car covered 10 km in the northern direction. After making a 30° turn, it covered another 20 km. By calculation or graphical method, determine the total displacement of the car

ANSW: consider a free body diagram below



let: 1cm represents 4 km

from the graph; R represents 7.3 cm

then; 1cm = 4 km

7.3cm = ? (R)

$$R = \frac{7.3 \text{ cm} \times 4 \text{ km}}{1 \text{ cm}} = 29.2 \text{ km}$$

when measuring, $\theta = 19^\circ$

therefore, total displacement = 29.2 km, at N 19° E

Alternatively; using cosine rule ie,.. $R^2 = a^2 + c^2 - 2ac\cos 150^\circ$

$$R^2 = 20^2 + 10^2 - 2 \times 20 \times 10 \times \cos 150^\circ = 846.4 \rightarrow R = \sqrt{846.4} = 29.1 \text{ km}$$

$$\text{Also; } a^2 = b^2 + c^2 - 2bc\cos\theta \rightarrow 20^2 = 29.1^2 + 10^2 - 2 \times 29.1 \times 20 \times \cos\theta$$

$$400 = 946.81 - 582 \times \cos\theta \rightarrow \cos\theta = \frac{(400 - 946.81)}{(-582)} = 0.9395$$

$$\theta = \cos^{-1}(0.9395) = 20^\circ$$

\therefore total displacement, R = 29.1 km, at N 20° E

Class Activity –1:1

1. A brick is pulled by a force of 4N acting northward and another force of 3N acting north-east. Find the resultant of these two forces. **ANS: Rs is 6.5 N**
2. Two forces, one 8 N and the other 6 N, are acting on a body. Given that the two forces are acting perpendicularly to each other, find the magnitude of the third force which would just counter the two forces. **ANS = 10 N**
3. Two vectors P and Q have magnitudes of 9 units and 16 units and make an angle of 30° with each other. Using triangle law of vector addition, find the magnitude and direction of resultant vector. [**ANS; R=24.2units, at 19.3°**]
4. A weight of 25 N is suspended from a beam by a string; what horizontal force must be applied to the weight to keep the string at an angle of 20° to the vertical. What is the tension in the string? (**ANS: $F_H = 9.1\text{N}$, $T_S = 26.6 \text{ N}$**)

Parallelogram Method (Parallelogram law of vector addition)

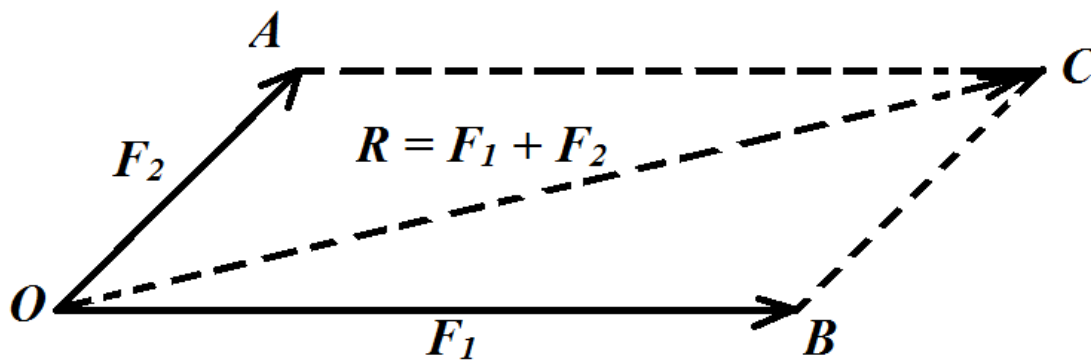
- The law states that:

“If two vectors are represented by two adjacent sides of a parallelogram, then the diagonal of the parallelogram through the common point represents the sum of the two vectors in both magnitude and direction”

OR

“If two vectors are represented by the two sides given and they include angle between them, then resultant of the two vectors will be represented by the diagonal from their common point of a parallelogram formed by the two vectors”

- To calculate the resultant force, we draw the vectors **F1** and **F2** as adjacent sides of a parallelogram **OABC** as shown in the figure below.
- The diagonal **OC** passing through point **O** represents the resultant force both in magnitude and direction.



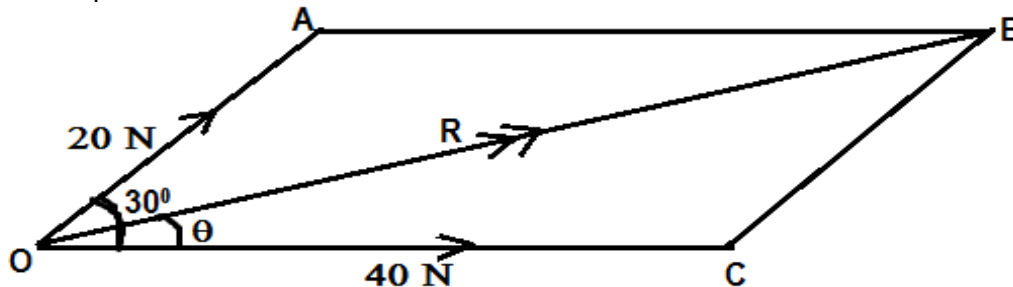
Addition of forces using the Parallelogram law

Worked examples:

- Two forces of 20 N and 40 N acts at a point and the angle between them is 30° . Find the resultant force and the angle it makes with the force of 20^o (ANS: R = 58 N, at 20°)

Solution:

Scale: 1cm represents 5cm



From the graph above, R represents 11.6 Cm

Then: If 1cm = 5 N

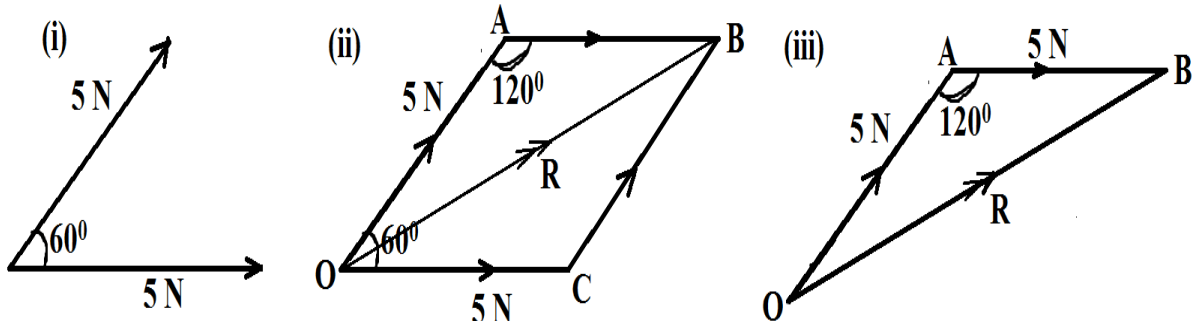
11.6 cm = R (By crossing multiplication R = 58 N)

\therefore The resultant force is 58 N at an angle of 20°

- Find by drawing or calculation the resultant of two forces each 5 N acting at a point at an angle of 60° with each other

ANS:

Consider a free body diagram below;



Consider $\triangle OAB$ from figure (iii); by Cosine Rule

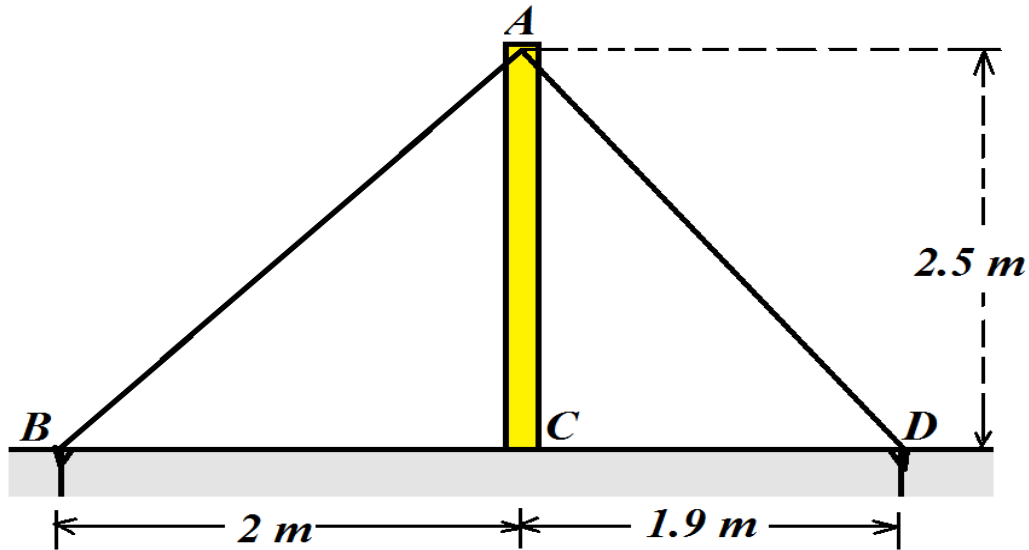
$$R^2 = \overline{OA}^2 + \overline{AB}^2 - 2\overline{OA} \times \overline{AB} \times \cos O\hat{A}B$$

$$R^2 = 5^2 + 5^2 - 2 \times 5 \times 5 \times \cos 120^\circ = 75 \rightarrow \therefore R = \sqrt{75} = 8.66 \text{ N}$$

Alternatively; by scale drawing

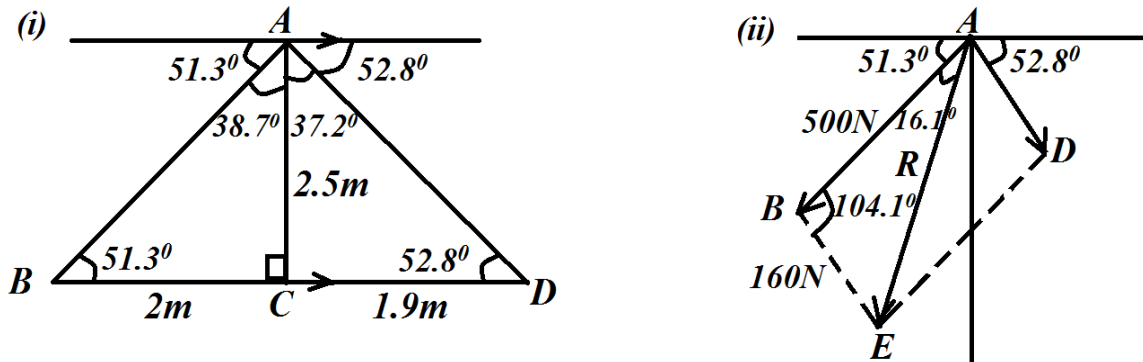
Let: 1cm represents 1N, from the figure (ii) above R = 8.7 cm $\therefore R = 8.7 \text{ N}$

3. Two cables are used to support a pole as shown in figure below.



If the tension in cables AB and AD is 500 N and 160 N respectively, determine the magnitude and direction of the resultant force exerted by the cables AB and AD at point A using the parallelogram law.

ANS; Consider a free body diagram below (by graphical method)



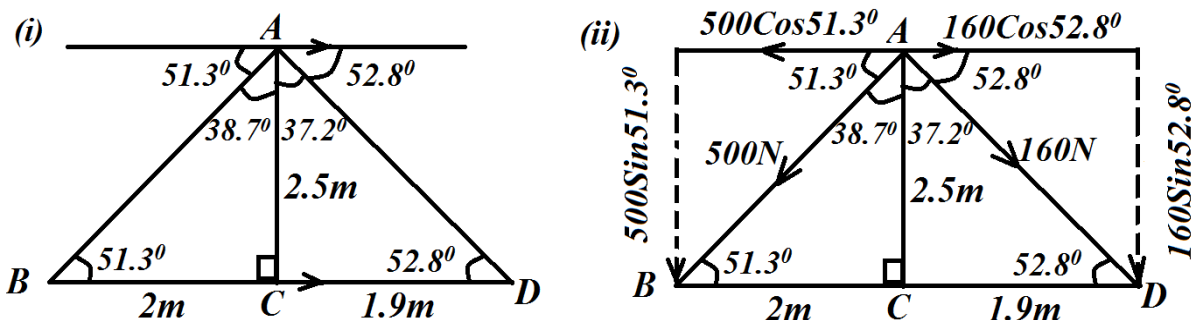
From figure (ii): Let 1cm represents 50 N, Then; when measuring; $R = 11.22\text{cm}$

Thus; $1\text{cm} = 50\text{N}$
 $11.22\text{cm} = ? (R)$ $\Rightarrow R = \frac{11.22\text{cm} \times 50\text{N}}{1\text{cm}} = 561\text{N}$

Angle of Direction can be measured from horizontal to R which is $51.3^\circ + 16.1^\circ = 67.4^\circ\text{W}$

Alternatively; (NB; alternative solution is based on resolution of vector which is next part of this chapter)

Consider free body diagrams below

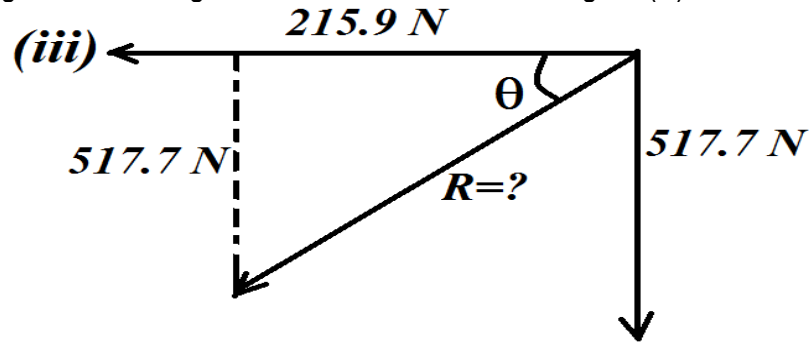


- Use the distances to find the horizontal angles (see figure (i) above)
- Now, resolve the angles vertically and horizontally (fig. (ii) above)

$$\sum F_x = -500\cos 51.3^\circ + 160\cos 52.8^\circ = -215.9 \text{ N (due West)}$$

$$\sum F_y = -500\sin 51.3^\circ - 160\sin 52.8^\circ = -517.7 \text{ N (due South)}$$

After resolving, then, the diagram will be as seen from the figure (iii) below



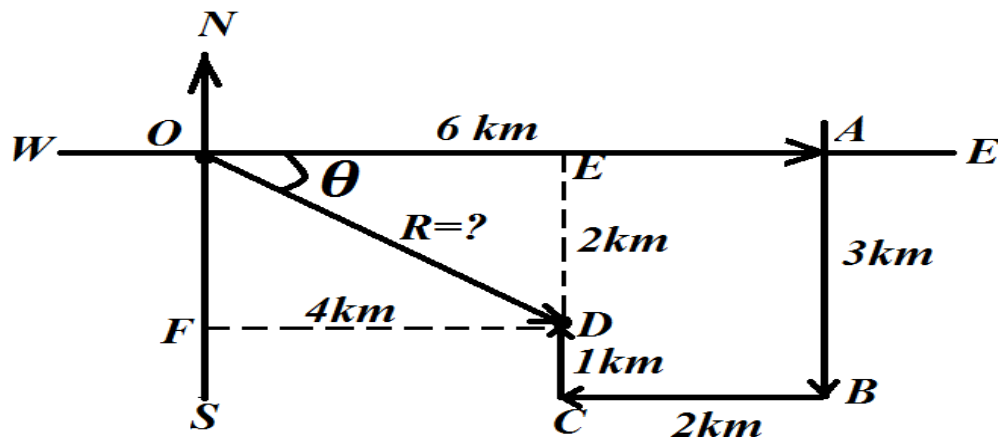
- From figure (iii) above, $\therefore R = \sqrt{(-215.9)^2 + (-517.7)^2} = 560.9 \text{ N}$

- Angle of direction, $\tan \theta = \frac{517.7}{215.9} \rightarrow \theta = \tan^{-1}(2.3979) = 67.36 \approx 67.4^\circ \text{SW}$

4. A truck goes 6 km east 3 km south, 2 km west and 1 km north. Find the resultant displacement.

Solution; Consider a free body diagram below

scale: 1cm represents 1km



From the figure above the resultant $R = 4.5 \text{ cm}$, since the scale used is $1\text{cm} = 1\text{km}$, then $4.5\text{cm} = ? \text{ km}$

When crossing multiplication, therefore $R = \frac{4.5 \text{ cm} \times 1\text{km}}{1\text{cm}} = 4.5 \text{ km}$

Direction of the truck when θ is measured is 26.6°

Hence, the resultant displacement is 4.5 km , 26.6° South of East.

Alternatively;

From the figure above, consider; ΔOED ; $R^2 = 2^2 + 4^2 = 20$

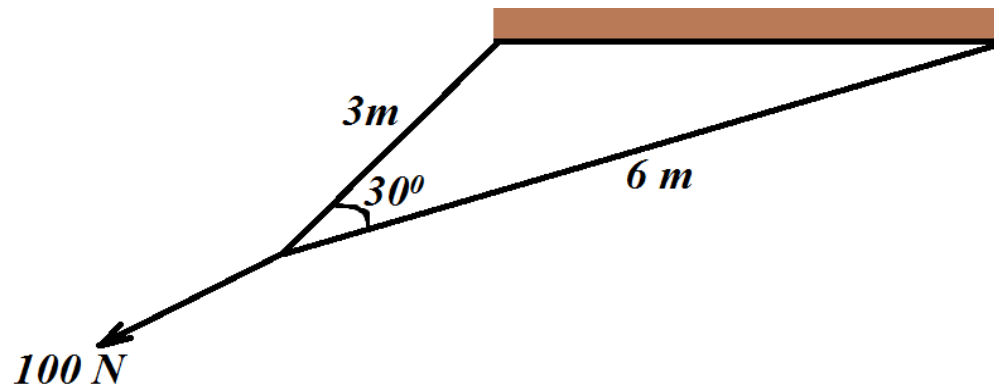
$$R = \sqrt{20} = 4.47 \text{ km}$$

Direction of the truck is $\tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{2}{4} \rightarrow \theta = \tan^{-1}(0.5) = 26.56^\circ$

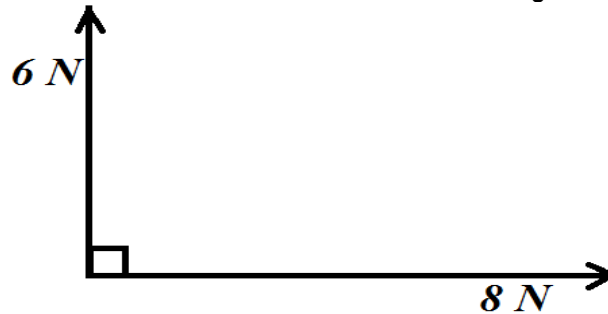
Class Activity – 1:2

- Two forces AB and AD of magnitude 40 N and 60 N respective are pulling a body on horizontal table. If the two forces makes an angle 30° between them, find the resultant force on the, body. (**ANS: R = 96.7N**)
- Find the resultant force, F, when two forces, 9 N and 15 N, act on an object with an angle of 60° between them. (**ANS: $F_R = 21 \text{ N}$**)

- Two forces of 6 N and 8 N are acting at a point such that the angle between them is 60 degrees. Find the resultant force. [ANS; R=12.16N]
- Two ropes of 3 m and 6 m long are tied to a ceiling and their free ends are pulled by a force of 100 N as shown in the figure below. Find the tensions in each rope if they make angle 30° between them. (A: $T_1 = 34.5\text{N}$, $T_2 = 69\text{N}$)



- Two forces, while acting on a particle in opposite; directions, have the resultant of 10N. If they act at right angles to each other, the resultant is found to be 50N. Find the two forces.
ANS; $F_1 - F_2 = R = 10\text{N}$ ---(i) $F_1^2 + F_2^2 = R^2 = 50^2$ ---(ii), Thus, $F_1 = 40\text{N}$, $F_2 = 30\text{N}$
- Two forces each of 10N act at an angle 60° with each other. Find the magnitude and direction of the resultant force. [ANS; R=17.3N, 30.1°]
- Find the resultant force when two forces act as shown in the figure below.



(ANS: $R_F = 10\text{N}$)

Relative Motion

- Relative Motion** is defined as the motion of an object when observed with respect to another object which may be either at rest or in motion.
- Example of Relative Motion:** Suppose you and your friend are traveling on a train sitting together and a boy standing alongside the track is observing you. Both of you don't change your position with respect to each other hence both of you are at rest with respect to each other but with respect to the boy who was standing outside you and your friend are in motion because your position is changing with time.
- Therefore, we observe that the state of motion is different for different observers making motion a relative concept rather. The difference in the observation of the boy standing outside and you sitting inside is because of the different **Frames of Reference**.
- Another example is**, if we consider a person sitting in an airplane, the person appears to be at a zero velocity relative to the airplane but actually moves with the same velocity as that of the plane taking into account the ground. In this case, the motion is dependent on the observer's **frame of reference**. The objects may be moving in the same or different directions with reference to each other.

Relative Velocity

- The relative velocity of the body comes into consideration when the other object moves either in the same or opposite direction. The speeds of the involved objects may be **increasing, decreasing, or constant** with reference to each other.
- Let us consider two objects **A** and **B** which are moving relative to each other. Then the relative velocity will be the velocity in which a body **A** would appear to body **B** and vice versa. Mathematically, we may say that the relative velocity will be **the vector difference between the velocities of two objects**.
- If the objects (**A and B**) are moving to the same direction, then **the vector difference between the velocities of two objects** is given by $V_{AB} = V_A - V_B$
- Similarly; when the objects (**A and B**) are moving to the opposite directions, then the velocity of one object say body B will be negative to signify the opposite direction relative to body A , ie,..
 $V_{AB} = V_A - (-V_B) \quad \therefore \mathbf{V_{AB} = V_A + V_B}$

Worked Examples

1. Two cars A and B moving in opposite direction along a straight road with constant velocities of 20 m/s and 40 m/s respectively. What is the velocity of B with respect to A?

ANS; $V_A = 20\text{m/s}$, $V_B = 40\text{m/s}$, $V_{BA}=?$

Since the cars are in opposite direction, then

$$\mathbf{V_{BA} = V_B - (-V_A) = V_B + V_A = 40 + 20 = 60 \text{ m/s}}$$

2. Two cars A and B are moving with velocities of 50 m/s and 30 m/s respectively. Find the relative velocity when they are in

(a) The same direction (b) Opposite direction

Soln:

Given: $V_A = 30 \text{ m/s}$, $V_B = 50 \text{ m/s}$

(a) In the same direction,

$$\text{From: } V_{AB} = V_A - V_B \quad \rightarrow \mathbf{V_{AB} = 50 - 30 = 20 \text{ m/s}}$$

(b) In opposite direction

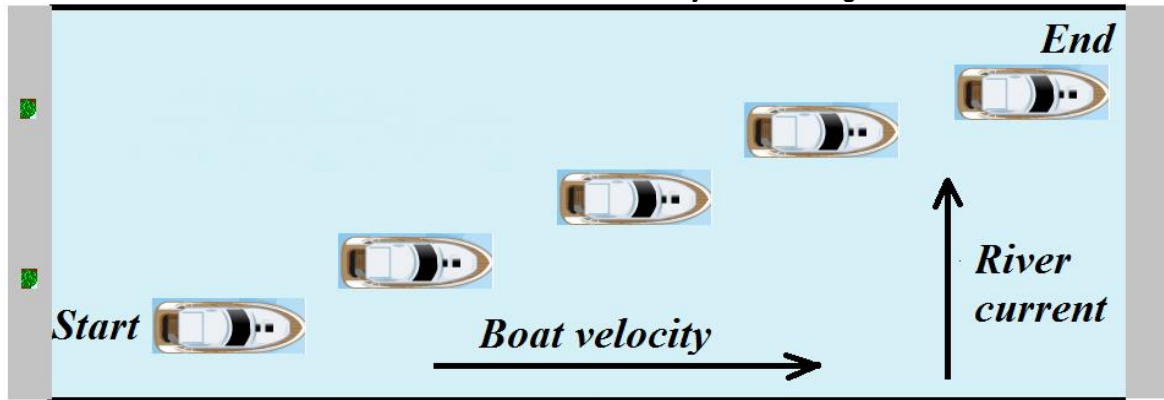
$$\text{From: } V_{AB} = V_A + V_B \quad \rightarrow \mathbf{V_{AB} = 50 + 30 = 80 \text{ m/s}}$$

- **On occasion objects move within a medium that is moving with respect to an observer.** For example, an airplane usually encounters a wind - air that is moving with respect to an observer on the ground below. As another example, a motorboat in a river is moving amidst a river current - water that is moving with respect to an observer on dry land.
- In such instances as this, the magnitude of the velocity of the moving object with respect to the observer on land will not be the same as the speedometer reading of the vehicle. That is to say, the speedometer on the motorboat might read 25 km/hr; yet the motorboat might be moving relative to the observer on shore at a speed of 30 km/hr.
- Motion is relative to the observer. The observer on land would measure the speed to be different than that of the person on the boat. The observed speed of the boat must always be described relative to who the observer is.

Relative Velocity and River Boat Problem

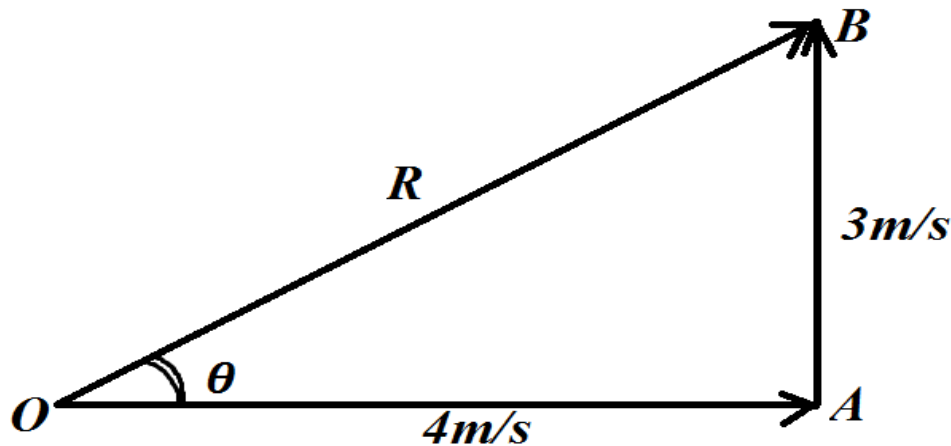
- Suppose that the river is moving with a velocity of 3 m/s, North and the motorboat is moving with a velocity of 4 m/s, What would be the resultant velocity of the motorboat?

- Since the boat heads straight across the river and since the current is always directed straight downstream, the two vectors are at right angles to each other. Thus, the **Pythagorean theorem** can be used to determine the Resultant velocity. See the figure below



motion of motor boat with current

- From the figure above, consider ΔOAB below

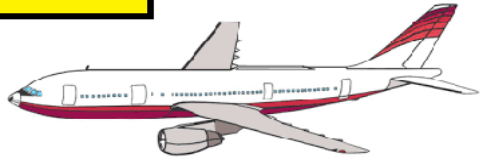


- Then, It follow that; $\overline{OA}^2 + \overline{AB}^2 = \overline{OB}^2 \rightarrow R^2 = 3^2 + 4^2 \rightarrow R = 5\text{m/s}$

Tailwinds, Headwinds, and Cross (Side) Winds

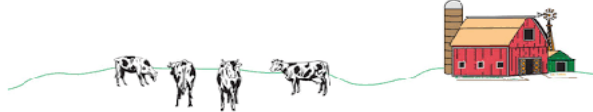
- There are three kinds of winds related to aerospace engineering i.e. an aircraft faces different types of winds in its journey; these **are** tailwind, headwind, **and** crosswind.
- **A** tailwind is defined as a type of wind which blows in the same direction of the object, as the wind blows in the same direction of the plane; so, the speed of the plane increases and it takes less time to reach the destination aimed.
- Consider, If the plane is traveling at a velocity of 100 km/hr with respect to the air, and if the wind velocity is 25 km/hr, then the plane travels with a resulting velocity of 125 km/hr relative to the ground as shown in the diagram below.

Tailwind



$wind\ speed = 25km/h$

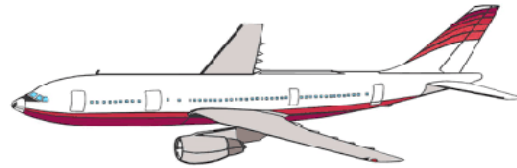
$plane\ speed = +100km/h$



$Ground\ speed, R = wind\ speed + plane\ speed = 125km/h$

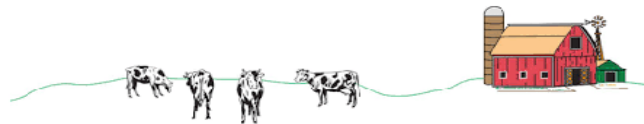
- A **headwind** in aerospace engineering is defined as a type of wind which blows in the opposite direction of the object, as the wind blows in the opposite direction of the plane; so, the speed of the plane decreases and it takes more time to reach the destination aimed.
- Suppose a plane traveling with a velocity of 100 km/hr with respect to the air meets a headwind with a velocity of 25 km/hr. In this case, the resultant velocity would be 75 km/hr; this is the velocity of the plane relative to an observer on the ground. This is depicted in the diagram below.

Headwind



$wind\ speed = -25km/h$

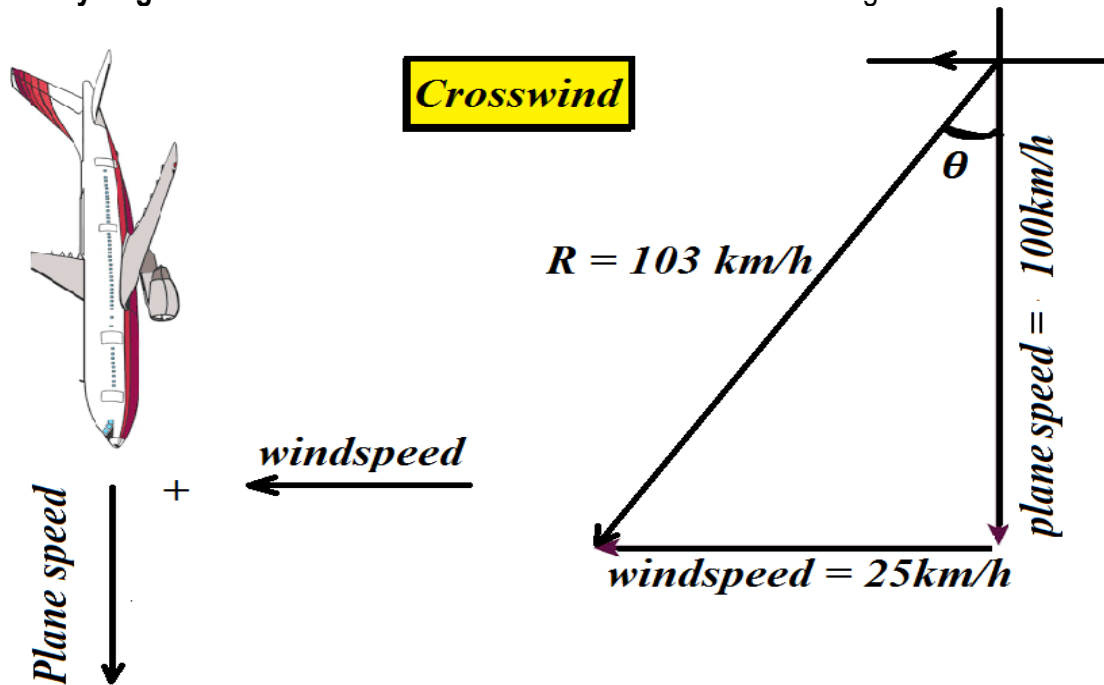
$plane\ speed = +100km/h$



$Ground\ speed, R = plane\ speed - wind\ speed = 75km/h$

- A **crosswind** in aerospace engineering is defined as a type of wind which blows in perpendicular components to the path of travel.
- Now consider a plane traveling with a velocity of 100 km/hr, South that encounters a **side wind** of 25 km/hr, The resulting velocity of the plane is the vector sum of the two individual velocities. To determine the resultant velocity, the plane velocity (relative to the air) must be added to the wind velocity.

- This is the same procedure that was used above for the headwind and the tailwind situations; only now, the resultant is not as easily computed. Since the two vectors to be added - the southward plane velocity and the westward wind velocity - are at right angles to each other, the **Pythagorean theorem** can be used. This is illustrated in the diagram below.



From Pythagoras theorem:

$$R^2 = 100^2 + 25^2 \Rightarrow R = \sqrt{10625} = 103\text{km/h}$$

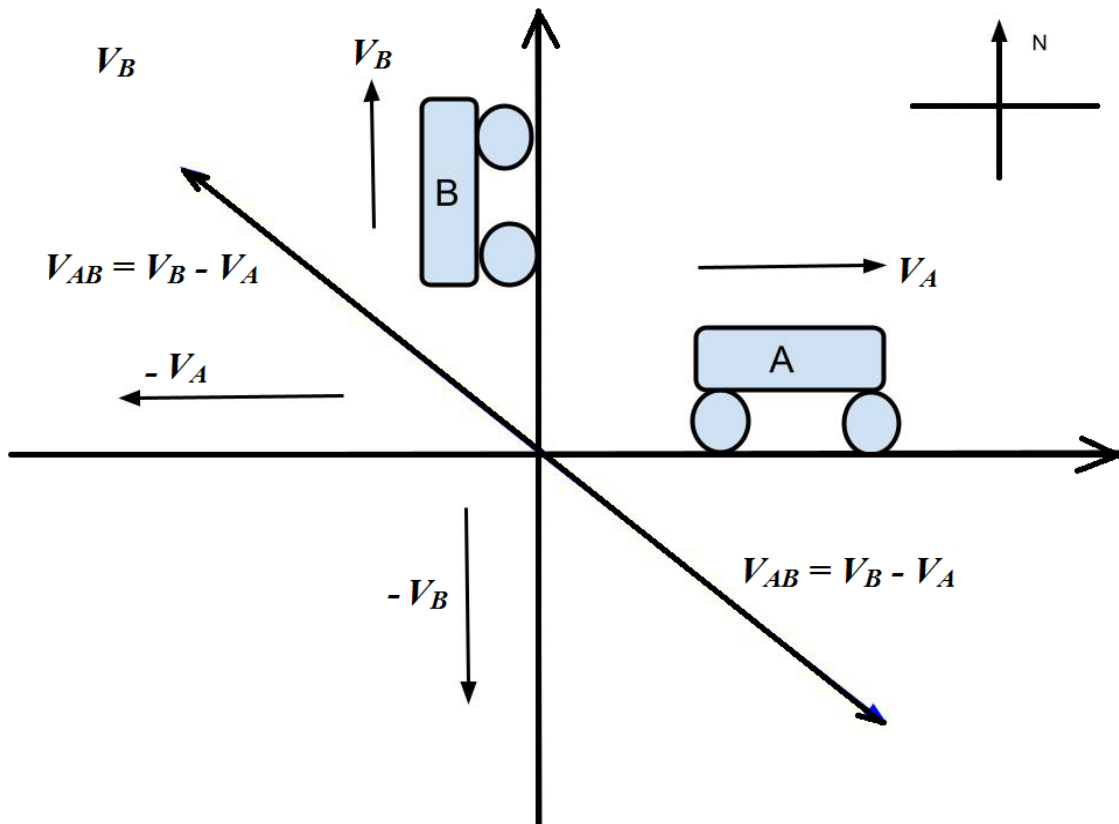
- The direction of the resulting velocity can be determined using a trigonometric function. Since the plane velocity and the wind velocity form a right triangle when added together in head-to-tail fashion, the angle between the resultant vector and the southward vector can be determined using the sine, cosine, or tangent functions.
- From the figure above, it follows that;

$$\sin\theta = \frac{W}{R} = \frac{25}{103} \rightarrow \theta = \sin^{-1}(0.2427) = 14^\circ$$
- If the resultant velocity of the plane makes a 14° angle with the southward direction, then the direction of the resultant is $270^\circ - 14^\circ = 256^\circ$. This is because the resultant vector's direction is measured as a counterclockwise angle of rotation from due East.

Class Activity – 1:3

- Two trains A and B are moving along parallel tracks with the same velocity in the same direction. If the velocity of each train be 50 km/h due east. Calculate the relative velocities of the trains **[ANS; $V_{AB} = 0\text{km/h}$ OR $V_{BA} = 0\text{km/h}$]**
- Suppose two trains A and B are moving with uniform velocities along parallel tracks but in opposite directions. Let the velocity of train A be 40 km/h due east and that of train B be 40 km/h due west. Calculate the relative velocities of the trains **[ANS; 89 km/h]**
- Two cars, initially 900 m distant apart, start moving towards each other with speeds 1 m/s and 2 m/s along a straight road. When would they meet?
ANS; they will meet after 300 sec
- Car A is moving with a velocity of 20 m/s while car B is moving with a velocity of 30 m/s. Calculate the velocity of car B relative to car A if:

- (a) They are moving in the same direction (**ANS: $V_{BA} = 10 \text{ m/s}$**)
 (b) They are moving in the opposite directions. (**ANS: $V_{BA} = 50 \text{ m/s}$**)
5. Two cars A and B are moving in opposite directions on the same road at 62 km/h and 31 km/h respectively, determine the velocity of car A relative to car B [**A;93km/h**]
6. Two trucks, standing a few distances apart, start moving towards each other with speeds 1 m/s and 2 m/s along a straight road. What is the speed with which they approach each other?
ANS; two trucks move towards each other each with a relative speed of 3 m/s.
7. A boat is headed north in a river that flows due east at a velocity of 5.0m/s. If the resultant velocity of this boat is 8.7m/s N35°E, what is the speed of the boat with respect to the water [**ANS; 7.12m/s**]
8. Jackson is driving 30km, west then he turns 40km south. Determine the resultant displacement.
[ANS; R = 50 km, S 53.1° W]
9. A Car is travelling at 60m/s due east and a lorry is travelling at 100m/s due north. What is the velocity of the car relative to the lorry? (**ANS: 116.6 m/s at 31° NE**)
10. An automobile A, travelling relative to the earth at 45km/h on a straight level road is ahead of motor cycle B travelling in the same direction at 90 km/h .What is the velocity of B relative to A?
(ANS: $V_{BA} = 45\text{km/h}$)
11. A passenger at the back of a train travelling at 15m/s relative to the earth , throws a hammer with a speed of 15 m/s in the opposite direction to the motion of the train .What is the velocity of the hammer relative to the earth (**ANS: $V_{HE} = 0 \text{ m/s}$**)
12. An airplane heads due north at 100 m/s and a 30 m/s cross wind blows from the east to the west. Find the resultant velocity of the airplane. [**A; $V_R = 104 \text{ m/s}$**]
13. A bus starting from a point travels towards the east with a velocity of 3 m/s. Another bus starting from the point travels towards the north with a velocity of 4 m/s. Find the magnitude of the relative velocity of one with respect to another. [**ANS; consider a free body diagram below**]



Relative velocity V_{BA} of Bus-B with respect to Bus-A is towards the west of north direction as shown in the figure above

Thus the magnitude of $V_{BA} = \sqrt{3^2 + 4^2} = 5\text{m/s}$

OR, Relative velocity V_{AB} of Bus-A with respect to Bus-B is towards the south of east direction as shown in the figure, so magnitude is

$$V_{AB} = \sqrt{4^2 + 3^2} = 5\text{m/s}$$

14. A motorboat traveling 5 m/s, East encounters a current traveling 2.5 m/s, South
- What is the resultant velocity of the motor boat?
 - If the width of the river is 80 meters wide, then how much time does it take the boat to travel shore to shore?
 - What distance downstream does the boat reach the opposite shore?

[ANS: 5.59m/s, 16s, 40m]

15. A boat heading due north crosses a wider river with a velocity of 36 m/s relative to the water. The river has a uniform velocity of 12 m/s due south

- Determine the velocity of the boat with respect to an observer on the river bank
- If the river was flowing due east, determine the velocity of the boat with respect to an observer on the river bank (ANS: (a) 24 m/s (b) 37.947m/s)

16. A person can swim in water with a speed of 13 km/hr in still water. If the speed of the stream is 4 km/hr, what will be the time taken by the person to go 68 km downstream? [ANS: 4hrs]

17. Consider yourself sitting on a train moving at 80 km/h.

- What is the velocity of the train relative to the track? [ANS; 80km/h]
- What is the velocity of the track relative to you? [ANS; -80km/h]
- Another train running on a parallel track moves past you in the opposite direction at a speed of 50 km/h. What is the velocity of this train relative to you? [ANS; 130km/h]

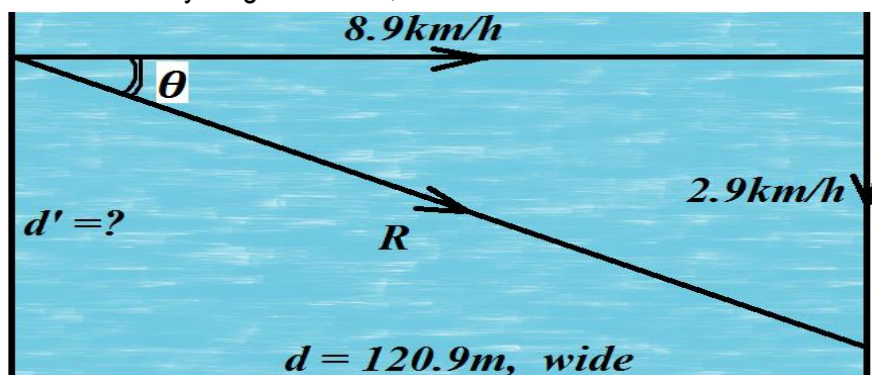
18. What is the wind velocity experienced by a cyclist travelling due east at 20 km/h if the wind is blowing from the west at 40 km/h? [ANS; 20 km/h due west]

19. With a headwind, an aircraft travels 235 nautical miles in an hour. With a tailwind, the aircraft travels 245 nautical miles in an hour. Determine the speed of the wind, and speed of the aircraft in still air. Knots are units of speed which are nautical miles per hour.

[ANS; $W_v = 5\text{knot}$, $A_v = 240\text{knots}$]

20. A boat is moving at 8.9km/h relative to the water in a river. Initially the boat at the bank aims straight to the opposite bank of the river which is 120.9 m wide. If the speed of the water in the river is 2.9km/h, how far downstream will the boat be when it reaches the opposite bank?

ANS; Consider a free body diagram below;



- The time a boat takes to travel shore to shore, $t = \frac{\text{distance}}{\text{velocity}}$

$$t = \frac{\text{distance}}{\text{velocity}} = \frac{d}{v} = \frac{120.9\text{m}}{8.9\text{km/h}} = \frac{0.1209\text{km}}{8.9\text{km/h}} = 0.0136\text{hrs}$$

- Downstream distance of a boat to reach the opposite shore, d'

Distance, $d' = \text{velocity of stream} \times \text{time}$

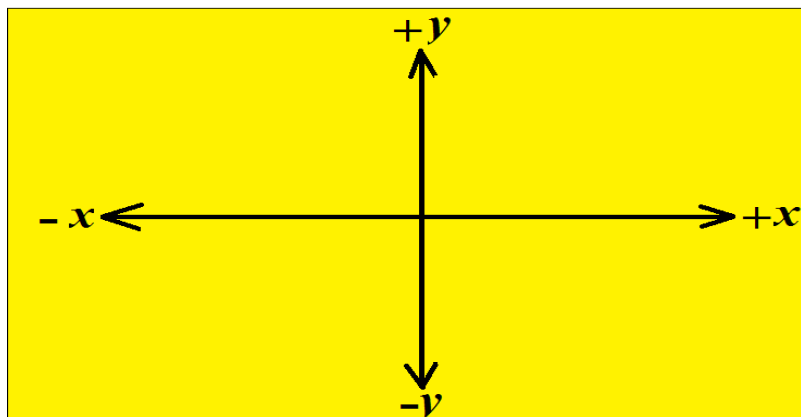
$$\therefore d' = 2.9 \times 0.0136 = 0.03944\text{ km (39.44 m)}$$

Application of relative motion

- Is used in navigation to determine the actual velocity of vessels in moving water or in air
- Is used to determine the velocities of stars and asteroids with respect to earth's velocity
- Used by structural engineers to design structures to avoid maximum deformation when earthquakes strike
- Helps us to calculate the velocity of an object in a fluid

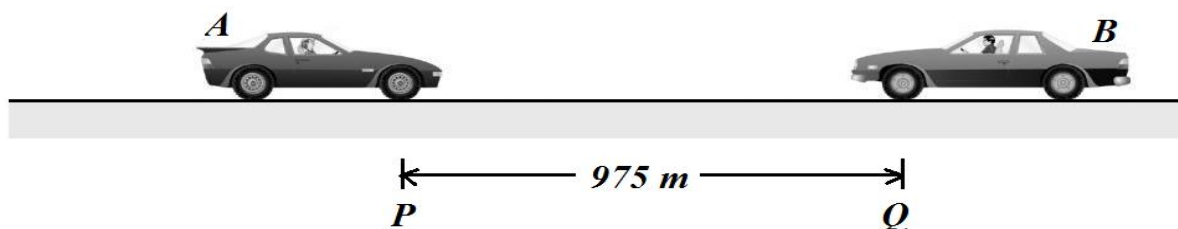
Coordinate Systems for One-Dimensional Motion

- In order to describe the direction of a vector quantity, you must designate a coordinate system within the reference frame. (see fig. below)
- **For one-dimensional motion**, this is a simple coordinate system consisting of a one-dimensional coordinate line.
- In general, when describing horizontal motion, motion to the **right is usually considered positive**, and motion to **the left is considered negative**.
- With vertical motion, **motion up is usually positive and motion down is negative**. In some cases, however, it can be more convenient to switch the positive and negative directions.
- **For example**, if you are analyzing the motion of falling objects, it can be useful to define downwards as the positive direction. If people in a race are running to the left, it is useful to define left as the positive direction. It does not matter as long as the system is clear and consistent. Once you assign a positive direction and start solving a problem, **you cannot change it**.



Worked examples

1. Two motorcars, A and B are approaching each other on a high way as seen from the figure below. At time $t=0$, car A and B are 975m apart at point P and Q, with their speeds being, $u_A=30\text{m/s}$ and $u_B = 17\text{m/s}$, respectively. Car A passes point Q after 40seconds and car B passes point P after 42seconds.



- (a) Find their acceleration.
- (b) After how long will the cars pass each other.
- (c) What is the speed of car B relative to car A when they pass each other.

ANS;

(a) $u_A = 30\text{m/s}$, $u_B = 17\text{m/s}$, $t_A = 40\text{s}$, $t_B = 42\text{s}$

From; $s = ut + \frac{1}{2}at^2$, But; when A passes Q and B passes P, $s = s_A = s_B$

then; $s = u_A t_A + \frac{1}{2} a_A t_A^2$, $s = u_B t_B + \frac{1}{2} a_B t_B^2$

For car A: $975 = 30 \times 40 + \frac{1}{2} a_A 40^2 \rightarrow (975 - 1200) = +\frac{1}{2} a_A 40^2$

$$-225 \times 2 = a_A 42^2 \rightarrow a_A = \frac{-450}{40^2} = -0.28\text{m/s}^2$$

For car B: $975 = 17 \times 42 + \frac{1}{2} a_B 42^2 \rightarrow (975 - 714) = +\frac{1}{2} a_B 42^2$

$$261 \times 2 = a_B 42^2 \rightarrow a_B = \frac{522}{42^2} = 0.3\text{m/s}^2$$

(b) Let t be the time whereby the cars will pass each other, ie, $t = t_A = t_B$

So; at that time they pass each other, Car A has already travelled distance x m, while Car B has travelled $(975 - x)\text{m}$ (ie, $s_A + s_B = 975\text{m}$)

Then; $s_A + s_B = 975\text{m} \rightarrow (u_A t + \frac{1}{2} a_A t^2) + (u_B t + \frac{1}{2} a_B t^2) = 975$

$$\left(30t + \frac{1}{2} \times -0.28 \times t^2\right) + \left(17t + \frac{1}{2} \times 0.3 \times t^2\right) = 975$$

$$(30t - 0.14t^2) + (17t + 0.15t^2) = 975 \rightarrow 47t + 0.01t^2 = 975$$

$$47t + 0.01t^2 = 975 \rightarrow 0.01t^2 + 47t - 975 = 0 \rightarrow t^2 + 4700t - 97500 = 0$$

By using general equation; $t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4700 \pm \sqrt{(4700)^2 - (4 \times 1 \times -97500)}}{2 \times 1}$

$$t = \frac{-4700 + 4741.3}{2} = \frac{41.3}{2} = 20.65 \text{ OR } t = \frac{-4700 - 4741.3}{2} = \frac{9441.3}{2} = -4720.65$$

Since time cannot be negative, we only consider a positive value, $\therefore t = 20.65\text{s}$

Alternatively;

From; $v = u + at \rightarrow t = \frac{\Delta v}{a} \rightarrow t_A = \frac{\Delta v_A}{a_A}$ also, $t_B = \frac{\Delta v_B}{a_B} - (ii)$, then, use any

equation (i) or (ii) to solve for t

NB; here you must find the speeds for each car at the time they meet. (v_A & v_B)

(c) Speed of car B relative to car A when they pass each other;

First, find the speeds of both cars at that time they meet;

$$v_A = u_A + a_A t = 30 + (-0.28 \times 20.7) = 30 - 5.8 = 24.2$$

$$v_B = u_B + a_B t = 17 + 0.3 \times 20.7 = 17 + 6.2 = 23.2$$

$$\therefore V_{BA} = V_B - (-V_A) = V_B + V_A = 23.2 + 24.2 = 47.4\text{m/s}$$

2. A man is walking inside a bus which is travelling at 56.2 km/hr. If the speed of the man relative to the ground is 55.8 km/h, is the man walking towards the front or the back?

ANS;

According to relative velocity fundamental definition,

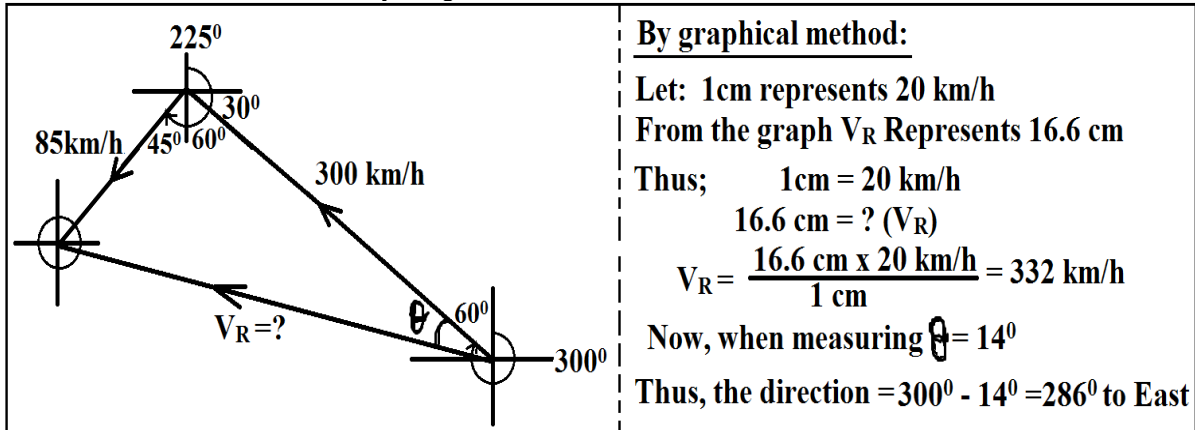
Relative velocity of the given man with respect to the given bus

$$\begin{aligned} & (\text{Velocity of the man w.r.t ground}) - (\text{velocity of the bus w.r.t ground}) \\ & = 55.8 \text{ km/hr} - 56.2 \text{ km/hr} = -0.4 \text{ km/hr} \end{aligned}$$

Therefore from above it is showing that; the relative velocity of the man with respect to the bus is in opposite direction (as indicated by the '-' sign), hence we conclude that the man is walking towards the back of the bus.

3. A plane is flying at a velocity of 300 km/h relative to the ground in the direction 300° from East. The plane is flying amidst a wind blowing at 85km/h relative to the ground in the direction 225° from East. What is the velocity of the plane relative to the air?

ANSW: Consider a free body diagram



Alternatively; By Cosine Rule:

$$V^2 = 300^2 + 85^2 - 2 \times 300 \times 85 \times \cos 105^\circ = 110,424.8$$

$$\therefore V_R = \sqrt{110,424.8} = 332.3 \text{ km/h}$$

$$\theta = \cos^{-1} \left(\frac{85^2 - 300^2 - 332.3^2}{-2 \times 300 \times 332.3} \right) = \cos^{-1}(0.969) = 14.3^\circ$$

Thus; the direction = $300^\circ - 14.3^\circ = 285.7^\circ$ from East

Class Activity: 1 – 4

- Two cars, A and B, are moving at speeds of 70km/h and 90 km/h, respectively. What is the speed of car A relative car B if the two cars are moving in:
 - The same direction [ANS; -20km/h]
 - The opposite direction [ANS; 160 km/h]
- If a man is walking to the back of the bus with a speed of 2.5mph with respect to the bus and the bus is travelling forward with a speed of 35.0 mph with respect to the ground. What is the velocity of the man with respect to the ground?
- A boat moving at a speed of 9.8km/h relative to the water in a river wants to get to a fishing camp that is 5.2 km upstream. If the speed of the water in the river is 6.0km/h, how long will it take the boat to reach the camp?

ANS;

Since the boat is moving in the opposite direction to the stream. In this case, the net speed of the boat is called the upstream speed

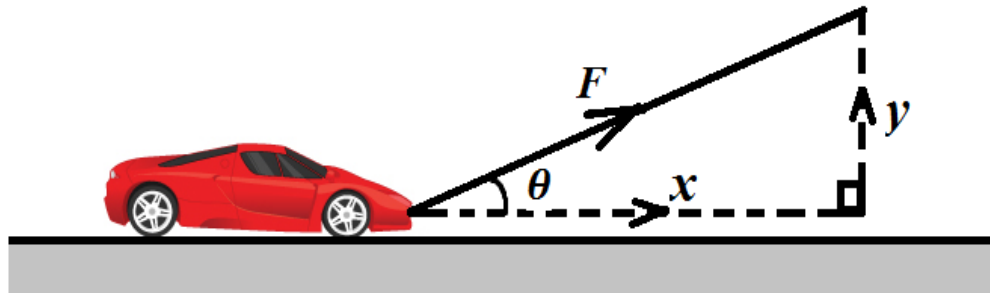
$$\text{Thus; the upstream speed} = v' = u - v = 9.8 - 6 = 3.8 \text{ km/h}$$

Whereby u speed of the boat in still water and v speed of the stream

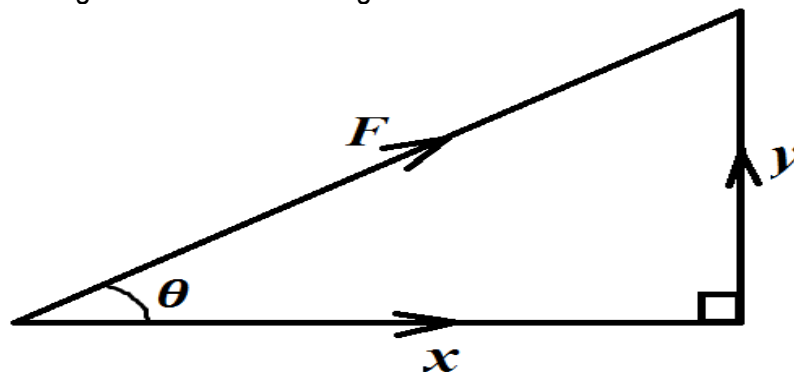
$$\text{But; speed, } v' = \frac{\text{distance}}{\text{time}} \therefore \text{time} = \frac{\text{distance}}{\text{speed}} = \frac{5.2}{3.8} = 1.37 \text{ hrs}$$

Resolution of the Vector

- **Resolution of vector, Is the process of splitting a vector into its components.** That is a horizontal and vertical components.
- Consider the diagram below where the toy car is pulled at a certain angle but it seems to move horizontally due to horizontal force, not only that but also vertical force is formed



- From the figure above: **Horizontal vector = x** and **Vertical vector = y**
- Then, extract the triangle from the above diagram



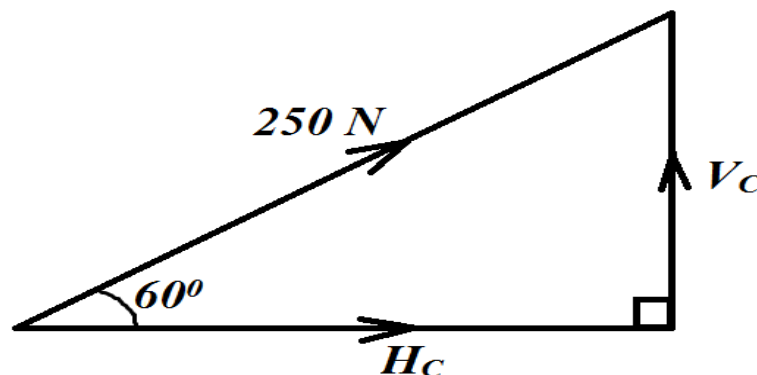
Horizontal vector is given by the formula; **From:** $\text{Cos}\theta = \frac{x}{F} \quad \therefore H_c(x) = F\text{Cos}\theta$

Vertical vector is given by the formula **From:** $\text{Sin}\theta = \frac{y}{F} \quad \therefore V_c(y) = F\text{Sin}\theta$

Worked examples

1. Resolve a force of **250 N** acting at **60°** to the horizontal into its horizontal and vertical components.

ANS;



From the figure above;

Horizontal component, $H_c = F\text{Cos}\theta = 250 \times \text{Cos}60^\circ = 125\text{N}$

Vertical component; $V_c = F\text{Sin}\theta = 250 \times \text{Sin}60^\circ = 216.5\text{N}$

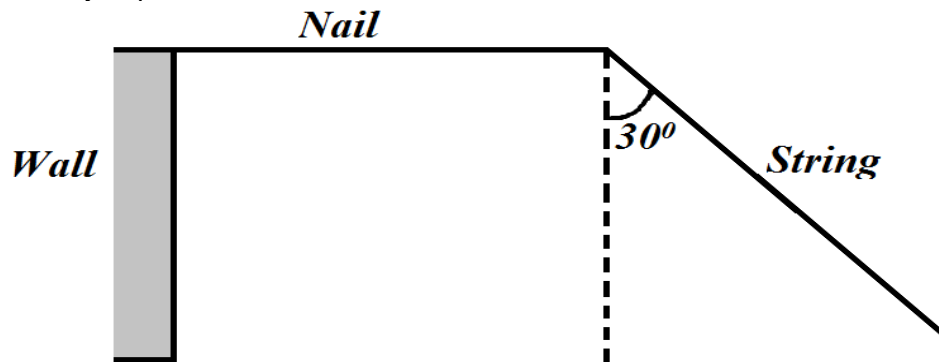
2. Resolve a force of **100 N** acting at **30°** to the horizontal into its horizontal and vertical components. **[ANS; $H_c(x) = F\text{Cos}\theta = 86.6\text{N}$, $V_c(y) = F\text{Sin}\theta = 50\text{N}$]**
3. Resolve a force of **250 N** acting at **60°** to the vertical into its horizontal and vertical components

ANS; $H_c(x) = F \cos \theta = 250 \times \cos 30 = 216.5 \text{ N}$, $V_c(y) = F \sin \theta = 125 \text{ N}$
OR; $H_c(x) = F \sin \theta = 250 \times \sin 60 = 216.5 \text{ N}$, $V_c(y) = F \cos \theta = 125 \text{ N}$

4. A horse exerts a pull of 600 N on a rope attached to a barge. The rope makes an angle of 30° with the direction in which the barge is travelling. Find
- the force which is pulling the barge forwards, **[ANS; 520N]**
 - the force which tends to pull the barge sideways. **[ANS; 300N]**

Class Activity– 1:5

1. A nail is being pulled using a string from a wall. The string forms an angle of 30° with the normal. If the force being used is 10 N, part of the force will tend to bend the nail while the other part will try to pull it out.



What is the magnitude of the force:

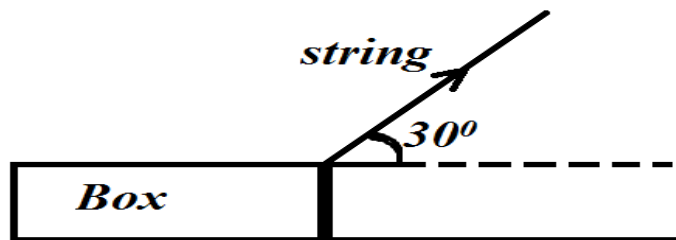
- Tend to bend the nails? **(ANS: $F_1 = 8.66 \text{ N}$)**
 - Tend to pull the nails out? **(ANS: $F_2 = 5.0 \text{ N}$)**
2. A body is being acted on by two forces: $F_1 = 18 \text{ N}$ acting at an angle of 25° and $F_2 = 30 \text{ N}$ acting at 140° from due East. Find the resultant of the two forces, F , by separating the forces into x- and y- components. **(ANS: $R_F = 27.70 \text{ N}$ at an angle of 103.94° to west)**
3. A trolley was pulled by a force of 60 N acting 30° to the horizontal. Find the vertical component of this force. **ANS: $V_c = 30 \text{ N}$**
4. A boy pulls his toy on a smooth horizontal surface with a rope inclined at an angle 60° to the horizontal. If the effective force pulling the toy along the horizontal surface is **5N**, calculate the tension in the rope? **[ANS; $H_c = 5\text{N}$, $T=?$, $\cos \theta = \frac{H_c}{T} \rightarrow T = \frac{H_c}{\cos \theta} = \frac{5}{0.5} = 10\text{N}$]**
5. A block of wood of weight 12 N rests on a slope angled at 30° to the horizontal. Find
- the component of the block's weight acting down the slope, **[ANS; 6N]**
 - the component of the block weight acting into the surface of the slope. **[ANS; 10.4N]**
6. A boy pulls a nail from the wall with a string tied to a nail. The string is inclined at angle 60° to the wall. If the tension in the string is 4N, what is the effective force used in pulling the nail? **[ANS; 3.46N]**
7. An artillery shell is fired with a velocity of 500 m/s at an angle of 37° to the horizontal. Find
- the vertical component of the initial velocity **[ANS; 300.9 m/s]**
 - the horizontal component of the initial velocity. **[ANS; 399.3m/s]**
8. A weight of 20 N rests on a plane inclined at 40° to the horizontal. What are the components of the weight parallel and perpendicular to the plane? **(AN: $C_{\text{Parallel}} = 12.865 \text{ N}$, $C_{\text{Perp}} = 15.32 \text{ N}$)**
9. A man pushes a lawnmower across a horizontal lawn with a force of 50 N at an angle of 50° to the horizontal. What is the component of the force which is moving the mower across the grass? **[ANS; 32N]**

Uses of Component Vectors

- ✓ Vector components are used in vector algebra to add, subtract, and multiply vectors.
- ✓ Vector components allow us to break a single vector quantity into two (or more) scalar quantities

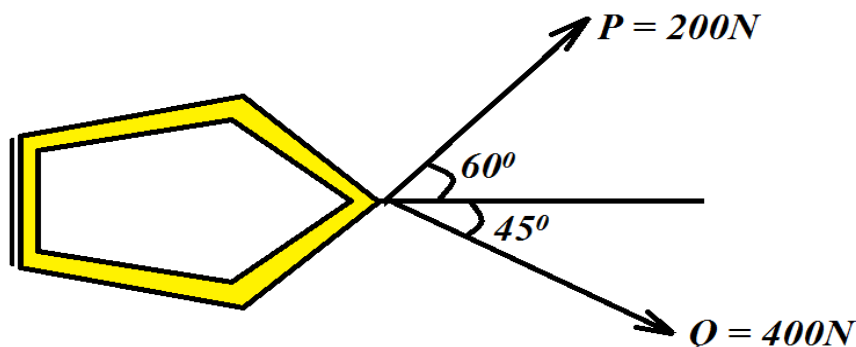
Self Assessment –1

- The velocity of car B relative to car A is 8 m/s when the two cars are moving in the same direction and 28 m/s when the two cars are moving in opposite directions. Determine the velocity of each car
- An aeroplane is taking off at a velocity of 20 m/s. Find the components of the plane's velocity if the take – off angle is (a) 70° (b) 45° (c) 60°
[ANS; (a) 6.84m/s and 18.79m/s, (b)14.14m/s and 14.14m/s, (c) 10m/s and 17.32m/s]
- A river is flowing at a velocity of 2m/s due south .A person in a boat wants to move across the river at 10 m/s.
 - In which direction should the person move?
 - At what velocity should the person move the boat?
- A box is being pulled on the floor using a string. The string makes an angle of 30° with the box as shown in the figure below



If the force being applied at the string is 200 N, find:

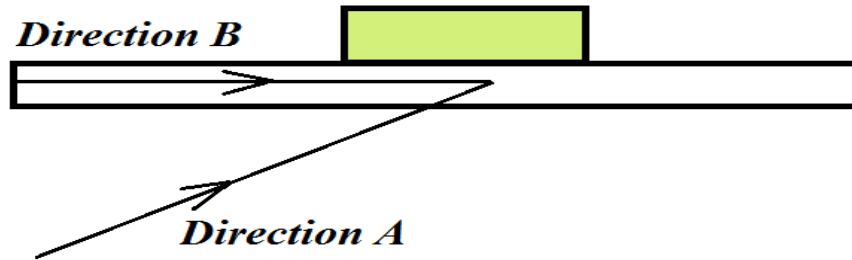
- The force which tends to lift the box [ANS; 100N]
 - The force which tends to pull the box forward [ANS; 173.2N]
- A car moves 5 km east 3 km south, 2 km west and 1 km north. Find the resultant displacement (ANS: 3.6 km 34° south of east)
 - An airplane is flying east at 200 km/h, which is its velocity relative to the air ,while a 100 km/h wind blows towards the north – east .What is its resultant velocity ? (A: $V = 280\text{km/h}$ at 14.6°)
 - A plane is flying due east with a velocity of 100 m/s when it encounters a wind moving at a velocity of 20 m/s . Find the resultant velocity of the plane if the direction of the wind is due
 - East
 - West
 - South
 - South – East
 - An aeroplane is taking off at a velocity of 20 m/s. Find the components of the plane's velocity if the take – off angle is (a) 70° (b) 45° (c) 60°
 - Two forces ,P and Q are applied on a small boat stuck in a shallow stream as shown below



Determine the magnitude and direction of the resultant of the two forces

[ANS; 398.5N, 16° below the horizontal]

13. The diagram in the figure below shows a block being pushed along a track .If a force of 20 N is applied in direction A at an angle of 60°, what is the resolved part of the force in direction B ?



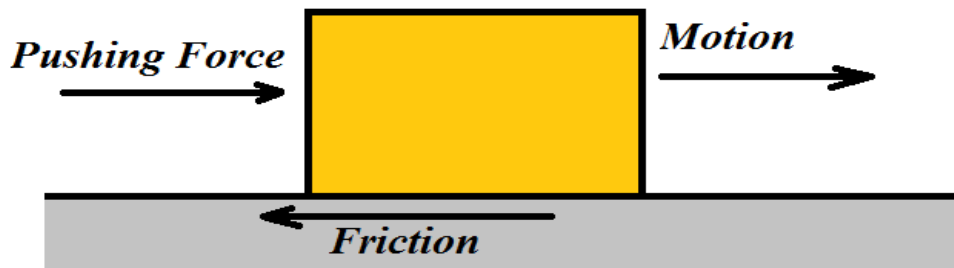
14. A Vehicle moving at a speed of 80km/hr emits smoke from its exhaust pipe in opposite direction at 50 km/hr with respect to the vehicle .Determine the speed of smoke with respect to the ground
15. When two motorbikes are approaching each other at constant speed ,the linear distance between them decreases at 7 km/hr .When moving in the same direction ,the linear distance between them decreases at 3 km/h Determine the velocity of each motorbike
16. **A car is travelling due north at 45 km/hr .It turns and then travels due east at 72 km/hr .Find the magnitude and direction of the resultant velocity of the car**
17. A Mass 3 kg hangs at the end of a string .Find the horizontal force needed to pull the mass sideways until the string is at 30° to the vertical .Find also the tension in the string
18. **An air craft heads north – west at 320 km/hr relative to the wind .The wind velocity is 80km/hr from the south .Find the velocity of the aircraft relative to the ground**
19. A deep sea diver dives at an angle of 30° with the surface of water and follows a straight – line path for a distance of 220 m. How far is the diver from the surface of water?
20. **A velocity of magnitude 40 m/s is directed at an angle of 40° east of north .Represent this velocity on paper**
21. A car travels 3 km due north ,then 5 km north–east .Represent these displacements graphically and determine the resultant displacement
22. Two forces , one of 12 N and another of 24 N, act on a body in such a way they make an angle of 30° with each other .Find the resultant of the two forces
23. **A plane can travel with a speed of 80 mil/hr with respect to the air .Determine the resultant velocity of the plane if it encounters a**
- (a) 10 mil/hr headwind (c) 10 mil/hr crosswind
(b) 10 mil/hr tailwind (d) 60 mil/hr crosswind
24. Find the horizontal and vertical components of a force of 10 N acting at 30° to the vertical
25. **A weight of 25 N is suspended from a beam by a string; what horizontal force must be applied to the weight to keep the string at an angle of 20° to the vertical .What is the tension in the string?**
26. **A car covered a displacement of 10 km due 30° ,then15 km due 120° and finally 8 km due 270° .Find the total displacement covered by the car**
27. **A man using a 70 kg garden roller on a level surface ,exerts a force of 200 N at 45° to the ground .Find the vertical force of the roller on the ground**
- (a) If he pulls (ANS: 560 N) (b) If he pushes the roller (ANS: 840 N)
28. **A plane mirror is approaching you at a speed of 10 m/s you can see your image in it. At what speed will your image approach you? (ANS: v = 20 m/s)**
29. How long will a boy sitting near the window of a train travelling at 36 km/h see a train passing by in the opposite direction with a speed of 18 km/h. The length of the slow-moving train is 90 m.[ANS; $t = \frac{s}{v_R} = \frac{90}{15} = 6s$]

Chapter – 02: Friction

Friction is a force between two surfaces that are sliding, or trying to slide, across each other.

OR; Friction – Is the force that opposes motion between any surfaces that are in contact

- **For example**, when you try to push a book along the floor, friction makes this difficult. Friction always works in the direction opposite to the direction in which the object is moving, or trying to move.
- Friction always slows a moving object down.



- The amount of friction depends on the materials from which the two surfaces are made. The rougher the surface, the more friction is produced.
- Friction also produces heat. If you rub your hands together quickly, you will feel them get warmer.
- Friction force which occurs in fluids (liquid or gas) is known as **viscosity**

Causes of Friction

Friction is caused by molecular adhesion, surface roughness and deformations

Adhesion between body surfaces:

- Adhesion is the molecular force resulting when two materials are brought into close contact with each other. Trying to slide objects against each other requires breaking these adhesive bonds.
- For years, scientists thought that friction was caused by surface roughness, but recent studies have shown that it is actually a result of adhesive forces between the materials

Surface roughness:

- When two rough surfaces come into contact with each other, they generate frictional force or an opposing force, which can sometimes be converted into heat.
- All solid materials have some degree of surface roughness. If you looked at what seems to be a smooth surface under a high-powered microscope, you would see bumps, hills and valleys that could interfere with sliding motion.

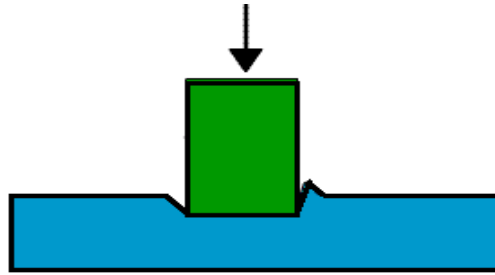


Close up - view of surface roughness

- At one time it was thought that the surface roughness of materials was the cause for friction. In reality, it only has a small effect on friction for most materials.

Body deformation:

- During motion, deformations in the body or the surface of the body in contact may cause friction. Soft materials will deform when under pressure. This also increases the resistance to motion.
- **For example**, when you stand on a rug, you sink in slightly, which causes resistance when you try to drag your feet along the rug's surface.
- **Another example** is how rubber tires flatten out at the area on contact with the road. When materials deform, you must "plow" through to move, thus creating a resistive force.



Pushing Object on soft surface

- When the deformation becomes large, such that one object sinks into the other, streamlining can affect the friction, similar to what happens in fluid friction.

Advantage of friction

- ✓ It aids in walking and movement
- ✓ Helps moving body to stop by applying brakes
- ✓ Used to wear unneeded layers of some materials
- ✓ Causes lighting in match stick
- ✓ Supports life on the earth by preventing burning asteroids
- ✓ Causes nail to stick on the wood
- ✓ Enables bottle stopper to stick on the bottle neck

Disadvantage of Friction

- ✓ It causes wear and tear
- ✓ Friction causes the wear and tear in mechanical parts of automobiles and vehicles. This is the reason why lubricants are used to smoothen the surfaces of brakes, clutches, or pads of vehicles
- ✓ It produces heat in various machine parts causing efficiency to decrease
- ✓ Due to friction noise is produced in machine
- ✓ Causes loss of energy in form of heat and sound
- ✓ The force of friction acts in the opposite direction of motion, so friction slows down the motion of moving objects.
- ✓ Forest fires are caused due to the friction between tree branches.
- ✓ Heat produced by friction can cause appliance to burn
- ✓ It causes wounding, when skin wearing

Methods of increasing Friction

- Increasing the normal force by increasing the weight of the body
- Increasing the roughness of the surface
- Use materials of high coefficient of friction. Example, rubber band
- Scrubbing equipment is made rough to increase friction e.g. Steel wire for scrubbing “surface”

Methods of reducing Friction

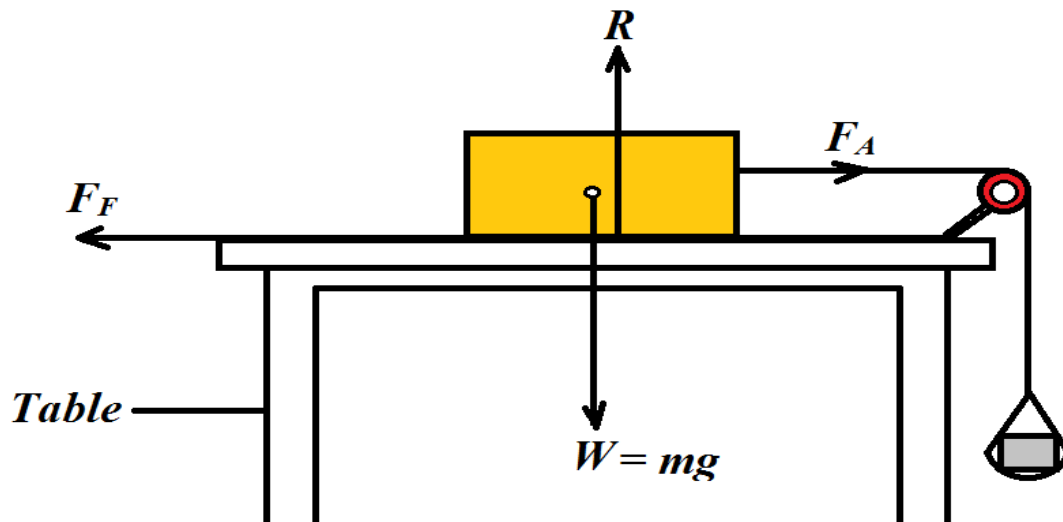
- **The use of streamlined bodies.** For objects that move in fluids such as boats, planes, cars, etc, the shape of their body is streamlined in order to reduce the friction between the bodies of the objects as the fluid.
- **By polishing the surface,** as polishing makes the surface smooth and friction can be reduced.
- **Lubrication.** Using lubricants such as oil or grease can reduce the friction between the surfaces.
- **Use of ball bearings or roller bearings.** When objects are rolled over the surface, the friction between the rolled object and surface can be reduced by using ball bearings.
- **Avoiding moisture.** When the moisture is present the friction is more, So, we must avoid moisture between the two surfaces,
- **The use of alloys.** Friction is reduced by lining the moving parts by an alloy because alloys have low coefficients of friction
- **Reduce pressure or weight on the object**

Class Activity – 2:1

1. A batsman hits a cricket ball which then rolls on a level ground. After covering a short distance, the ball comes to rest. Explain
2. Why it is convenient to pull luggage bags fitted with rollers?
3. When we try to push a very heavy box kept on ground, it does not move at all. Which force is preventing this box to move forward? Where does this force act?
4. Explain why most of vehicles have their engines directly over the drive wheel?
5. In which case will there be more friction between the truck and the road: when the truck is empty or when it is loaded?
6. Jack has to push a lighter box and Joseph has to push a similar heavier box on the same floor. Who will have to apply larger force and why?
7. You might have noticed that when used for a long time, slippers with rubber soles become slippery. Explain the reason.
8. We use ball bearings between the hub and axle of ceiling fan and bicycles. Why?
9. Imagine that friction suddenly vanishes. How would life be affected? List ten such situations.

Normal Force and Friction

- Let us consider a body of weight **mg** lying on a horizontal surface as shown in the figure. When a body presses against a surface, the surface deforms even if it appears to be rigid. The deformed surface pushes the body with a **normal force *R*** that is perpendicular to the surface. This is called normal reaction force which is equal and opposite to the weight of the body.
ie, . $R = mg$



forces acting on an object resting on a horizontal plane

- If you push on an object with a force that is smaller than friction force, then the static friction force is exactly equal and opposite to your force (*ie*, $F_A = F_F$), and the object stays at rest.
- But if you increase your force so that it exceeds F_F , then the maximum friction force is not enough to keep the object at rest. So it will move, and the friction force will abruptly drop to the kinetic value of $\mu_k N$. **(It turns out that μ_k is always less than or equal to μ_s).** At this situation $F_F \neq F_A$ so we have to find the net force, *ie*, $F_{NET} = F_A - F_F$

NB:

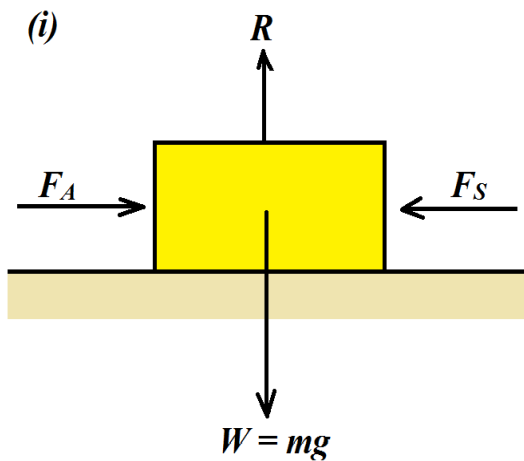
- When the body starts to move static friction force is equal to limiting friction then the minimum force applied tends to start the motion
- When the body starts to move kinetic friction force is not equal to the minimum force applied, then the body tends to start the motion
- If limiting friction is less than the force applied, the body will move
- If limiting friction is greater than the force applied, then the body cannot move
- **Limiting friction:** Is the maximum possible value of static friction

Laws of friction forces

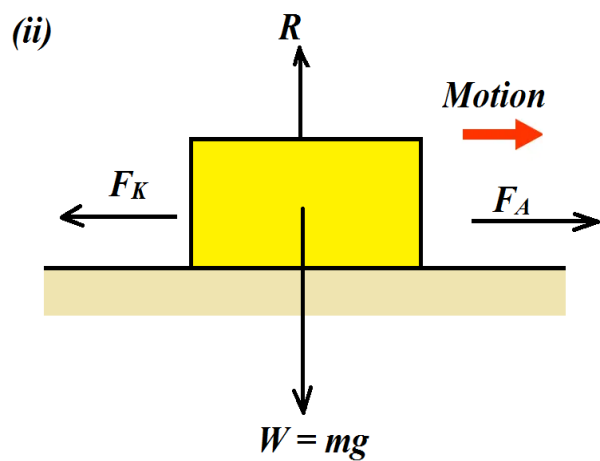
1. Frictional force is directly proportional to the normal force between the two surfaces in contact.
($F_f \propto R$)
2. Friction depends on the nature (roughness) of surfaces in contacts.
3. Friction does not depend on the surface areas in contact. For example If you slide a book lying flat or turn it on edge, the force of friction is the same.
4. Frictional force is independent of the speed once an object has been set in motion

Types of Friction

- Solid surfaces are subjected to two types of friction: **Static Friction and Kinetic (Dynamic) Friction.**
- Imagine, for example, trying to slide a heavy box across a concrete floor—you might push very hard on the box and not move it at all. **This means that the static friction responds to what you do — it increases to be equal to and in the opposite direction of your push.**



Static friction keeps the box from moving until the applied force (F_A) exceeds the static friction force (F_S).



Once the static friction is overcome, the box will move. Now, the friction opposing movement is much lower and is referred to as kinetic friction (F_k).

- If you finally push hard enough, the box seems to slip suddenly and starts to move. **Now static friction gives way to kinetic friction.** Once in motion, it is easier to keep it in motion than it was to get it started, indicating that the kinetic frictional force is less than the static frictional force.

Static Friction

- Static friction – Is the friction which occurs when the two objects are not moving relative to each other

From: $F \propto R \rightarrow F = \mu_s R \rightarrow \mu_s = \frac{F}{R}$

Whereby; μ_s – is the coefficient of static friction

∴ **Coefficient of static friction** is the ratio of limiting friction force to the normal reaction

ie., $\mu_s = \frac{(\text{static frictional force})}{(\text{Normal reaction})} = \left(\frac{F}{R}\right)$

Dynamic (kinetic) Friction

- Kinetic friction – Is the friction that occurs when objects are moving relative to each other and rub against each other

From: $F \propto R \rightarrow F = \mu_k R \rightarrow \mu_k = \frac{F}{R}$

Whereby; μ_k – is the coefficient of kinetic friction

∴ Coefficient of kinetic friction is the ratio of kinetic friction force to the

Normal reaction. *ie.,* $\mu_k = \frac{\text{Kinetic friction Force}}{\text{Normal Reaction}} = \left(\frac{F}{R}\right)$

- Kinetic friction is always slightly less than the limiting friction. **This is because** once, the motion starts actually; inertia of rest has been overcome. Also, when motion has actually started, irregularities of one surface have little time to get locked again into the irregularities of other surface.

NB:

- When a body rolls on the surface of another, the form of kinetic friction that exists between the surfaces is called '**ROLLING FRICTION**'. For example, when a wheel, a circular disc or a ring or a sphere or a cylinder rolls over a surface, the force opposing it is the **rolling friction**.
- **Sliding Friction** is the kind of kinetic friction that is caused by two bodies rubbing or sliding against each other. For example, when a flat block is moved over that flat surface of a table, the opposing force is called **sliding friction**.
- It is easy to roll a body than to slide it on the ground, **This is because Rolling friction is always less than Sliding friction**
- **The coefficient of kinetic friction is always less than the coefficient of static friction**

Worked Examples:

1. A block of mass 500g is pulled along a horizontal surface. If the coefficient of kinetic friction between the block and the surface is 0.8. What is the friction force acting on the block as it slides?

Soln:

Given: $m = 500 \text{ g} = 0.5 \text{ kg}$, $\mu_k = 0.8$, $F = ?$

$$\text{From: } \mu_k = \frac{F}{R} \quad \rightarrow F = \mu_k R = \mu_k mg = 0.5 \times 0.8 \times 10 = 4\text{N}$$

2. A block of mass 1000 kg lying steady on the horizontal surface of a table needs 200 N horizontal force to come into motion. What is the coefficient of static friction between the block and the surface of table?

$$[\text{ANSW: } \mu = \frac{F}{mg} = \frac{200}{1000 \times 10} = 0.02]$$

3. A box of 12 kg is being pulled across a horizontal floor by a force of 60 N. If the acceleration of the box is 2m/s^2 , what is the force of friction acting between the box and the floor?

ANS: $m = 12\text{kg}$, $F_A = 60 \text{ N}$, $a = 2\text{m/s}^2$, $ma = 12 \times 2 = 24 \text{ N}$

From; $F_{\text{NET}} = F_{\text{APP}} - F_F \rightarrow ma = 60 - F_F \rightarrow F_F = F_A - ma = 60 - 24 = 36\text{N}$

4. **The brakes of a car moving at 20m/s along a horizontal road are suddenly applied and it comes to rest after travelling some distance. If the coefficient of friction between the tyres and the road is 0.90 and it is assumed that all four tyres behave identically, find the shortest distance the car would travel before coming to a stop.**

ANS; $u = 20\text{m/s}$, $v = 0 \text{ m/s}$, $\mu = 0.9$, $s = ?$

From; retardation, $a = \mu g = 0.9 \times 10 = -9\text{m/s}^2$

$$\text{Also; } v^2 = u^2 + 2as \rightarrow s = \frac{v^2 - u^2}{2a} = \frac{0^2 - 20^2}{2 \times -9} = \frac{-400}{-18} = 22.2 \text{ m}$$

Alternatively;

k.e = work done against friction $\rightarrow \frac{1}{2}mv^2 = Fs$, whereby $F = \mu mg$

$$\frac{1}{2}mv^2 = \mu mgs \rightarrow s = \frac{v^2}{2g\mu} = \frac{20^2}{2 \times 10 \times 0.9} = \frac{400}{18} = 22.2 \text{ m}$$

5. Consider an object of mass 50 kg at rest on the floor. A Force of 5 N is applied on the object but it does not move. What is the frictional force that acts on the object?

Solution

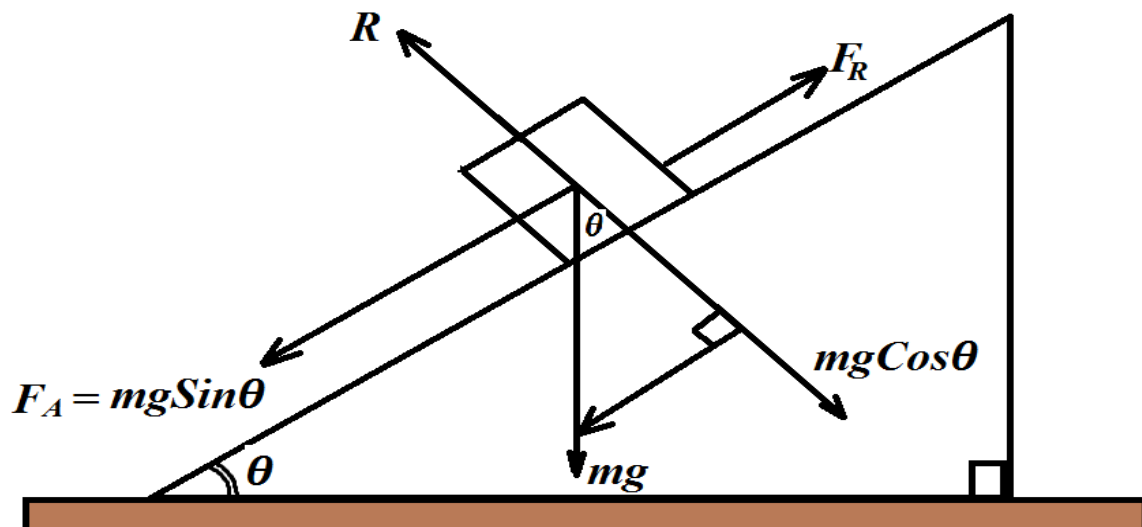
- When the object is at rest, force applied and the static frictional force are equal and opposite. The magnitudes of these two forces are equal, **ie, . $F_A = F_F = 5\text{N}$**
- Therefore, the static frictional force acting on the object is, **$F_S = 5 \text{ N}$**

Class Activity– 2:2

1. A block of mass 270kg is pulled along a horizontal surface. If the coefficient of kinetic friction between the block and the surface is 0.4. What is the friction force acting on the block as it slides? (ANS: $F_r = 1,080\text{N}$)
2. A box of mass 2kg rest on a horizontal surface, a force of 4.4 N is required to just start the box moving. What is the coefficient of static friction between the block and the surface? (ANS: $\mu = 0.22$)
3. A box weighing 2000 N is sliding across a cement floor. The force pushing the box is 500N, and the coefficient of sliding friction between the box and the floor is 0.20. What is the acceleration of the box? [ANS; $a = 0.49\text{m/s}^2$]
4. An alluminium block of mass 2.1kg rests on a steel platform. A horizontal force of 15N is applied to the block
 - (a) Given that coefficient of limiting friction 0.6, will the block move?
 - (b) If will move, what will be its acceleration. Given that coefficient of kinetic friction is 0.47
 (ANS: (a) Since: $F_A > F_F$, hence the car will move (b) $a=2.44 \text{ m/s}^2$)
5. A brick starts sliding with 6m/s across a concrete horizontal surface floor and the coefficient of friction between the two surfaces is 0.4. How far will it travel before coming to rest? (ANS: $S = 4.5 \text{ m}$)
6. Find the static friction between a block of wood of mass 10kg placed on a table. A minimum force of 50N is required to make the block just move on the top. (ANS: $\mu = 0.5$)

Friction force at Inclined Plane

- Consider the diagram below, a body of mss, m sliding down the inclined plane at θ



- When the object begins to slide (from rest) there will be kinetic friction between the object and the incline .Thus the net force will be:

$$F_{net} = F_A - F_R$$

$$\text{Whereby: } F_A = mg \sin \theta \text{ and; } F_R = F_k = \mu_k R = \mu_k mg \cos \theta$$

$$F_{net} = mg \sin \theta - \mu_k mg \cos \theta = mg(\sin \theta - \mu_k \cos \theta)$$

Thus acceleration will be given as: From

$$F_{net} = ma = mg(\sin \theta - \mu_k \cos \theta)$$

- Therefore, the acceleration is given by ,

$$a = g(\sin\theta - \mu_k \cos\theta), \text{ For downward motion}$$

$$a = g(\sin\theta + \mu_k \cos\theta), \text{ For upward motion}$$

- When $F_r = 0$ (when the incline is frictionless) , then, $a = g\sin\theta$
- At constant speed (at rest), $a = 0 \text{ m/s}^2$,
 $F = mg\sin\theta$ and $R = mg\cos\theta$

$$F = \mu R \quad \rightarrow \quad \mu = \frac{F}{R} = \frac{mg\sin\theta}{mg\cos\theta} = \tan\theta$$

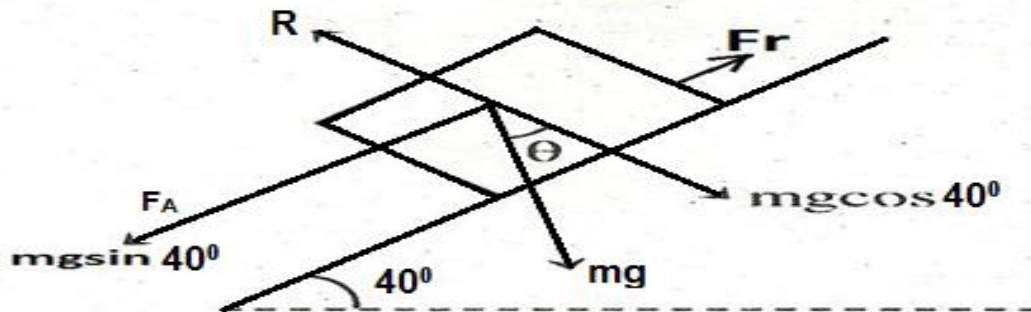
$$\therefore \text{ At constant speed (at rest), } \mu = \tan\theta$$

Worked Examples:

1. A block of wood of 4kg just slides without acceleration down an inclined plane of 40° to the horizontal. What is the coefficient of dynamic friction?

Soln: Given: $m = 4 \text{ kg}$, $\theta = 40^\circ$

Consider the fig below



From: *At constant speed* ($a = 0$), $\mu = \tan\theta \therefore \mu = \tan\theta = \tan 40 = 0.84$

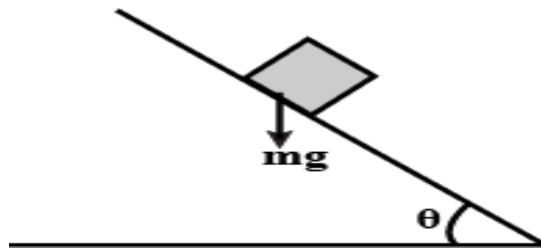
2. A rectangular box of mass 10 kg rests on an incline with coefficients of static and kinetic friction of 0.55 and 0.25 respectively.
 - (a) At what angle will the box begin to slide?
 - (b) If the incline is kept at that angle after the box begins to slide, what will be the box's acceleration?

ANS;

(a) From: $\tan\theta = \mu \rightarrow \theta = \tan^{-1}(0.55) = 28.8 \approx 29^\circ$

(b) $a = g(\sin\theta - \mu\cos\theta) = 2.66 \text{ m/s}^2$

3. A plank with a box on it at one end is gradually raised about the other end. As the angle of inclination with the horizontal reaches 30° the box starts to slip and slides 4.0 m down the plank in 4.0s. What will be the coefficients of static and kinetic friction between the box and the plank, respectively?



Solution:

$$\mu_s = \tan\theta = 0.5774,$$

$$\text{from; } s = ut + \frac{1}{2}at^2, \rightarrow 4 = 0 \times 4 + \frac{1}{2} \times a \times 4^2 \rightarrow a = 0.5 \text{ m/s}^2, \text{ also; } a = g(\sin\theta - \mu_k \cos\theta)$$

$$0.5 = 10(\sin 30 - \mu_k \cos 30) \rightarrow \mu_k = \frac{0.05 - \sin 30}{-\cos 30} = 0.5196$$

4. A block of mass 1 kg slides down on a rough inclined plane of inclination 60° starting from its top. If the coefficient of kinetic friction is 0.5 and length of the plane is 1m, what is the value of work done against friction?

Soln;

Wd = Force x distance = friction force x length of the plane

But; Frictional force, $F = \mu R = \mu mg \cos\theta$, length of plane, $s = 1\text{m}$

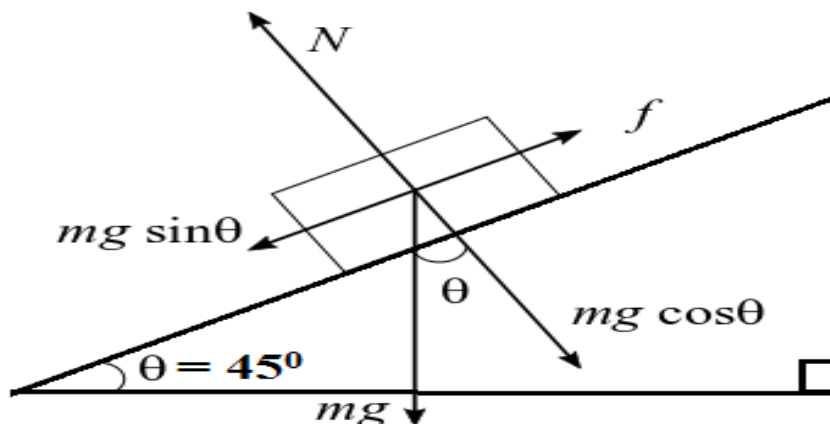
Therefore; Work done = $F \times S =$

$$\mathbf{Wd = \mu mg \cos\theta = 0.5 \times 1 \times 10 \times \cos 60 \times 1 = 2.5\text{J}}$$

5. Starting from rest, the time taken by a body to slide down a 45° inclined plane with friction, is twice the time it takes to slide down the same distance in the absence of friction. Determine the coefficient of friction between the body and the inclined Plane

Soln: $u = 0\text{m/s}$, $\theta = 45^\circ$, $t_1 = 2t_2 = 2t$, $t_2 = t$

Consider the various forces acting on the body have been shown in the figure below.



NB:

- The force on the body down the inclined plane in presence of friction, is given by; $ma = mg \sin\theta - f = mg \sin\theta - \mu mg \cos\theta$

this gives, $a = g(\sin\theta - \mu \cos\theta)$

- The force on the body down the inclined plane in absence of friction, is given by; $ma = mg \sin\theta \rightarrow a = g \sin\theta$

Since the block is at rest thus initial velocity $u = 0$, and $S_1 = S_2 = S$

$$\text{Then, } s = ut + \frac{1}{2}at^2 \rightarrow s = \frac{1}{2}at^2$$

Thus; time taken to slide down the plane for both cases is given by

$$\mathbf{S = \frac{1}{2}at_1^2 \rightarrow t_1 = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2s}{g(\sin\theta - \mu \cos\theta)}} \quad \text{----(i),}$$

$$S = \frac{1}{2}at_2^2 \rightarrow t_2 = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2s}{g\sin\theta}} \quad \text{----(ii) (when no friction)}$$

Now; compare equation (i) and (ii) ie,.. $t_1 = 2t_2$

$$\sqrt{\frac{2s}{g(\sin\theta - \mu\cos\theta)}} = 2 \times \sqrt{\frac{2s}{g\sin\theta}} \rightarrow \frac{4}{g\sin\theta} = \frac{1}{g(\sin\theta - \mu\cos\theta)}$$

$$4g\sin\theta - 4\mu\cos\theta = g\sin\theta \rightarrow 4\mu\cos\theta = 3g\sin\theta$$

$$\therefore \mu = \frac{3}{4} \frac{\sin\theta}{\cos\theta} = \frac{3}{4} \tan\theta = \frac{3}{4} \tan 45 = \frac{3}{4} = 0.75$$

Class Activity – 2:3

1. A mass is placed on an inclined plane such that it can move at constant speed, when slightly tapped. If the angle of the plane makes with the horizontal plane is 30° . Find the coefficient of kinetic friction. **(ANS: $\mu = 0.56$)**
2. A mass of 5 kg is placed on a plane inclined at an angle of 30° to the horizontal. What is the accelerating force required to pull the mass up the plane if the coefficient of friction is 0.5? **(ANS: $F_A = 46.65\text{N}$)**
3. Block of mass 10kg is moving on inclined plane with constant velocity 10 m/s. What is the coefficient of kinetic friction between incline plane and block **[ANS; $\mu = \tan\theta =$]**
4. **A block of wood of mass 5kg is placed on a rough plane inclined at 60° . Calculate its acceleration down the plane if coefficient of friction between the block and the plane is 0.32 (ANS: $a = 7.1 \text{ ms}^{-2}$)**
5. Why mountain roads are winded up rather than keeping them straight?
6. You are playing with your younger sister in the snow. She is sitting on a sled and asking you to slide her across a flat, horizontal field. You have a choice of pushing her from behind, by applying a force at 30° below the horizontal or attaching a rope to the front of the sled and pulling with a force at 30° above the horizontal. Which would be easier for you and why?

Overall summary;

1. Acceleration of a block (body) against Friction

(a) Acceleration of a block on horizontal surface

When a body is moving under application of force P, then kinetic friction opposes its motion, if a is the net acceleration of the body, then

$$ma = P - F_k \rightarrow a = \frac{P - F_k}{m}$$

$$\text{When, } P = 0 \text{ N, } ma = F \rightarrow a = \frac{F}{m} = \frac{\mu mg}{m} = \mu g$$

(b) Acceleration of a block down a rough inclined plane

From; $ma = mg\sin\theta - F$, but $F = \mu mg\cos\theta$

From; $ma = mg\sin\theta - \mu mg\cos\theta = mg(\sin\theta - \mu\cos\theta)$

$$\therefore a = g(\sin\theta - \mu\cos\theta)$$

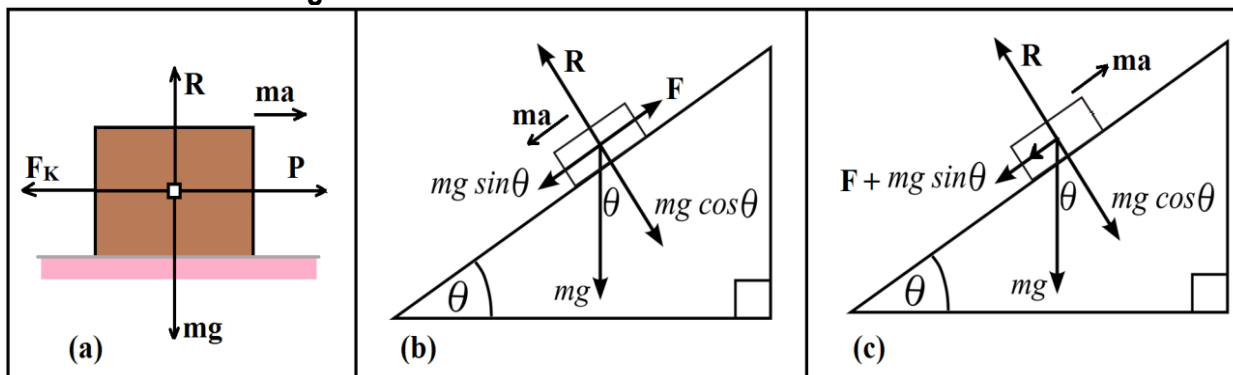
(c) Retardation of a block up a rough inclined plane

$ma = mg\sin\theta + F$, but $F = \mu mg\cos\theta$

$ma = mg\sin\theta + \mu mg\cos\theta = mg(\sin\theta + \mu\cos\theta)$

$$\therefore a = g(\sin\theta + \mu\cos\theta)$$

See the figure below



(d) For smooth inclined plane, (ie,..when $\mu = 0$) $\therefore a = g \sin \theta$

(e) When the top object keeps sliding with constant velocity, the tangent of that angle is equal to μ (ie,.. $\tan \theta = \mu_k$)

2. The work done against friction

(a) Work done over a rough inclined surface

- If a body of mass m is moved up on a rough inclined plane through distance s , then;

$$\text{Work done (Wd)} = Fs = ma \times s = mg (\sin \theta + \mu_k \cos \theta) S$$

- For body of mass m moves down rough inclined plane through distance s , then;

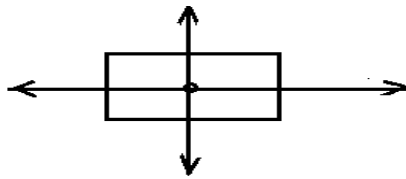
$$\text{Work done (Wd)} = Fs = ma \times s = mg (\sin \theta - \mu_k \cos \theta) \times S$$

(b) Work done in sliding a body over a horizontal surface is given by;

$$\text{Work done} = F_f \times S = \mu_k mgS$$

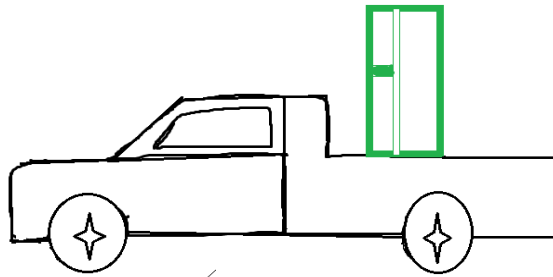
Self Assessment–2

1. Calculate the coefficient of kinetic friction between the surface of a table and a block of wood when 5 kg block of wood is moving on the table and experiencing a frictional of 5 N.
(ANS: $\mu = 0.1$)
2. A box weighing 2 kg is at rest on a wooden floor. The coefficient of static friction is 0.6 and the coefficient of kinetic friction is 0.35.
 - a) What minimum force is required to start the box sliding?
 - b) What minimum force is required to keep it sliding at a constant velocity?
3. A 12 kg box is being pulled across a level floor by a force of 60 N. If the acceleration of the box is 2 ms^{-2} , What is the force of friction between the box and the floor
4. A 0.5 kg object is given an initial velocity of 3 m/s after which it slides a distance of 8 m across a level floor. What is the coefficient of kinetic friction between the object and the floor?
5. The coefficient of kinetic friction between a block of wood and a wooden inclined plane at an angle of 40° is 0.126. If the friction acting on the sliding prism is 42 N, calculate the mass of the prism. (ANS: mass = 43.4 kg)
6. Calculate the friction force acting on a carton box of mass 9 kg which is moving over a surface. The coefficient of kinetic friction between the two surfaces is given as 0.45.
(ANS: $F_R = 40.5 \text{ N}$)
7. The coefficient of friction between a particle of mass 8 kg, and a rough horizontal plane is 0.4. Given that a horizontal force of 29 N acts on the particle as shown in the figure below. Would it start to move (ANS: Since $F_A(29 \text{ N}) < F_R(32 \text{ N})$, No motion)



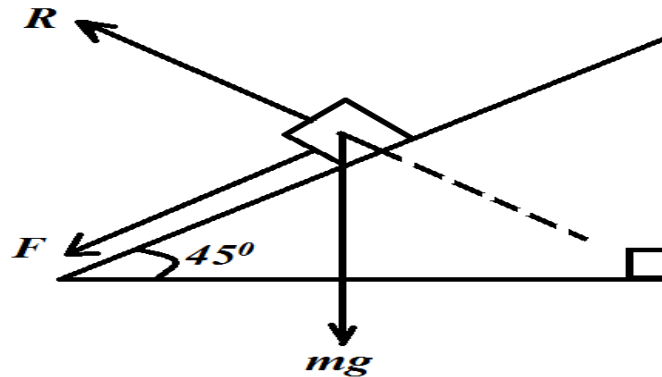
8. A wooden block of mass 8 kg is resting on a wooden table. If the coefficient of static friction between the pair is 1.3 , Calculate the minimum horizontal force required to just slide the box .Given that $g = 10 \text{ N/ kg}$
9. A 3 tones lorry is resting on a tarmac road .The lorry requires a minimum force of 12000 N in order for it to just move .Determine the coefficient of static friction between the lorry's tyres and the road
10. A crate of soda with mass 40 kg will just begin to slide with constant speed down a rough ramp (slope) at 30° to the horizontal .What is the coefficient of static friction.
(ANS: $\mu = \tan\theta = 0.5774$)
11. A box of mass 5 kg is at rest on a wooden floor. If the coefficient of static friction between the box and the floor is 0.6, what minimum external force is required to set the box sliding?
12. Define the following (a) Rolling friction (b) Sliding Friction
13. A 0.5 kg object is given an initial velocity of 3 m/s after which it slides a distance of 8 m across a level floor, What is the coefficient of kinetic friction between the object and the floor?
14. A box weighing 2 kg is at rest on a wooden floor .The coefficient of static friction is 0.6 and the coefficient of kinetic friction is 0.35
(a) What minimum force is required to start the box sliding
(b) What minimum force is required to keep it sliding at a constant velocity?
15. In a car, The brakes stop the tyres while friction between the tyres and the road surface stops the car .On a wet road the coefficient of kinetic friction between the road surface and the tyre is 0.1 .Two cars, A and B, are travelling at a speed of 15 m/s and 30 m/s ,respectively .Brakes are suddenly applied on each of the cars .How far will each of the cars travel before coming to rest?
16. A rectangular box of mass 10 kg rests on an incline with a coefficient of static friction of 0.55 and coefficient of kinetic friction of 0.25
(a) At what angle will the box begin to slide?
(b) If the incline is kept at that angle after the box begins to slide, what will be the box's acceleration?
17. The coefficient of kinetic friction between the tyres of a car and the road is 0.7. The car brakes are applied and it travels a distance of 120 m before stopping .What was the car's velocity just before the brakes were applied?
18. A box of mass 5 kg is at rest o a wooden floor. The coefficient of static friction is 0.42 and the coefficient of dynamic friction is 0.15. Find its acceleration if a force of :
(a) 15 N is applied to the box (b) 25 N is applied to the box
19. A block of metal with a mass of 20 kg requires a horizontal force of 50 N to pull it with uniform velocity along a horizontal surface. Calculate the coefficient of friction between the surface and the block. (ANS: $\mu = 0.25$)
20. A Car of weight 1000 N is moving with uniform speed .If the kinetic friction acting on the car is 500 N , calculate the coefficient of kinetic friction
21. A 42 kg refrigerator is sitting on the back of a stationary pick – up .The coefficient of static friction between the refrigerator and the pick – up bed is 0.44 .At what rate can the pick – up accelerate without the refrigerator sliding off the back?

$$[A; ma = F_s \rightarrow a = \frac{F_s}{m} = \frac{\mu mg}{m} = \mu g = 4.4 \text{ m/s}^2]$$



22. A 6 kg mass is resting on a horizontal surface .It is determined that a force of 20 N will start the object sliding and keep it sliding with an acceleration of 0.83 m/s^2 .What are the coefficients of static and kinetic friction between the mass and the surface ?
23. What is the normal normal reaction of the body of mass 10kg placed on an inclined plane of angle 30° ?
24. A concrete block of mass 10kg rests on a table. It is found that when a horizontal force of 4kg weight pulls the mass, it is just begins to slide on the table. Find the coefficient of static friction
25. A block of wood rests on a sloping plank which makes an angle of 31° with the horizontal. If the block suddenly begins to slide down hill, what is the coefficient of static friction?
26. A box of mass 50 kg is dragged on a horizontal floor by means of a rope tied to its front. If the coefficient of kinetic friction between the floor and the box is 0.30, what is the force required to move the box at uniform speed? (ANS: $F = 150 \text{ N}$)
27. A car of mass 1200 kg is brought to rest by a uniform force of 300 N, in 80 sec. What was the speed of the car? (ANS: $u = 20 \text{ m/s}$)
28. A loaded trailer weighing 10kg is being towed across level ground .The coefficient of dynamic friction is 0.25. What is the frictional force of the trailer?
29. A block of wood just slides without acceleration down an inclined plane of 25° to the horizontal. What is the coefficient of dynamic friction?
30. Define the following terms
 (a) Limiting friction (b) Normal reaction
 (c) Viscosity (d) Coefficient of Friction
31. (a) State the laws of friction (b) Explain, why Friction is friend and foe?
32. A brick is sliding at 8m/s across a concrete horizontal surface floor and the coefficient of friction between the two surfaces is 0.5 How far will it travel before coming to rest?
33. Show that the acceleration of a stone sliding at a velocity ,v across a concrete horizontal surface floor is given by $a = \mu g$ where μ is the coefficient of friction between the stone and the floor and g = acceleration due to gravity
34. A mass of 5 kg is placed on a plane inclined at an angle of 30° to the horizontal .What is the accelerating force required to pull the mass up the plane if the coefficient of friction is 0.5?
35. A wooden box of mass 30 kg rests on a rough floor. The coefficient of friction between the floor and the box is 0.6. Calculate
 (a) The force required to just move the box. (AN: $F_F = 180 \text{ N}$, $a = 0.67 \text{ m/s}^2$)
 (b) If a force of 200 N is applied to the box, with what acceleration will it move?
36. Describe how friction is minimized by the following methods: (a)Lubrication
 (b) Use of bearings (c) Streamline flow
37. A boy is pulling a box of mass 10 kg. What is the normal force and the frictional force if the coefficient of static friction? ($g = 10 \text{ N/kg}$)

38. A 50 g mass is placed on a straight track sloping at an angle of 45° to the horizontal as shown from the figure below calculate
- Acceleration of the load as it slides down the slope
 - The distance moved from rest in 0.2 seconds

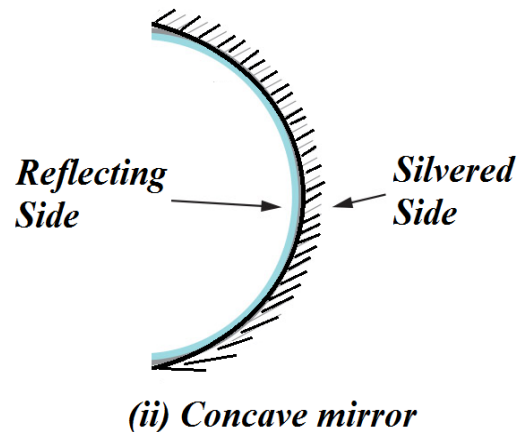
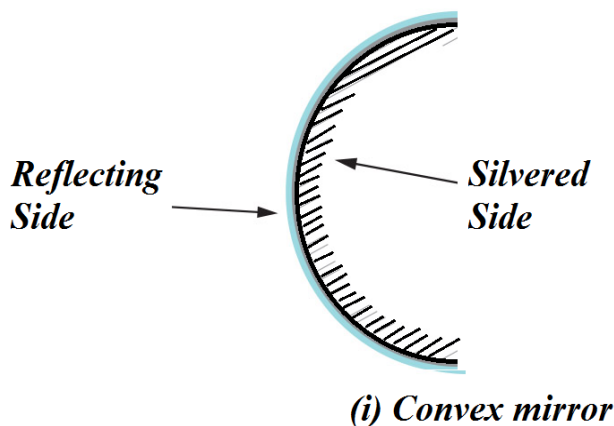


39. A 5 kg block is resting on a horizontal surface. Given that the coefficient of static friction is 0.57 $g = 10 \text{ N/kg}$
- What is the frictional force required to just move the block?
 - What force must be applied to the block to keep it moving at constant velocity?
 - Determine its acceleration if a force of 35 N is applied
40. A force of 8.0 N gives a 3.0 kg mass an acceleration of 0.6 m/s^2 to the right
- What is the limiting friction on the block?
 - Determine the coefficient of static friction required to produce a net kinetic force of 6.0 N? ($g = 10 \text{ N/kg}$)**
41. A 53.0 kg block slowed by friction has an acceleration of -0.1 m/s^2 . Determine the force of friction on the block
42. A 10.0 kg solid sliding along a horizontal surface is brought to rest after 30 minutes
- Name the force that caused it to stop
 - Determine the magnitude of the force that caused it to stop
- (Given that: $\mu_k = 0.45$, $g = 9.8 \text{ N/kg}$)**

Chapter – 03: Light

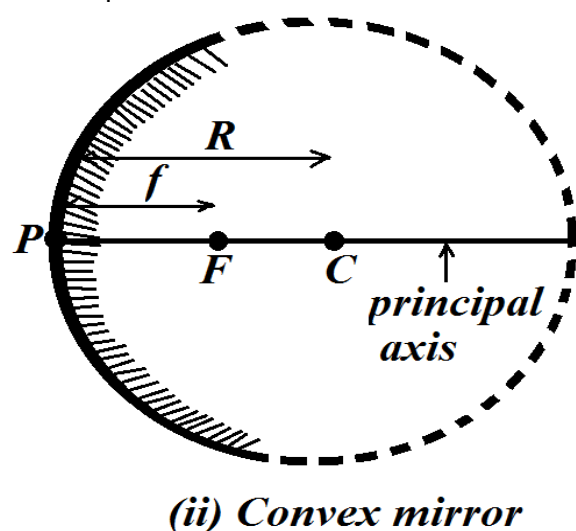
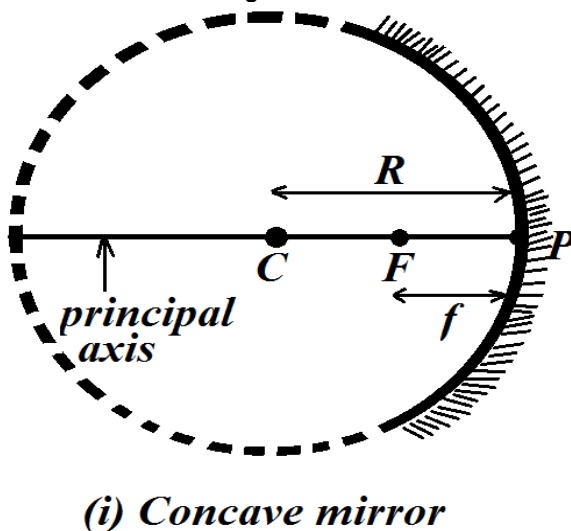
Reflection of Light from Curved Mirrors

- When a shell of a hollow sphere of glass is made out of a piece of glass and then silvered, a curved or spherical mirror is obtained. These mirrors due to their curvature form images that are quite different from plane mirrors.
- If the glass is **silvered from outside** so that light can be reflected from inside, it is called **concave or converging mirror**.
- If the coating is done so that the reflection is from outside, it is called **convex or diverging mirror**.
- Therefore, There are two types of curved mirrors; concave and convex mirror.



Terms related to Spherical Mirrors

- In order to understand the image formation in spherical mirrors, we need to know about some of the terms related to them.
- Consider the diagram below shows the terms related to a spherical mirror



Center of Curvature, C

It is the center of the sphere from which the mirror is made. It is denoted by the letter C in the ray diagrams.

Pole, P

It is the geometric centre of the spherical mirror. It is denoted by the letter **P**.

Radius of Curvature, R

It is the distance between the center of the sphere and the vertex. It is shown by the letter **R** in ray diagrams (The vertex is the point on the mirror's surface where the principal axis meets the mirror. It is also called as 'pole').

Principal Axis

The line joining the pole of the mirror and its center of curvature is called principal axis.

Principal focus, F

- For a concave mirror, it is the point at which all rays parallel and close to the principal axis **appears to converge at after reflection**.
- In the case of a convex mirror, it is the point at which all rays parallel and close to the principal axis **appears to diverge from after reflection**. It is also called the **focal point** or **Focus**. It is denoted by the letter **F** in ray diagrams.

Focal length, f

- The distance between the pole and the principal focus is called **focal length (f)** of a spherical mirror.
- There is a relation between the focal length of a spherical mirror and its radius of curvature. The focal length is half of the radius of curvature.

$$\text{ie. focal length} = \frac{\text{Radius of curvature}}{2} \rightarrow f = \frac{R}{2}$$

Location of Image using Ray diagrams

The following are the rules used to locate image in the curved mirrors.

- A ray of light travelling to the mirror parallel to the principal axis, a ray is reflected through the principal focus
- A ray of light travelling to the mirror through the centre of curvature is reflected along its own path
- A ray of light travelling to the mirror through the principal focus is reflected parallel to the principal axis
- An incident ray not parallel to the principal axis, but strikes the pole of the mirror, is reflected back making the same angle with the principal axis. That is, the angle of incidence equals the angle of reflection

Note: Any two of these rays are sufficient to locate the image.

Procedure to draw ray diagrams

- **Choose an appropriate scale so that the ray diagram fits on the available space.**
- Draw a horizontal line to represent the principal axis of the mirror. Mark the focal point of the mirror.
- Using the chosen scale, draw the object in position along the principal axis. The object is drawn as a vertical line from the principal axis.
- Locate the position of the image by drawing rays from the object to the mirror. Use the rules for drawing ray diagrams to draw the reflected rays.
- At the point of intersection of the reflected rays, draw the image in position

NB:

- When rays are produced behind the mirror, they are indicated using dotted lines. This means that they are imaginary or virtual. Hence the focal point and focal length of a concave mirror are **real** while the focal point and focal length of a convex mirror are **virtual**
- A real focal length is given a **positive sign** while a negative focal length is given a **negative sign**.

Images formed in Curved mirror

Terms used to describe the images formed by curved mirrors:

Position

- A real image is that image formed by actual intersection of real rays while a virtual image is formed by imaginary rays. Furthermore, a real image can be formed on a screen while a virtual image cannot be formed on a screen.
(ie., **Real image** is on the same side of the mirror as the object while **Virtual image** is on the opposite side of the mirror compared to the object)

Nature

- **Upright image** has the same orientation as the object.
- **Inverted image** is oriented in an upside down position compared to the object.

Size

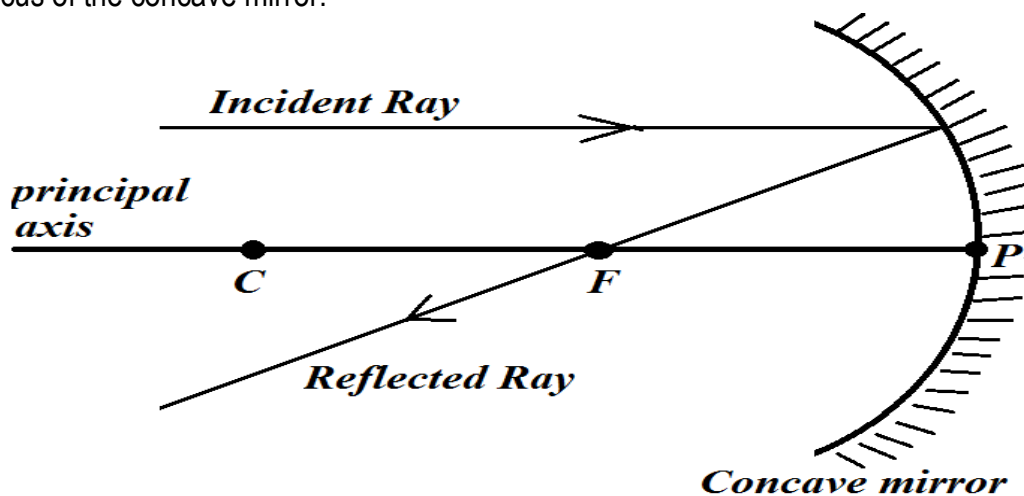
- **Enlarged image** is bigger than the object.
- **Diminished image** is smaller than the object

Rules for Image formation for Concave Mirror

- Each of the rules below describes what happens to an incident ray when it reflects off a concave mirror. Used together, they can help you accurately predict what type of images will be formed by concave mirrors.

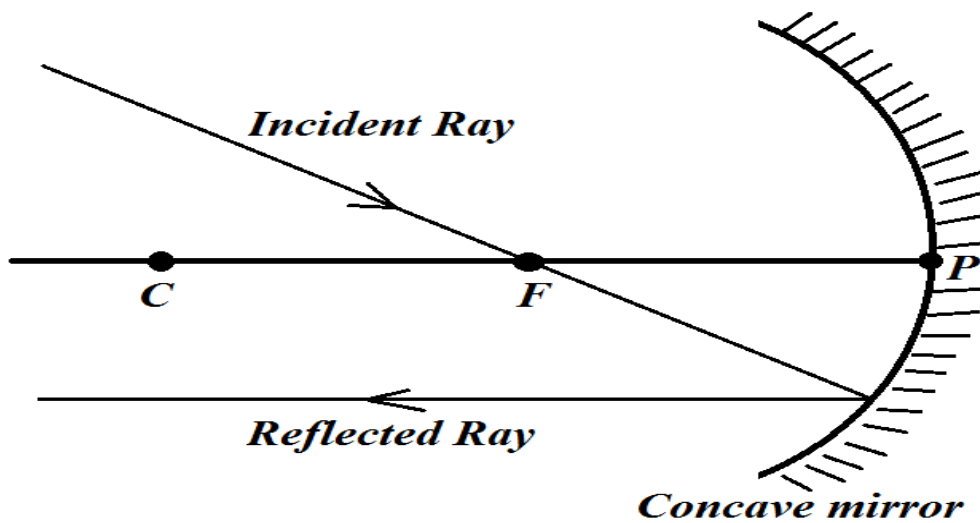
Rule 1:

An incident ray traveling *parallel to the principal axis* creates a reflected ray that passes through the principal focus of the concave mirror.



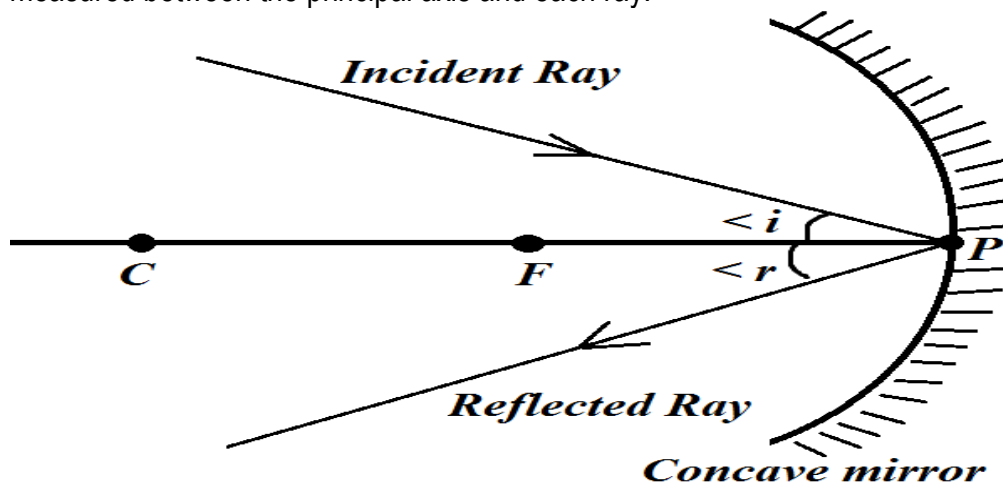
Rule 2:

An incident ray of light *passing through the principal focus* of a concave mirror creates a reflected ray that is parallel to the principal axis of the mirror.



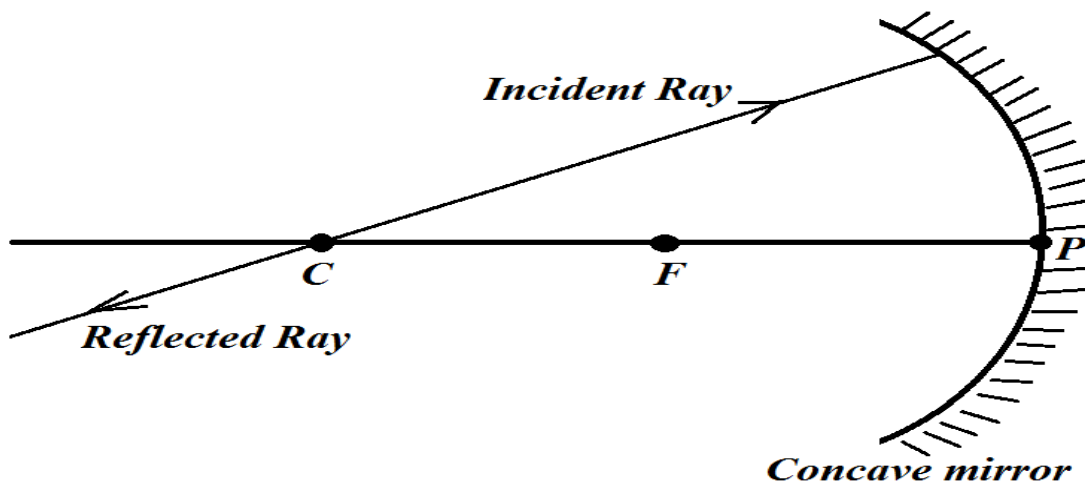
Rule 3:

An incident ray of light that is not parallel or perpendicular to the principal axis, which strikes the pole (P) of a concave mirror creates a reflected ray with an angle of reflection equal to the angle of incidence measured between the principal axis and each ray.



Rule 4:

An incident ray of light passing through the center of curvature creates a reflected ray that travels back along the same path. This is because the incident rays fall on the mirror along the normal line, perpendicular to the reflecting surface.

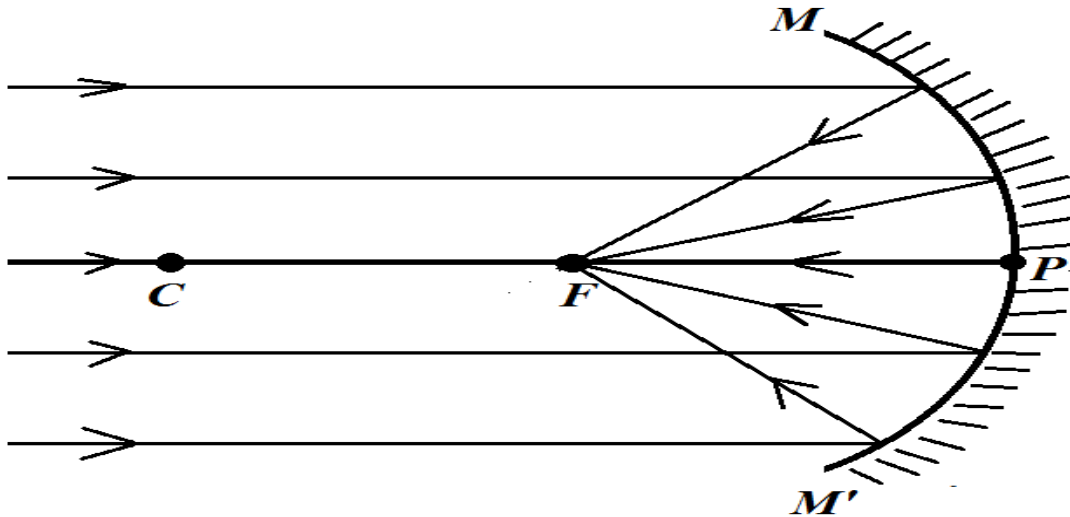


Note: Any two of these rays are sufficient to locate the image.

Images formed by Concave mirrors

Case I: Image formed by concave mirror when object placed at infinity

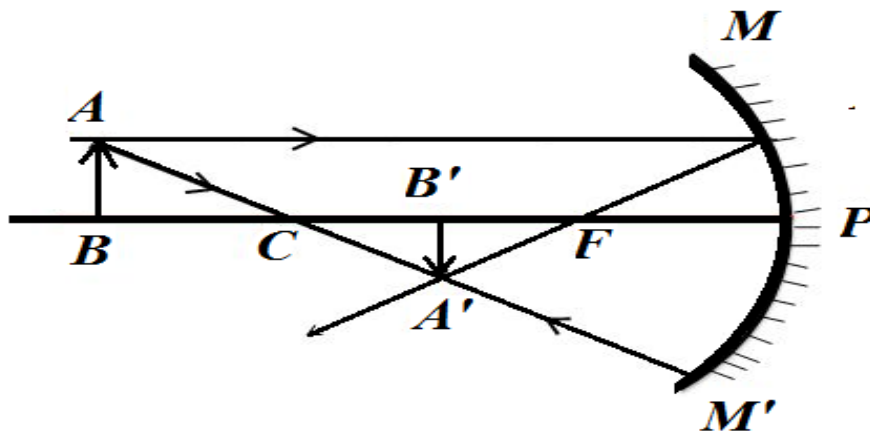
- Considering the light rays coming from an object placed at infinity to the reflecting surface MM' . Let the pole of reflecting surface is P , the center of curvature is C and the principal focus is F .



Case II:

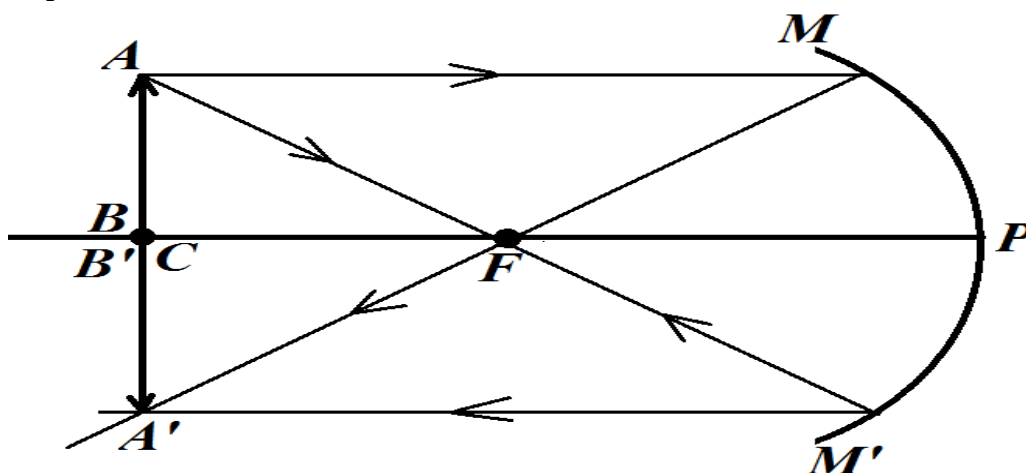
Image formed by concave mirror when the object is beyond the center of curvature (C)

- As you can see in the figure that object AB is placed at B beyond C . In addition, the image $A'B'$ is constructed at point B' between C and F .



Case III: Image formed by concave mirror when the object is placed at center of curvature (C)

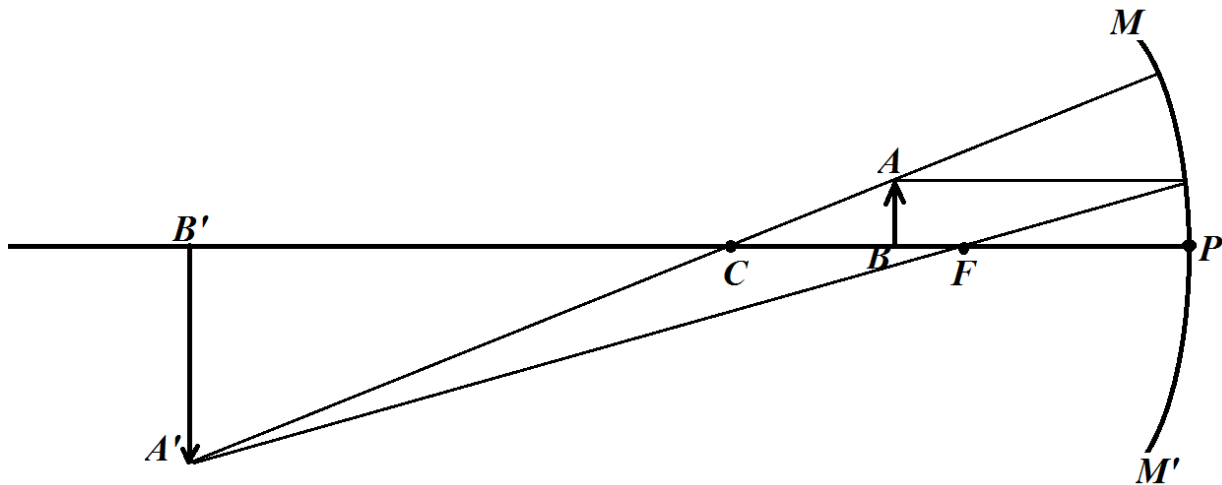
In the given figure you can see the object AB is placed at the center of curvature C . Equal and inverted image $A'B'$ is formed at C .



Case IV:

Image formed by concave mirror when the object is placed between center of curvature (C) and principal focus (F)

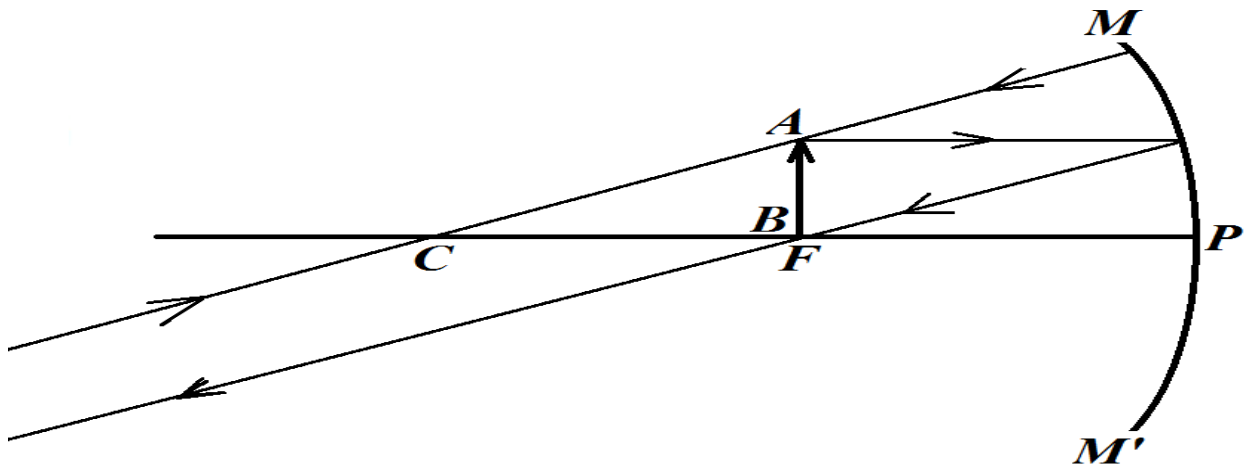
In that given image, you can see the object is placed between the center of curvature (C) and focus (F).



Case V:

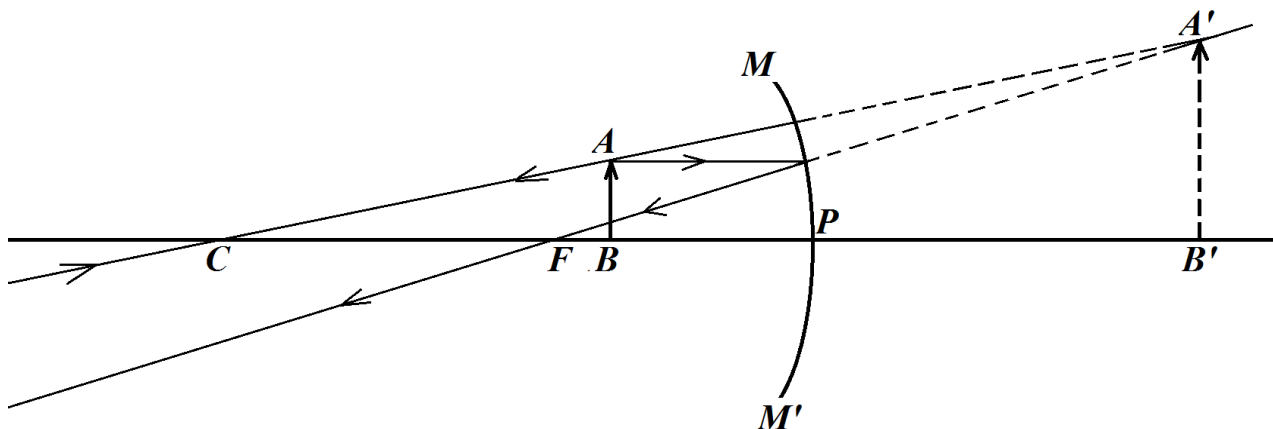
Image formation by concave mirror when the object is placed at principal focus (F)

In that case, object AB is placed at focus F and the image $A'B'$ forms at infinity.



Case VI: Image formation by concave mirror when the object is placed between pole P and the principal focus (F)

- In that figure below, the object AB is placed between the pole and focus. The image of the object is formed behind the mirror which is magnified.

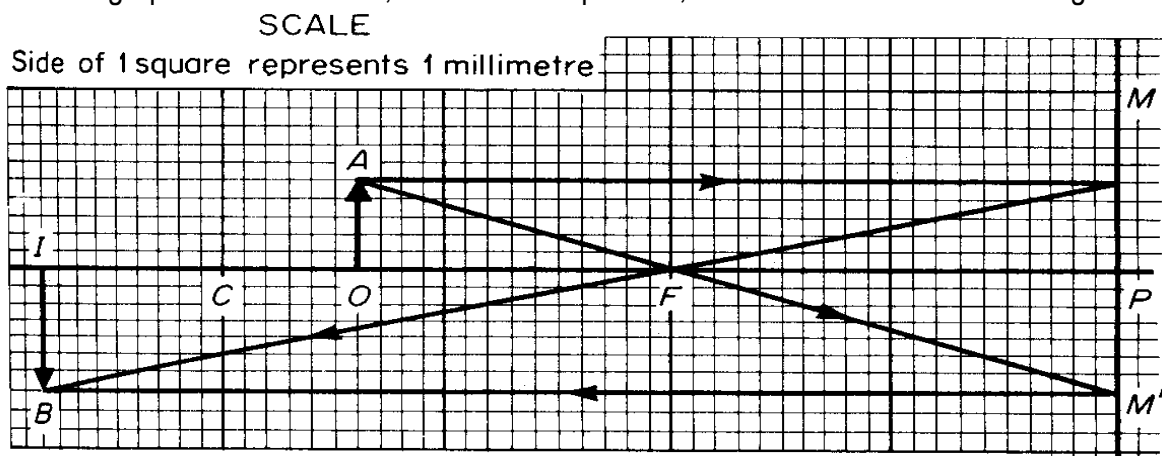


The table below summarizes the image characteristics formed by a concave mirror for different positions of the object as seen from the figures above.

Position of object	Position of image	Nature of image	Image size
At Infinity	At focus	Real and inverted	Highly Diminished
Beyond C	Between F and C	Real and inverted	Diminished
At C	At C	Real and Inverted	Same size as that of the object
Between F and C	Beyond C	Real and Inverted	Enlarged
At F	At Infinity	Real and Inverted	Highly Enlarged
Between F and P	Behind the mirror	Virtual and erect	Enlarged

Worked Examples;

1. An object 5 cm tall is placed 34 cm from a concave mirror of focal length 20 cm. By means of an accurate graphical construction, determine the position, size and the nature of the image formed.



Position of image = $(49 \times 1) = 49$ cm from the mirror

Size of image = $(7 \times 1) = 7$ cm

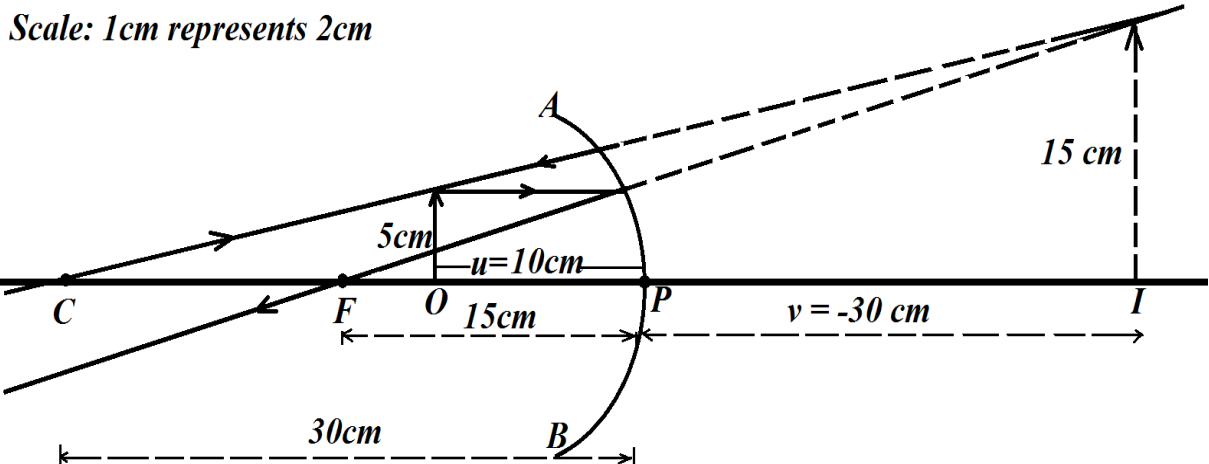
Nature of image: Image is real and inverted

2. An object of 5cm tall is placed at a distance of 10cm from a concave mirror of radius of curvature 30cm

- (i) Draw the ray diagram to represent the above information.
- (ii) From the diagram above, find the nature, position and size of the image formed

ANS;

- (i) Consider a free body diagram below

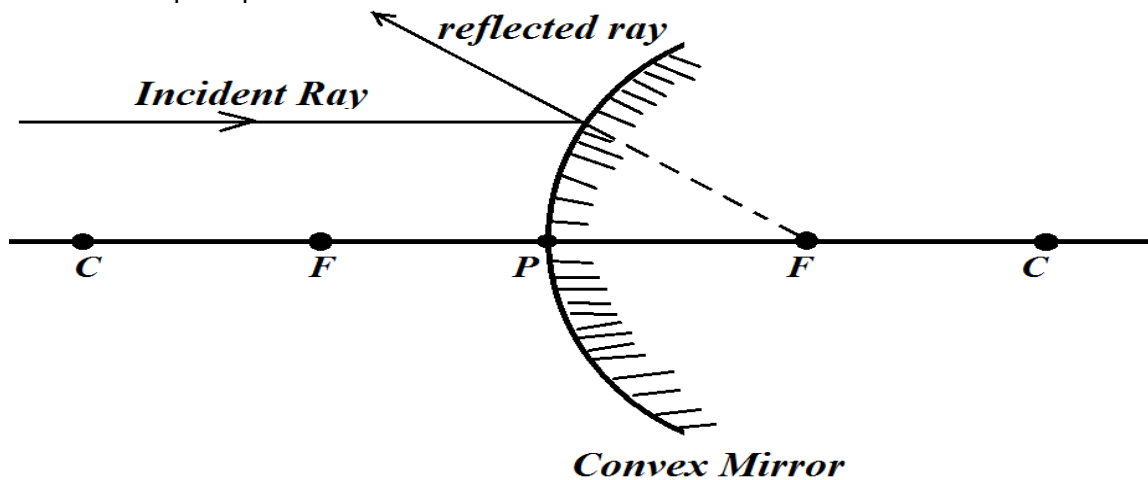


- (ii) Therefore, the image formed is virtual, it is formed 30 cm behind the mirror and it is 15cm high, larger than the object

Rules for Image formation – Convex Mirror (Diverging mirror)

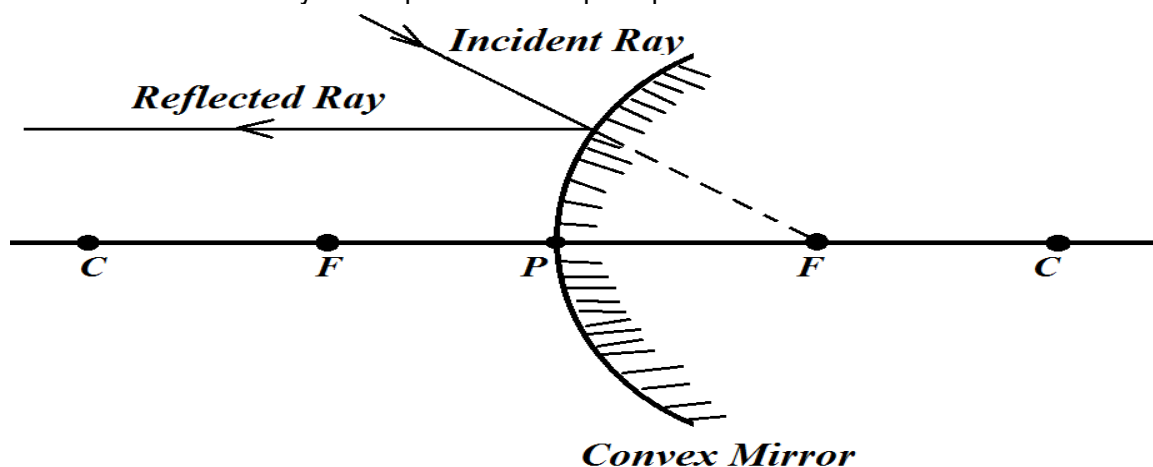
Rule 1:

An incident ray of light *traveling parallel to the principal axis* creates a reflected ray that appears to originate from the principal focus on the other side of the mirror.



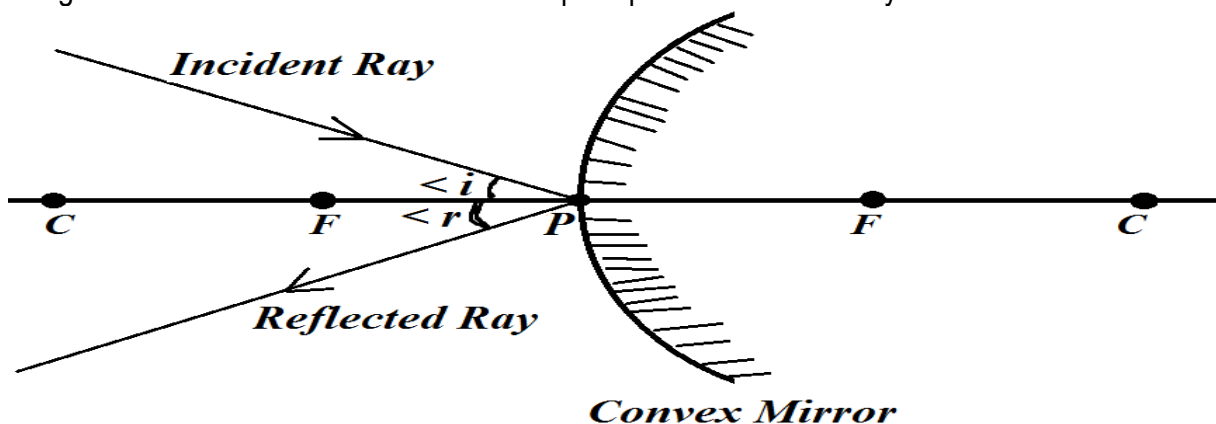
Rule 2:

An incident ray of light heading *towards the principal focus* on the other side of a convex mirror creates a reflected ray that is parallel to the principal axis.



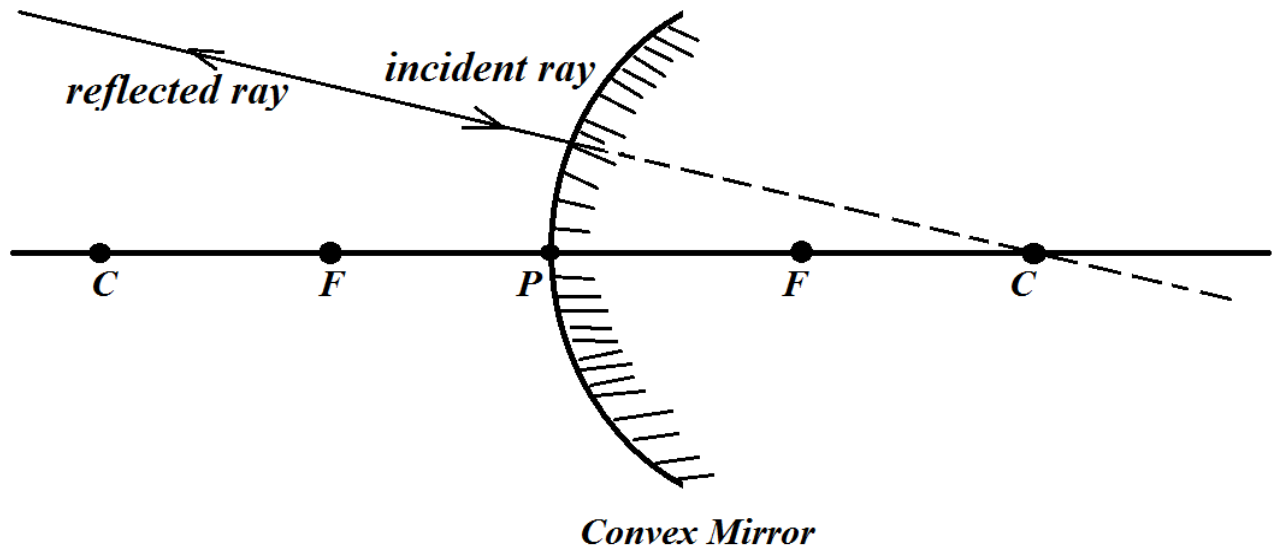
Rule 3:

An incident ray of light *that is not parallel or perpendicular to the principal axis*, heading straight for the pole (P) of the convex mirror creates a reflected ray with an angle of **reflection** equal to the angle of incidence measured between the principal axis and each ray.



Rule 4:

A ray of light heading towards the centre of curvature on the other side of the mirror creates a reflected ray that travels back along the same path.



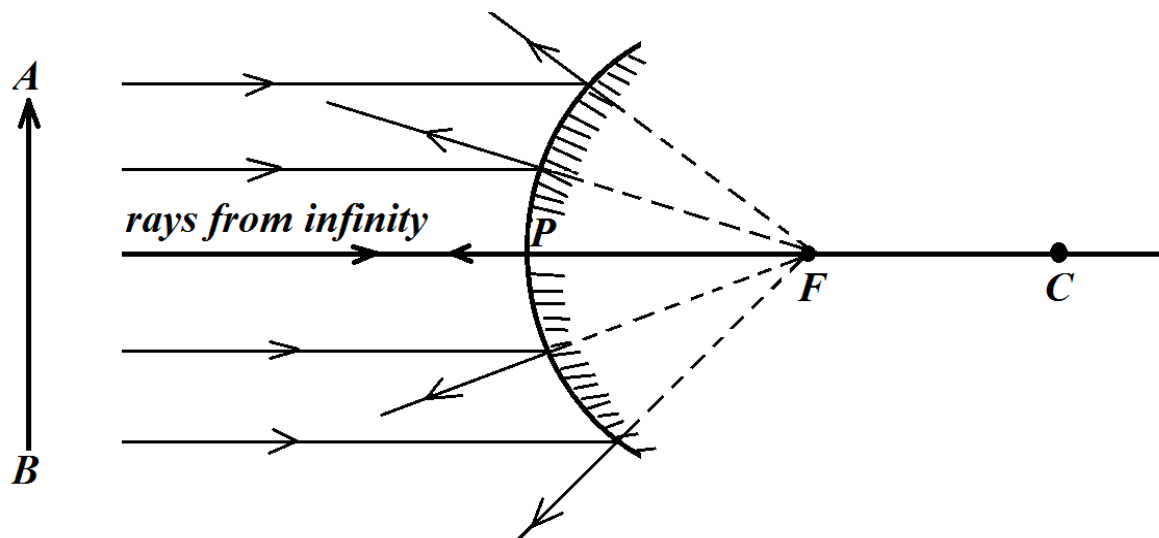
Note: Any two of these rays are sufficient to locate the image.

Images formed by convex mirror

- In **convex mirrors**, the reflecting surface is on the outside of the sphere, making the center of curvature and the focal point on the opposite side of the mirror from the object. Since the focal point is on the opposite side of the mirror from the object, the focal length is assigned a **negative number**.
- The images formed are always **virtual, erect and diminished** for all object positions.

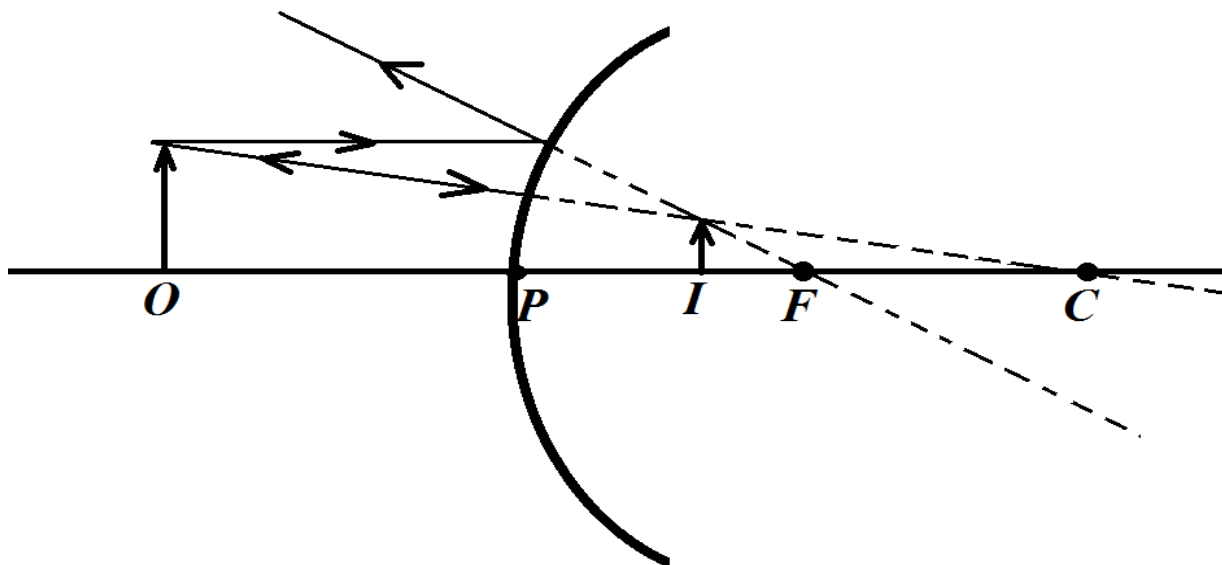
Case I: When an object is placed at Infinity

For a convex mirror, if the object is at infinity the image will be **a dot on the focal point**. As the object moves from infinity toward the mirror, the image moves along the principal axis toward the mirror. When the object is right next to the mirror, the image will be right next to the mirror on the other side.



Case II: When an object is placed *anywhere on the principal axis*.

All ray diagrams for convex mirrors look essentially like the figure below, where the image formed appears somewhere **between the mirror and the focal point**.



NB:

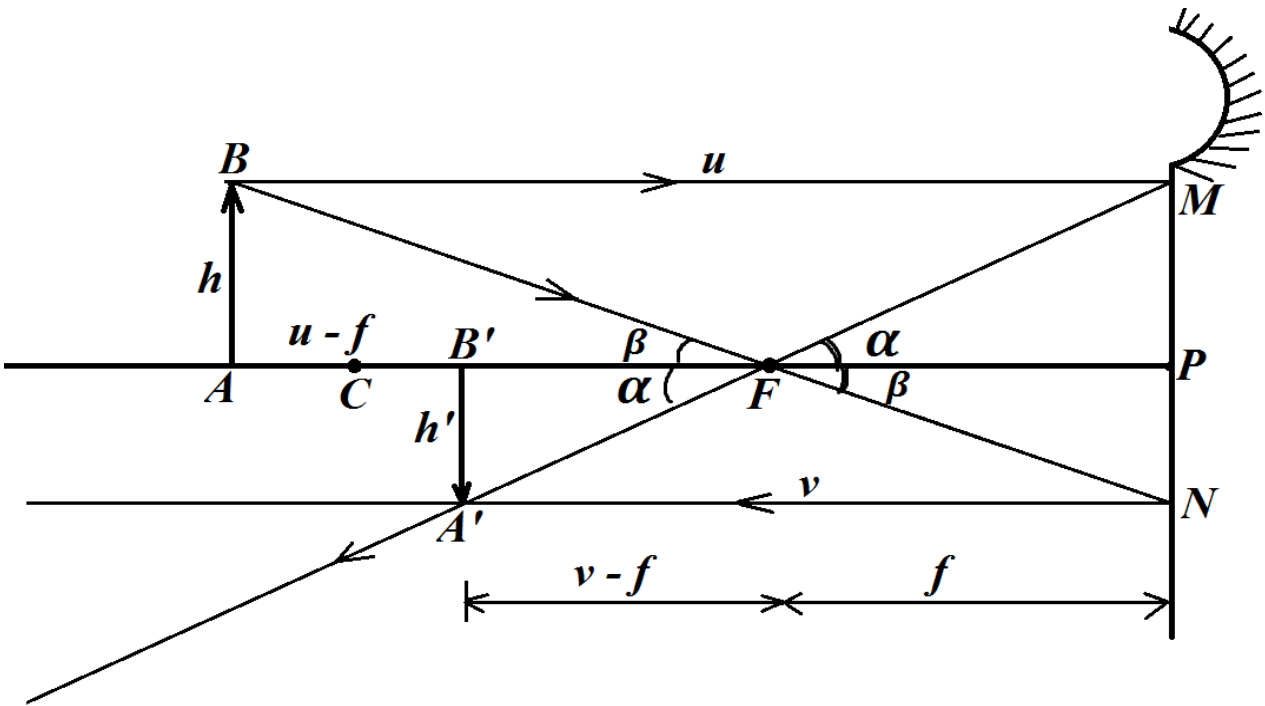
All images in convex mirrors are **upright, virtual, and diminished**. As the object moves toward the mirror, the image also moves toward the mirror and increases in size.

Class Activity – 3:1

1. An object 5 cm tall is placed 15 cm in front of a concave mirror of focal length 10 cm. By means of accurate graphical construction, determine the position and nature of the image formed. (ANS: The image is real and is 30 cm in front of the mirror)
2. An object 5 cm tall is placed 12 cm in front of a convex mirror of focal length 20 cm. By means of accurate graphical construction, determine the position and nature of the image formed. (ANS: The image is virtual and is formed 7.5 cm from the mirror)
3. In automobiles, why do the rear-view mirrors have a warning written as 'objects seen in the mirror are closer than they appear'?
ANS; The phrase "Objects in the mirror are closer than they appear" is a safety warning for drivers as Convex mirror is generally used as rear view mirrors which give a wide field of view, it also makes objects appear smaller. Since smaller appearing objects seem further away than they actually are, the driver may make mistake in changing the lane of thinking an adjacent vehicle is a safe distance behind, when in fact it is quite a bit closer.
4. A person is standing before a concave mirror cannot see his image, unless he is beyond the center of curvature? Why?
5. How will you distinguish between a plane mirror, a concave mirror and a convex mirror, without touching them?
6. When a concave mirror is used as a shaving mirror, where is the person's face in relation to the focus of the mirror?
7. If an object far away from a convex mirror moves towards the mirror, the image also moves. Does it move faster, slower or at the same speed as compared to the object?

The Curved Mirror Formula

- Consider the figure below showing an object placed beyond C in a concave mirror whose focal length is f , whereby the object and image distances are u and v respectively



- From the figure above, consider ΔABF and ΔPNF also $\Delta A'B'F$ and ΔMPF
- Therefore, the named triangles are equiangular (ie., $\Delta ABF \sim \Delta PNF$ and $\Delta A'B'F \sim \Delta MPF$)

Now consider, $\tan \alpha = \frac{h}{f} = \frac{h'}{v-f} \rightarrow \frac{h'}{h} = \frac{v-f}{f} \dots \dots \dots (i)$,

Also consider, $\tan \beta = \frac{h}{u-f} = \frac{h'}{f} \rightarrow \frac{h'}{h} = \frac{f}{u-f} \dots \dots \dots (ii)$,

Then compare equation (i) and (ii)

$$\frac{v-f}{f} = \frac{f}{u-f} \rightarrow f^2 = (v-f)(u-f)$$

$$f^2 = uv - fv - fu + f^2 \rightarrow uv - fv - fu = 0$$

$$uv = fv + fu = f(v+u)$$

$$uv = f(v+u) \rightarrow \frac{uv}{f} = v+u$$

$$\frac{1}{f} = \frac{v+u}{uv} = \frac{v}{uv} + \frac{u}{uv} = \frac{1}{u} + \frac{1}{v}$$

\therefore the mirror formula is given by: $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

Magnification of the image

- Magnification (m) is the ratio of the image size to the object size.

i.e $m = \frac{\text{image height}}{\text{object height}} = \frac{IH}{OH}$

OR

Also; Magnification **can be expressed as** the ratio of the image distance (v) from the mirror to the object distance (u) from the mirror.

$$\text{That is: } m = \frac{\text{image distance}}{\text{object distance}} = \frac{v}{u}$$

Example 1: An object is placed 20 cm in front of a concave mirror of focal length 12 cm. Find the position and the nature of the image formed

Answer; $u = 20$ cm, $f = 12$ cm, $v = v$

$$\text{From; } \frac{1}{f} = \frac{1}{u} + \frac{1}{v} \rightarrow \frac{1}{12} = \frac{1}{20} + \frac{1}{v} \rightarrow \frac{1}{v} = \frac{1}{12} - \frac{1}{20} = \frac{5-3}{60} = \frac{2}{60} \rightarrow \frac{1}{v} = \frac{1}{30}$$

Therefore, $v = 30$ cm, A positive value of v means a real image is formed. Hence a real image is formed 30 cm away from the mirror on the same side as the object

N.B

- The image formed is sometimes in front of the curved mirror and sometimes behind it. In order to make the formula applicable to spherical mirrors and various images formed. One of the following sign conventions can be used to solve the problems:-

(a) Real – is – positive Convention (b) New Cartesian Convention

(a) In Real – is – positive

- (i) All distances are measured from the pole of the mirror as the origin
- (ii) Distances of virtual objects, virtual images and virtual lengths from the pole of the mirror are negative (-)

(b) In New Cartesian Convention

- (i) All distances are measured from the pole of the mirror as the origin
- (ii) Distances measured to the right of the mirror from the pole are positive (+)
- (iii) Distances measured to the left of the mirror from the pole are negative (-)

Now we can summarize that:-

- (i) Focal length, (f) for a concave mirror is positive (+)
- (ii) Focal length (f) for a convex mirror is negative (-)
- (iii) The image distance, (v) is negative (-) for a virtual image
- (iv) The image distance, (v) is positive (+) for real images

Class Activity – 3:2

1. An object 3 cm high is placed 30 cm away from a concave mirror of focal length 12 cm. using the mirror formula, find the position, the height and the nature of the image formed.
(ANS: $V = 20$ cm, The image is real, diminished $H_i = 2$ cm)
2. **A concave mirror with a radius of curvature of 30 cm produces an inverted image 4 times the size of an object placed on its principal axis. Determine the position of the object and that of the image.** **ANS: $v = 75$ cm**
3. An object 30 cm high is placed 20 cm away from a convex mirror of focal length 25 cm. Describe the image formed. **(ANS: $H_i = 16.8$ cm, The image is diminished)**
4. An object is placed at 20 cm from a convex mirror of focal length 10 cm. Find position and nature of the image formed **[ANS; $v = 6.7$ cm behind the mirror, the image is virtual]**
5. A 1.50 cm tall object is placed 60.0 cm from a concave mirror whose radius of curvature is 30.0 cm. What is the image distance and what is the image height? **[A; $v=20$ cm, $H_i = 0.5$ cm]**
6. If a 3 cm tall object is held 15 cm away from a concave mirror with a radius of 20cm, describe its image. **[ANS; $H_i = 6$ cm]**

7. Describe the nature of image formed when an object is placed at a distance of 30 cm from a convex mirror of focal length 15 cm. **[ANS; the image formed is virtual at 10cm behind the mirror]**
8. If you have a concave mirror whose focal length is 100 cm, and you want an image that is upright and 10 times as tall as the object, where should you place the object?

ANS; $f = 100\text{cm}, m = 10, u = ?$

Also; $m = -\frac{v}{u} \rightarrow v = -mu = -10u$

From; $\frac{1}{f} = \frac{1}{u} + \frac{1}{v} \rightarrow \frac{1}{f} = \frac{1}{u} + \frac{1}{-10u} = \frac{10-1}{10u} = \frac{9}{10u} \rightarrow \frac{1}{100} = \frac{9}{10u} \rightarrow u = \frac{900}{10} = 90\text{cm}$

9. A concave mirror has a radius of curvature of 20 cm. Locate the image for an object distance of 40 cm. Indicate whether the image is real or virtual, enlarged or diminished, and upright or inverted. **[ANS; the image is real, smaller than the object and inverted]**
10. An object is placed at a distance of 15 cm from a convex mirror of focal length 20 cm. List four characteristics (nature, position, etc.) of the image formed by the lens.
ANS; image is virtual, smaller than the object, formed behind the mirror at 8.6cm, it is erect
11. A dentist uses a small concave mirror of focal length 3 cm and holds it at a distance of 2cm from the tooth. What is the magnification of the image? **[ANS $m=3$]**
12. A dentist uses a concave mirror to examine a tooth that is 1cm in front of the mirror. The image of the tooth forms 10cm behind the mirror.
- What is the mirror's radius of curvature? **[ANS; $f = 1.1 \therefore r = 2f = 2.2\text{cm}$]**
 - What is the magnification of the image? **[ANS; $m = -\frac{v}{u} = -\left(-\frac{10}{1}\right) = 10$]**
13. When a man stands 1.52 m in front of a shaving mirror, the image produced is inverted and has an image distance of 18.0 cm. How close to the mirror must the man place his face if he wants an upright image with a magnification of 2?

ANS; $u = 1.52\text{m} = 152\text{cm}, v = 18\text{cm} \rightarrow \frac{1}{f} = \frac{1}{u} + \frac{1}{v} \rightarrow f = 16.1\text{cm}$

Also; $m = -\frac{v}{u} \rightarrow v = -mu = -2u, \text{ since } m = 2$

From; $\frac{1}{f} = \frac{1}{u} + \frac{1}{v} \rightarrow \frac{1}{f} = \frac{1}{u} - \frac{1}{2u} = \frac{1}{2u} \rightarrow \frac{1}{16.1} = \frac{1}{2u} \rightarrow u = \frac{16.1 \times 1}{2} = 8.05\text{cm}$

14. A concave mirror is designed so that a person 20.0 cm in front of the mirror sees an upright image magnified by a factor of two. What is the radius of curvature of this mirror?

ANS; $u = 20\text{cm}, m = 2, r = ?$

From; $m = -\frac{v}{u} \rightarrow v = -mu = -2 \times 20 = -40, v$ is negative since an upright image was formed

Also, $\frac{1}{f} = \frac{1}{u} + \frac{1}{v} = \frac{1}{20} + \left(-\frac{1}{40}\right) = \frac{2-1}{40} = \frac{1}{40} \rightarrow f = 40\text{cm} \therefore r = 2f = 80\text{cm}$

NB:

- **Convex mirrors produce diminished images but have a very wide field of view compared to plane mirrors**
- **Concave mirrors magnify images**

Uses of Convex mirrors

- Convex mirrors can be used as driving mirrors in cars and motorcycles. This is because convex mirrors provide a wide field of view compared to a plane mirror. However, a convex mirror used as a driving mirror has one limitation. It forms a virtual diminished image which appears to be farther away from the observer than it actually is. This may lead to misjudgment by the observer which can result into accidents
- They are used to see around corners. To avoid the crashing of vehicles or supermarket trolleys at the corners

- (iii) They are used for Supermarket surveillance. For surveillance in business establishments and security installations

Uses of Concave mirrors

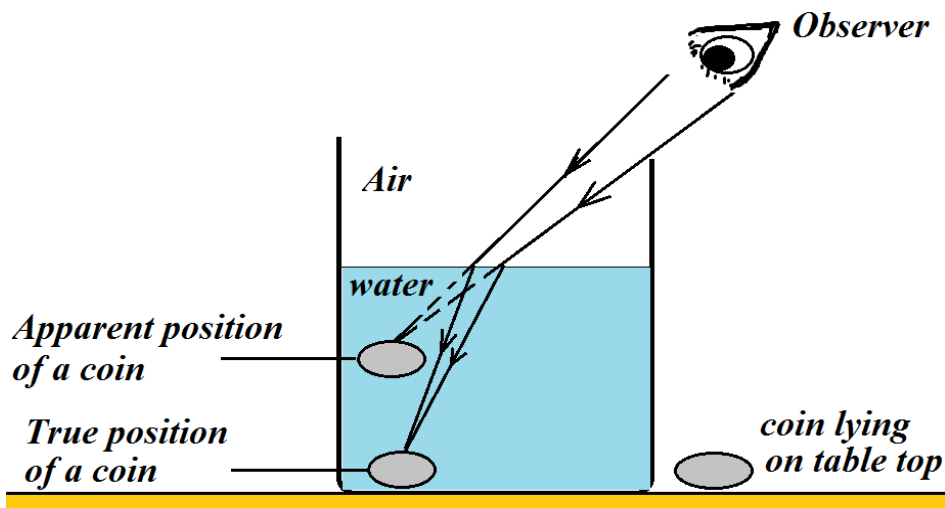
- (i) They used as shaving mirrors. Due to magnification and erect image
- (ii) They are used in reflecting telescopes. To see distant stars
- (iii) They are used in solar cookers.
- (iv) They are used in making car headlamp and torch (by using parabolic mirrors)
- (v) They are used by dentist to see enlarged image of patient teeth

Refraction of Light through Plane Media

- **Straight** things often look bent when they stand in water. The science behind this illusion is called **refraction**. Light rays usually travel in straight lines, but when they pass from one material to another they can bend. This happens **because** light travels at different speeds in different materials (medium). For example, when light moves from water to air, it changes speed and bend in a new direction. This can make an object in water appear to be in a different position
- **Therefore; Refraction of light** – Is the process by which the direction of a ray of light changes when passes obliquely from one medium into another
- It is possible to observe refraction through a “set up” of the type described below

Activity 1

- Take two coins and place one coin at the bottom of a beaker and another alongside the beaker, now half fill the beaker with water.



- Next observe the two coins by looking vertically downwards from the beaker

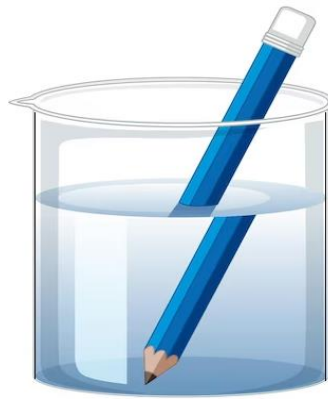
Question

What do you observe about the position of the coin in the beaker compared with the coin kept alongside the beaker?

- The coin in the beaker appears to be “**raised up**” with respect to the coin alongside the beaker

Activity 2

- Dip a pencil in a glass of water at about 45° to the surface of the glass.



- Examine the appearance of the pencil when viewed at an angle to the surface of the glass

Question

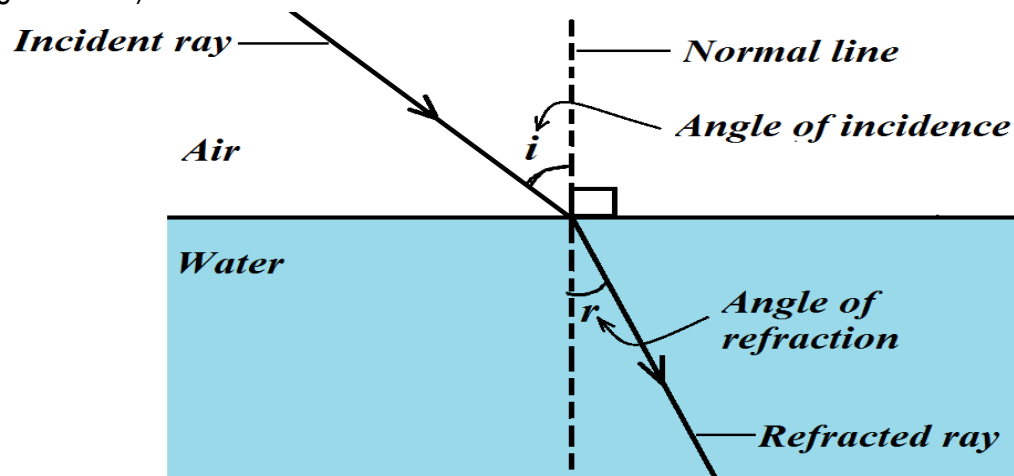
What do you observe about the appearance and shape of the pencil?

- The pencil appears “bent” near the surface of water due to change of direction of rays of light at the boundary between water and air as shown above

All of the above activities (and many other similar phenomenon) can be easily understood in terms of refraction of light

Angle of incidence and angle of refraction

- The angle of incidence is the angle between a ray of light which hits the surface and the normal line. The ray of light is called the incident ray.
- The angle of refraction is the angle between the refracted ray and the normal line. (see the figure below)



Refraction of light at air - water interface

Laws of Refraction

When a beam of light travels from two medium then it follow two laws of refraction. The laws state that:-

1. “At the point of incidence, the *Incident ray*, the *normal* and the *refracted ray* all lie in the *same plane*”
2. The ratio between the Sine of the angle of incidence and Sine of the angle of refraction is **constant** to the interference of the medium *i. e.*, $\frac{\sin\theta_i}{\sin\theta_r} = \text{constant}$

Refractive Index (index of refraction)

- As described above, when light is traveling from air into another medium **Snell's Law** states the relationship between the angle of incidence and angle of refraction is governed by the constant,

$$\eta = \frac{\sin\theta_i}{\sin\theta_r}$$

- Where θ_i is the angle of incidence, θ_r is the angle of refraction, and η is the ratio of the two Sines and is called the **index of refraction** or the **refractive index**.
- The index of refraction is also related to the relative speeds of light in a vacuum and in the medium. That is called an **absolute index of refraction** that can be measured for each material.

$$\eta = \frac{\text{Speed of light in a vacuum}}{\text{Speed of light in the medium}} = \frac{c}{v}$$

- For a light ray passing from a medium with refractive index η_1 to a medium with refractive index, η_2 the angle of incidence θ_i is related to the angle of refraction, θ_r by

$$\eta_1 \sin\theta_i = \eta_2 \sin\theta_r$$

- When a light ray moves from air to a denser medium, the value of η_1 can be taken as **approximately equal to 1**. We can then write Snell's law as $1 \times \sin\theta_i = \eta_2 \sin\theta_r \rightarrow \eta_2 = \frac{\sin\theta_i}{\sin\theta_r}$
- Whereby η_2 is the refractive index of the denser medium
- Therefore, **Refractive index** between air and the medium is given by $n_m = \frac{\sin\theta_i}{\sin\theta_r}$

Worked Examples:

- A ray of light travels from water to glass. What will be the angle of refraction in glass if the angle incidence in water is 20° ? If the refractive index of the glass is 1.5

Soln:

Given: $\theta_i = 20^\circ$, $n_g = 1.5$, $\theta_r = ?$ From: $n_g = \frac{\sin i}{\sin r}$

$$n_g = \frac{\sin i}{\sin r} \rightarrow 1.5 = \frac{\sin 20}{\sin r} \rightarrow \sin r = \frac{\sin 20}{1.5} = 0.2280$$

$$\therefore r = \sin^{-1}(0.2280) = 13.2^\circ$$

- What is the constant value if the angle of incidence is 50° and the angle of refraction is given to be 30° ?

Solution; From; $\eta = \frac{\sin\theta_i}{\sin\theta_r} = \frac{\sin 50}{\sin 30} = 1.53$

- A ray of light travels from water to glass. The refractive index of water is 1.3 and that of glass is 1.5. The angle of incidence is 60° . What would be the angle of refraction?

Solution; given $n_w = 1.3$, $n_g = 1.5$, $\theta_i = 60^\circ$

From; $n_w \sin\theta_i = n_g \sin\theta_r$, 48.63°

$$n_w \sin\theta_i = n_g \sin\theta_r \rightarrow 1.3 \times \sin 60 = 1.5 \times \sin\theta_r$$

$$\sin\theta_r = \frac{1.3 \times \sin 60}{1.5} = 0.75 \quad \therefore \theta_r = \sin^{-1}(0.75) = 48.64^\circ$$

NB:

- Any material has its own refractive index due to the fact that each has **individual optical density**
- **When light rays pass through rarer to a denser medium**, the light rays bend towards the normal. Due to this the angle of refraction is smaller than the angle of incidence. e.g. In the case when light rays pass from air to water or from air to glass, **it bends towards normal**. It is because of the reason that the speed of light rays reduces while passing from air to glass or water.
- **When light rays pass from denser to rarer medium**, the light rays bend away from the normal. Due to this the angle of refraction becomes more than the angle of incidence. e.g. In case when light rays pass from water to air or glass to air, **light rays bend away from the normal**. The speed of light rays becomes greater while passing from glass or water to air.
- The speed of light in a medium is low if the refractive index of the medium is high and vice versa.

Refractive Index of Different Medium

Medium	Refractive Index	Medium	Refractive Index
Diamond	2.417	Air (at s.t.p)	1.00029
Ethyl alcohol	1.360	Glycerin	1.47
Glass/crown	1.520	Paraffin	1.44
Quartz	1.553	Crown(flint)	1.65
Water (200°C)	1.333	Ice	1.31

Principle of Reversibility of Light

- **The principle of reversibility of light states that the paths of light rays can be reversed**
- If the refractive index for a ray of light moving from air (**a**) to glass (**g**) is represented as $a\eta_g$ and the refractive index for a ray moving from glass to air is represented as $g\eta_a$, then by principle of reversibility of light:

$$g\eta_a = \frac{1}{a\eta_g}$$

For Example;

The refractive index for a ray of light travelling from air to water is 1.33. What is the refractive index for a ray travelling from water to air

Answer:

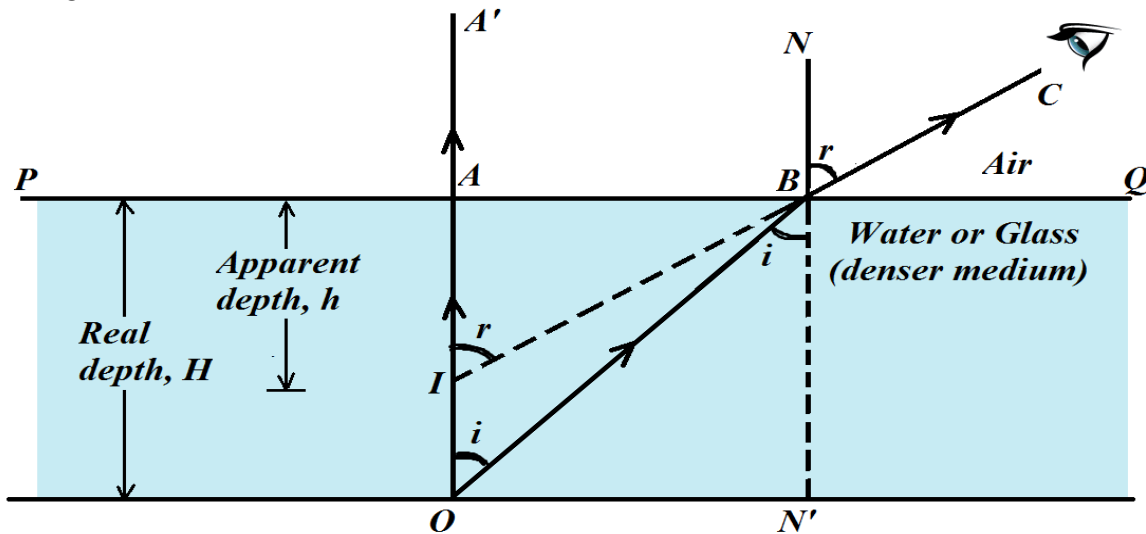
Given: $a\eta_w = 1.33$

From: $w\eta_a = \frac{1}{a\eta_w} = \frac{1}{1.33} = 0.75$

Calculating the Refractive index by the real and apparent depth method

- We see the objects closer than their real depth to the surface. We see objects only if the rays coming from them reaches our eyes. In the figure below, ray coming from the object O reaches the observer's eye after refraction. Thus, observer sees the image of the object at the distance **AI** from the surface which is the apparent depth of the Object. This is the result of the refraction of light.
- Consider a point object **O** kept at the bottom of a transparent medium (such as water or glass) separated from air by the surface **PQ**.

- A ray of light **OA**, starting from the object **O**, is incident on the surface **PQ** normally, so it passes undeviated along the path **AA'**. Another ray **OB**, starting along the object **O**, strikes the boundary surface **PQ** at **B** and suffers refraction. Since, the ray travels from denser to rarer medium so it bends away from normal **N'BN** drawn at the point of incidence **B** on the surface **PQ** and travels along **BC** in air.
- When viewed by the eye, the ray **BC** appears to be coming from point **I** which is the virtual image of **O**, obtained on producing **A'A** and **CB** backwards.
- Thus, any point seen from air will appear to be at **I**, which is lesser depth **AI** than its actual depth **AO**.



- From the figure above; $\angle AOB = \angle OBN' = i$
- **Also;** $\angle BIA' = \angle CBN = r$
- Consider a right angled triangle, **BAO**, $\text{Sini} = \frac{AB}{AO}$ ----- (i)
- Also, Consider a right angled triangle, **AIB**, $\text{Sinr} = \frac{AB}{AI}$ ----- (ii)
- Now divide equation (i) into equation (ii)

$$\frac{\text{Sini}}{\text{Sinr}} = \frac{AB}{AO} \div \frac{AB}{AI} = \frac{AB}{AO} \times \frac{AI}{AB} = \frac{AI}{AO} = \frac{h}{H}$$

- Thus; the refractive index from medium to air, by Snell's law

$$m\eta_a = \frac{\text{Sini}}{\text{Sinr}} = \frac{h}{H}$$

- **But the** refractive index from air to medium is given by,

$$a\eta_m = \frac{1}{m\eta_a} = \frac{1}{\frac{h}{H}} = \frac{H}{h} \quad (\text{Principle of reversibility of light})$$

- Therefore, the refractive index is given by $a\eta_w = \frac{\text{Real depth (H)}}{\text{Apparent depth (h)}}$
- The difference between real and apparent depth is known **as vertical displacement**

Worked Examples

1. A fish at the bottom of a pond appears to be 1.2 m from the water surface. What is the depth of the pond? The refractive index of water = 1.33
 Solution;
 Apparent depth, $h = 1.2\text{m}$, Depth of the pond, $H = ?$ $a\eta_w = 1.33$

From; refractive index, $a\eta_w = \frac{\text{real depth}}{\text{apparent depth}} = \frac{H}{h}$

$$1.33 = \frac{H}{1.2} \rightarrow H = 1.2 \times 1.33 \approx 1.6m$$

2. A coin is placed at the bottom of a tall glass jar. When the jar is filled with paraffin to a depth of 32.4 cm, the coin is apparently seen displaced 9.9 cm from the bottom. What is the refractive index of paraffin?

Answer

Apparent depth = (32.4 – 9.9) cm = 22.5 cm, Real depth (H) = 32.4 cm

$$\text{Refractive index} = \frac{\text{Real depth}}{\text{Apparent depth}} = \frac{32.4}{22.5} = 1.44$$

Refractive Index in terms of Velocity of Light

- For a ray of light travelling from medium 1 to medium 2, refractive index is the ratio of velocity of light in medium 1 to velocity of light in medium 2.

From Absolute Refractive index; *i.e.* $\eta = \frac{c}{v}$

$$\text{Thus; } \eta_1 = \frac{c}{v_1} \rightarrow c = \eta_1 v_1 \text{ ----- (i)} \quad \eta_2 = \frac{c}{v_2} \rightarrow c = \eta_2 v_2 \text{ ----- (ii)}$$

Now; compare the two equations

$$\text{Therefore; } \eta_1 v_1 = \eta_2 v_2 \rightarrow \frac{\eta_2}{\eta_1} = \frac{v_1}{v_2}$$

- Whereby the ratio, $\frac{\eta_2}{\eta_1}$ is referred as the relative refractive index of medium 2 to medium 1
- For example, for a ray of light travelling from air to water

$$\frac{\eta_w}{\eta_a} = \frac{\eta_w}{1} = a\eta_w = \frac{\text{velocity of light in air}}{\text{velocity of light in water}}$$

Worked Examples

1. The speed of light in air is 3.0×10^8 m/s. What is the speed of light in glass? Take refractive index of glass = 1.5

Answer

Given: $a\eta_g = 1.5$, $C = 3.0 \times 10^8$ m/s

$$\text{From: } a\eta_g = \frac{\text{velocity of light in air}}{\text{velocity of light in glass}} \rightarrow a\eta_g = \frac{3 \times 10^8}{v_g}$$

$$v_g = \frac{3 \times 10^8}{1.5} = 2 \times 10^8 \text{ m/s}$$

2. What is the speed of light in a diamond? The refractive index of diamond is 2.42

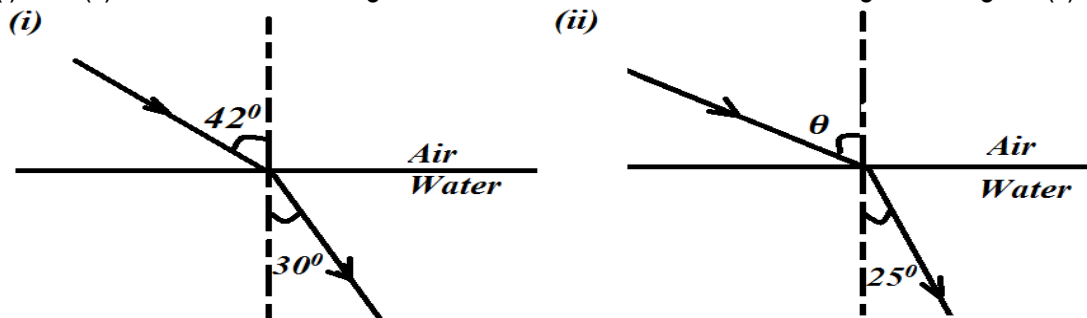
Given: $a\eta_d = 2.42$, $C = 3.0 \times 10^8$ m/s

$$\text{From: } a\eta_d = \frac{\text{velocity of light in air}}{\text{velocity of light in diamond}} \rightarrow a\eta_d = \frac{3 \times 10^8}{v_d}$$

$$v_d = \frac{3 \times 10^8}{2.42} = 1.24 \times 10^8 \text{ m/s}$$

Class Activity – 3:3

- When a ray of light is travelling from air to glass, the angle of refraction is 30° . If the refractive index of the glass is 1.5. Determine the angle of incidence, i . (ANS: $i^\circ = 48.6^\circ$)
- A fish appears to be 0.9 m below the surface of water of refractive index $4/3$ when viewed directly from above. What is the true depth the fish is? (ANS: $R = 1.2\text{m}$)
- Light is incident on an air-glass boundary at an angle of incidence of 40° . If the refractive index of the glass is 1.7, determine the angle of refraction [ANS; 22.2°]
- Fig (i) and (ii) show refraction of light at air-water interface. Determine angle θ in figure (ii)

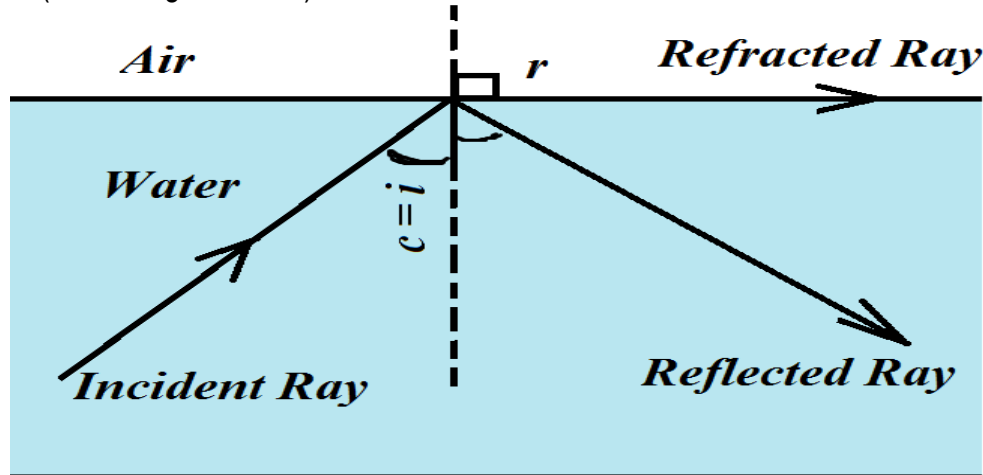


ANS; from; $n_W = \frac{\sin i}{\sin r} \rightarrow \frac{\sin 42}{\sin 30} = \frac{\sin \theta}{\sin 25} \rightarrow \theta = \sin^{-1} \left(\frac{\sin 25 \times \sin 42}{\sin 30} \right) = 34.4^\circ$

- A beaker of height 10 cm is filled with water. An optical pin which is at the bottom of the beaker is then viewed from the top of the beaker. How far does the pin appear from the surface, if the refractive index of water is $4/3$? (Answer: 7.5 cm)
- Refractive index of glass is 1.5. If the speed of light in vacuum is 3×10^8 m/s, find the velocity in medium (ANS: 2×10^8 m/s)
- Paraffin has a greater refractive index than that of water. Comment about the relative velocity of light in paraffin and in water.
- Calculate the refractive index of glass given that the velocity of light in air is 3×10^8 m/s and velocity of light in glass is 2.4×10^8 m/s. [ANS; $n_g = \frac{c}{v} = \frac{3 \times 10^8}{2.4 \times 10^8} = 1.25$]
- Calculate the refractive index of the medium if the speed of light in that medium is given as 2.76×10^8 m/s. [ANS; $n_m = \frac{c}{v} = \frac{3 \times 10^8}{2.76 \times 10^8} = 1.09$]
- Optical fibers are generally composed of silica, with an index of refraction around 1.44.
 - How fast does light travel in a silica fiber, and [ANS; $v = 2.08 \times 10^8$ m/s]
 - How long will it take for that light to travel from St. Petersburg, Florida to Anchorage, Alaska 6091 km apart? [ANS; $t = \frac{s}{v} = \frac{6091000}{2.08 \times 10^8} = 0.029\text{s}$ (29 ms)]
- The refractive index for a ray of light travelling from air to oil (n_o) is $5/3$, while that for a ray travelling from air to glass (n_g) is $3/2$. What is the refractive index for a ray travelling from glass to oil? (Answer: 10/9)
- A coin at the bottom of a jar of glycerin appears to be 13.2 cm below the surface of the glycerin. Calculate the height of the column of glycerin in the jar given that the refractive index of glycerin is 1.47. (ANS: $H = 19.4$ cm)
- The refractive index of water is $4/3$. Find the speed of light in water given that the speed of light in air is 3.0×10^8 m/s (Answer: 2.25×10^8 m/s)
- Water is poured into a beaker to a depth of 24 cm. To an eye looking vertically down through the water, the bottom of the beaker appears to be raised 6 cm from the bottom of the beaker. Determine the refractive index of the water. ANS: $n_g = 1.33$

Critical Angle

- The critical angle is the angle of incidence where the angle of refraction is 90° . The light must travel from an optically more dense medium to an optically less dense medium as in Figure below: When the angle of incidence is equal to the critical angle, the angle of refraction is equal to 90° .
- OR – Critical angle is the angle of incidence beyond which rays of light passing through a denser medium to the surface of a less dense medium are **NO** longer refracted but totally reflected (see the figure below)

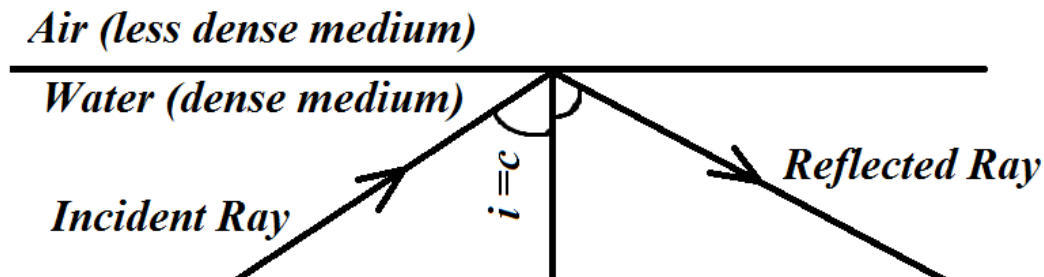


Whereby: $c = \text{critical angle}$ $r = \text{refracted angle} = 90^\circ$

- If the angle of incidence is bigger than this critical angle, the refracted ray will not emerge from the medium, but will be reflected back into the medium. This is called **total internal reflection**.

Total Internal Reflection

- **Total internal reflection** is the reflection due to the angle of incidence exceeding the critical angle
- **OR** is a phenomenon that occurs when light travels from a more optically dense medium to a less optically dense one. E.g, glass to air or water to air
- **OR** is a phenomenon that occurs when a propagated wave strikes a medium boundary at an angle larger than a particular critical angle with respect to the normal to the surface



NB:

- The reflected ray goes back to more dense medium
- When total internal reflection occurs, there is **no refraction** at all

Relationship between Critical Angle and Refractive Index

Consider the equation below;

$$w\eta_a = \frac{\sin i}{\sin r} \quad \text{but: } r = 90^\circ \text{ and } i = c$$

$$\text{Then, } w\eta_a = \frac{\sin c}{\sin 90} = \frac{\sin c}{1}$$

According to the principle of reversibility of light (the path can be reversed)

$$\text{Therefore: } a\eta_w = \frac{1}{\sin c}$$

- This is true for all material media where, water, w represent material media

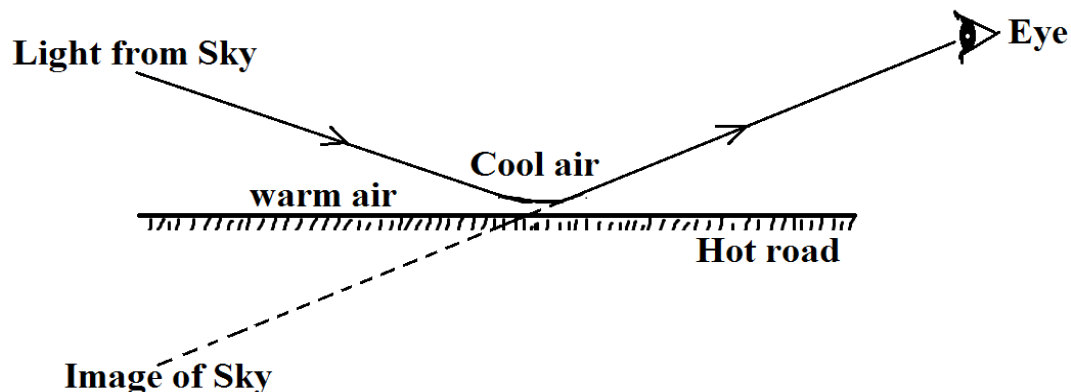
Conditions for total internal reflection to occur

- Light must be travelling from an optically denser medium (higher refractive index) to an optically less dense medium (lower refractive index) (e.g glass to air)
- The angle of incidence should be equal or greater than the critical angle for the two media

Some effects of total Internal Reflection of Light

(a) Mirage

- This is an impression of the presence of imaginary water some distance away
- **OR - Mirage** Is an optical phenomenon in the atmosphere that makes an object appears to be displaced from its true position.
- This happens when air nearer the surface of the ground is less dense than the air above. Cool air is dense than warm air



- Light from the sky is gradually refracted away from the normal as it passes from denser layer of air to less dense layers
- When light meets a layer at angles of incidences greater than the critical angle. It suffers total internal reflection
- Thus the reflection of the sky forms an image which appears as a pool of water on the road
- Mirages are often seen during hot sunny days.

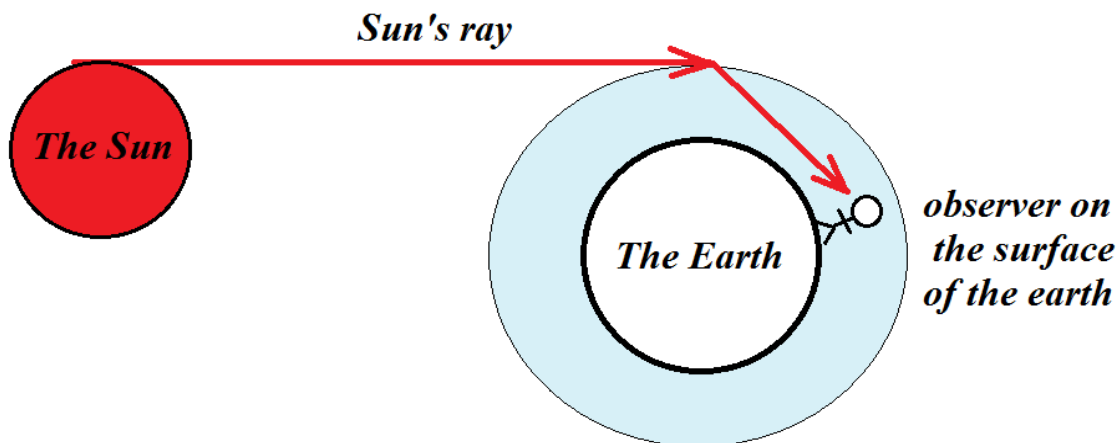
(b) Atmospheric Refraction

- Since we know the change in medium causes refraction likewise our earth is covered with the atmosphere having different layers, and these layers possess different temperatures at different heights, **some atmospheric layers are warm and some are cold.**

- Now the warmer layer of the atmosphere behaves like an optically rarer medium whereas the cooler layer behaves like an optically denser medium.
- So now we again have different media thus when light passes through different layers of the atmosphere it gets refracted.

Examples of Atmospheric Refraction

- **Twinkling of stars.** This happens when the atmosphere refracts more light to us the star **seems bright**. However, when the atmosphere refracts less light than **the star seems dim**, and this process happens so fast that it seems to us twinkling.
- **The apparent position of the stars:** As we discussed in the twinkling of a star that our atmosphere has different layers which cause light to refract, so our atmosphere bends the starlight towards normal, which causes the apparent position of the star to appear to be slightly different from its actual position.
- **Bending of Sun's Rays:**
Air is slightly more optically dense than a vacuum, this means that when sunlight passes from the vacuum of space into our atmosphere it bends slightly towards the normal. When the sun is below the horizon and thus not visible on a direct line, some of the light entering the atmosphere will refract and remain visible to us. Observers can see the sun before it actually comes up over the horizon, or for a short time after it sets.

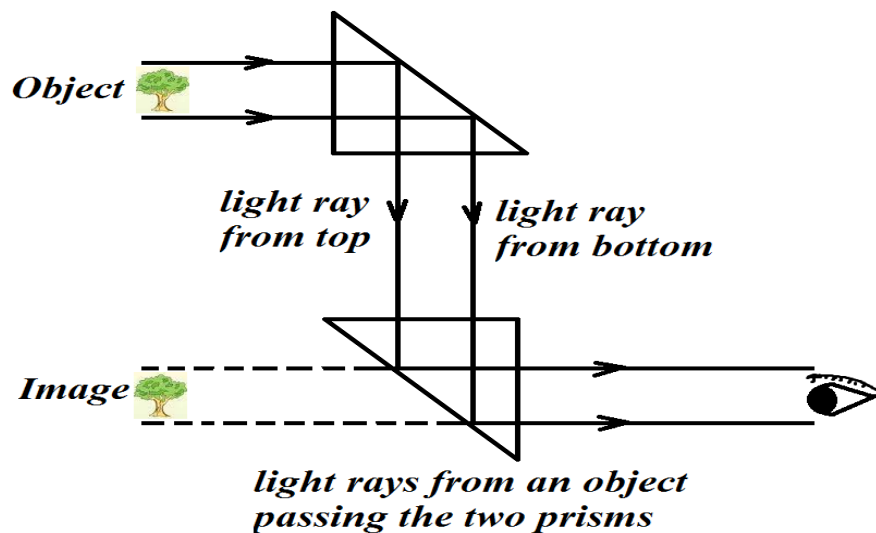


- **The Rainbow:** The formation of a rainbow involves Reflection, Refraction, and Dispersion. Firstly, the sunlight enters into the water droplet during which it undergoes refraction as well as dispersion. So, the light bends inside the droplet (due to change in medium) and performs total internal reflection and then comes out where the light again refracts which results in the formation of a rainbow.
- (c) A swimming pool always seems or looks much shallower than it really is because of the light that comes from the bottom of the pool bends at the surfaces due to the refraction of light.

Applications of total internal Reflection

1. Prism periscope

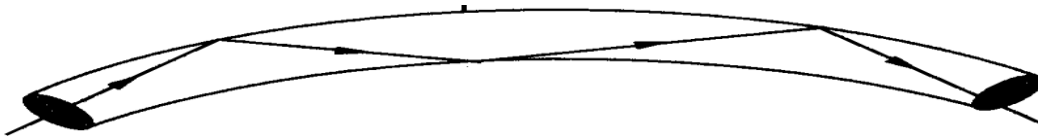
- A periscope is a device which enables us to see over the top of an obstacle
- A prism periscope consists of two $45^\circ - 90^\circ - 45^\circ$ prisms (See the fig below)



- Prisms are used in the construction of a periscope. The image seen through a periscope is upright and the same size as the object.
- The first prism rotates the image by 90° and the second prism rotates it back to normal.

2. Optical fibres

- An optical fibre is a thin rod of high-quality glass designed to guide light along its length by total internal reflection.
- Light inside these fibres hits the sides at an angle greater than the critical angle and is transmitted by being repeatedly totally internally reflected.



Some uses of optical fibres

- Are used in telecommunications and networking to send message signals.
- They are also used as light guides in medical and other applications
- Optical fibres are also used in imaging optics. A bundle of fibres along with lenses are used to make a long imaging device called an **endoscope**. An endoscope is an instrument made of a fibre optic cable. It is used by doctors to see the inside of the human body such as the stomach and the duodenum.

NB:

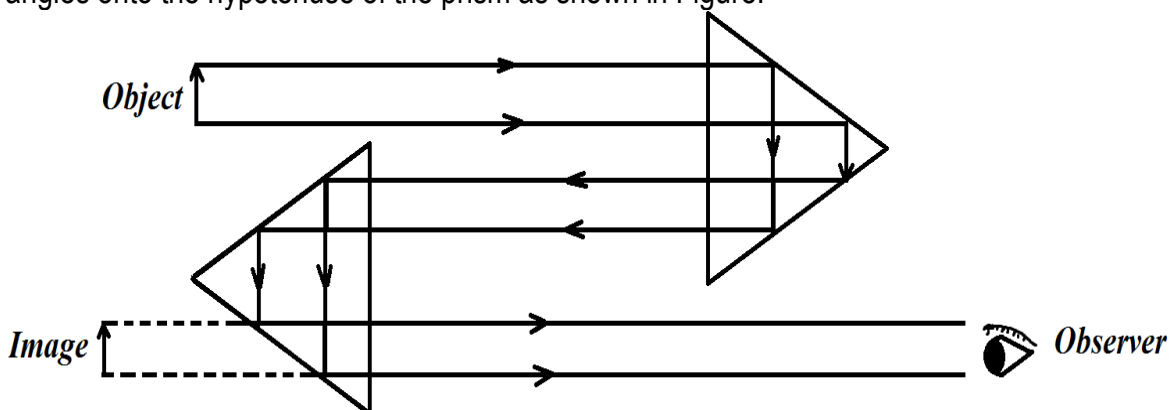
- ✓ Medical endoscopes are used in minimally invasive surgical procedures.
- ✓ Industrial endoscopes are used for inspecting machine parts
- ✓ **Advantages of using optic fiber rather than copper cables**
 - Have high speed.** Fiber optic cables transmit data much faster than copper wires do because fiber optics uses the speed of light rather than the speed of electrons. Fiber optic Internet connections can range from 5 Mbps to 100 Gbps. This ensures no wasted time or productivity due to a slow Internet connection. Fiber optic cables also enable “symmetric speed,” which means that uploading and downloading speeds are equal.
 - High security.** Because fiber optic cables cannot be tapped easily, this makes them the ideal option for high Internet security. The only way to penetrate a fiber-optic Internet connection is to cut the wires, which will make the signal disappear

This is important for sensitive applications such as financial transactions and government communications.

- (iii) Copper cables are typically thicker and heavier than fiber optic cables, making them less convenient for installations that require lightweight and slim cabling.
- (iv) **Distance:** Fibre optic cabling is capable of transmitting data over significantly longer distances without signal degradation. They are often used for long-distance communication, including undersea cables. Copper cabling, on the other hand, has distance limitations, with the quality of data transmission diminishing over greater distances.
- (v) **High Bandwidth:** Fiber optic cables have a much higher bandwidth than copper wires, which means they can carry more data at faster speeds. This enables the transmission of high-bandwidth content such as high-definition video and large data files. This makes optical fiber an essential component of modern communication networks.

Binoculars and telescopes

- Binoculars are just a pair of telescopes one for each eye.
- A 45°/45° prism can cause light rays to bend through 180° when the light is incident at right angles onto the hypotenuse of the prism as shown in Figure.



- This property of prisms makes them useful in the construction of binoculars.
- Two prisms are arranged to bend the light rays from an object through 360°.
- A pair of binoculars is much more compact than a telescope.
- Binoculars are used to see distant objects

Class Activity – 3:4

1. A certain glass material has a refractive index of 2.5. What is its critical angle? (**Ans: 23.57°**)
2. What is the critical angle for a light ray traveling in water with a refractive index of 1.33 that is incident on the surface of water above which air is with a refractive index of 1.00? [**ANS; 48.8**]
3. The critical angle of paraffin is 45°. What is the refractive index of paraffin? (**Answer: 1. 414**)
4. What is the critical angle for a light ray traveling in water with a refractive index of 1.33 that is incident on the surface of water above which there is ice with a refractive index of 1.31?

Solution; $\eta_1 = 1.33, \eta_2 = 1.31, \theta_c$

From; $\text{Sin}\theta_c = \frac{\eta_2}{\eta_1} \rightarrow \text{Sin}\theta_c = \frac{1.31}{1.33}$ Therefore; $\theta_c = \text{Sin}^{-1} \left(\frac{1.31}{1.33} \right) = 80^\circ$

5. An Eskimo walking along an Iceland observed an inverted image in the sky of a polar bear standing some distance away. Explain

ANS; The layers of air above the ice are such that warm air is above and temperature decreases downwards. When light from the bear travels across the layers of air it is refracted downwards until total internal reflection occurs making the apparent position of the image to be in sky.

6. Explain what you understand by the term critical angle as applied in optics
7. State two conditions necessary for total internal reflection to occur
8. Tanzania launched the use of optical fibres in communication recently. State why optical fibres are preferred to ordinary cables.
9. Critical angle of a material is 42° , determine the angle of refraction of light in the material if the incidence angle is 30° . $n_m = \frac{1}{\sin c} = \frac{1}{\sin 42} = 1.49$, $n_m = \frac{\sin i}{\sin r}$
 $1.49 = \frac{\sin 30}{\sin r} \rightarrow r = \sin^{-1} \left(\frac{\sin 30}{1.49} \right) = 19.6^\circ$
10. The refractive index for air-water boundary is $4/3$. Calculate the critical angle for water-air interface. [**ANS;** $n_w = \frac{1}{\sin c} \rightarrow C = 48.7^\circ$]
11. Explain with the aid of a diagram, how a suitable glass prism may be used to turn a ray of light 180°
12. Explain with an aid of a diagram why to a diver under water, most of the surface looks silvery. Bubbles of air rising from the diver look silvery.

Refraction of Light by Lenses

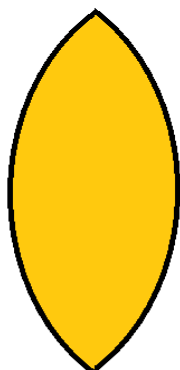
- Lens is a transparent or a translucent medium that alters the direction of light passing through it

Types of Lenses

- Convex lenses
- Concave lenses

Convex (Converging Lenses)

- In a **convex lens** (sometimes called a positive lens), the glass (or plastic) surfaces bulge outwards in the center giving the classic lentil-like shape.
- A convex lens is also called a converging lens because it makes parallel light rays passing through it bend inward and meet (converge) at a spot just beyond the lens known as the **focal point**.
- A convex lens is thicker at its centre than at its edge.
- Convex lenses converge light.
- A common application of convex lenses is that they are used in optical devices such as microscopes, telescopes, eyeglasses, magnifying glasses, etc.
- Convex lenses can be biconvex, plano-convex or converging meniscus



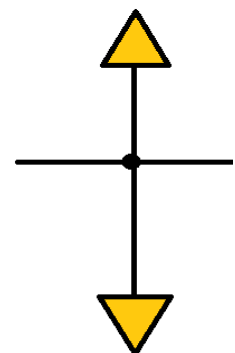
Biconvex



Plano - convex



Converging meniscus

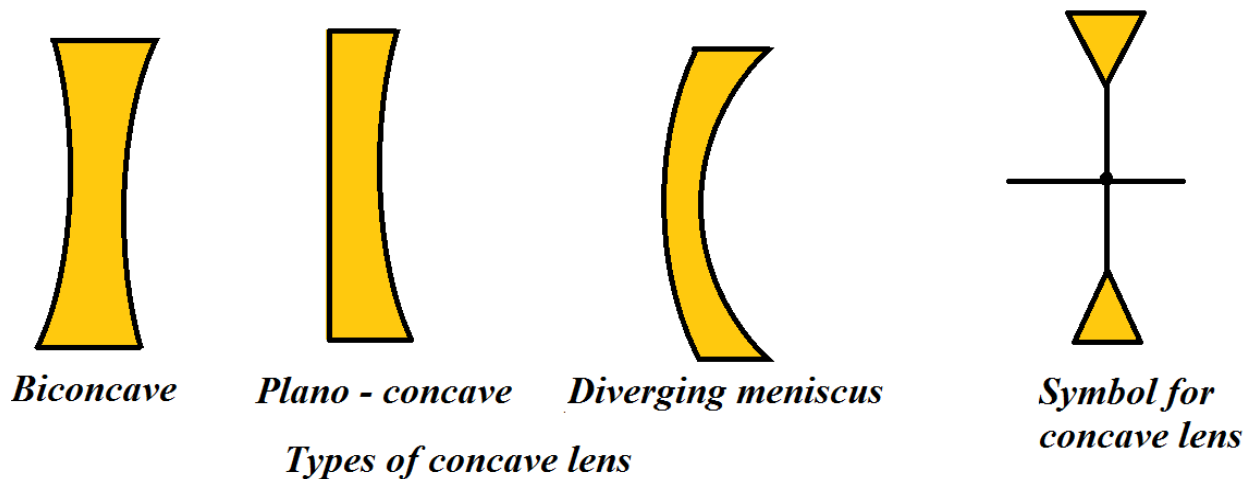


Symbol for convex lens

Types of convex lenses

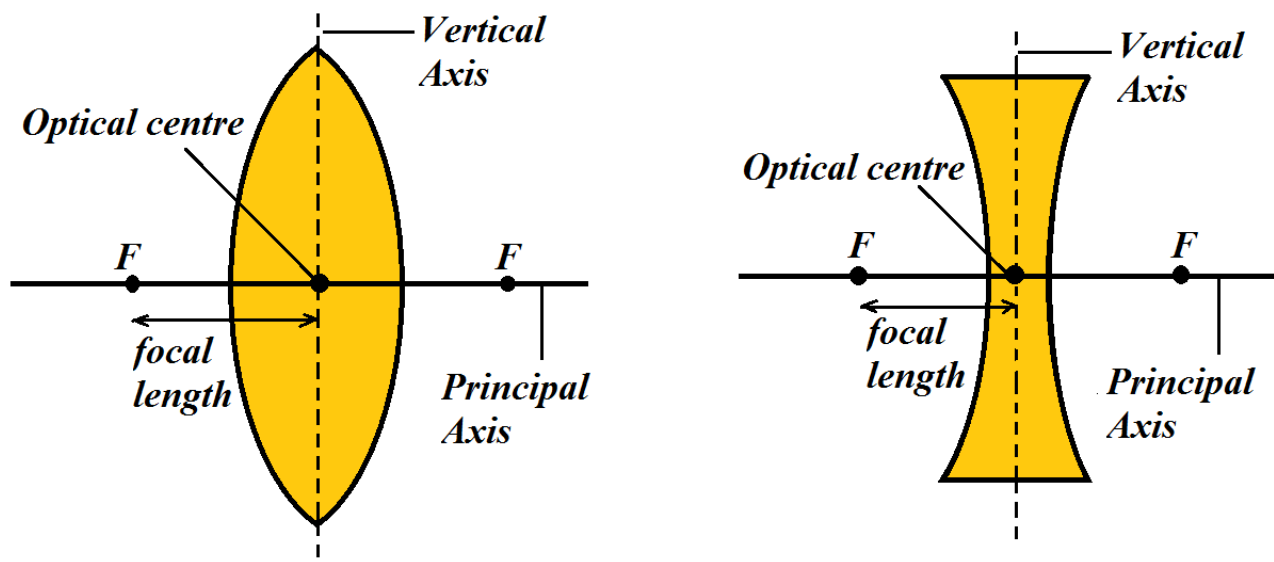
Concave (Diverging Lenses)

- A **concave lens** is thicker at the edges than at the center. Every concave lens causes all rays to diverge. Rays that approach the lens parallel to the principal axis refract as if they came from the focal point.
- Concave lenses include biconcave, plano-concave and diverging meniscus



Terms used on thin Lenses

Consider the diagrams below



- **Optical centre:** Is the geometric centre of a lens.
- **Centre of curvature, C:** Is the geometric centre of the sphere of which the lens surface is a part of.
- **Principal axis:** Is an imaginary line which passes through the optical centre of the lens at a right angle to the lens.
- **Radius of curvature, R:** Is the distance between optical centre and the centre of curvature.
- **Principal focus (focal point), F:** Is a point through which all rays travelling close and parallel to the principal axis pass through the lens.
- **Aperture:** Is the width of the lens, from one edge to another
- **Focal length, f:** Is the distance between the optical centre and the principal focus

Construction of ray diagrams

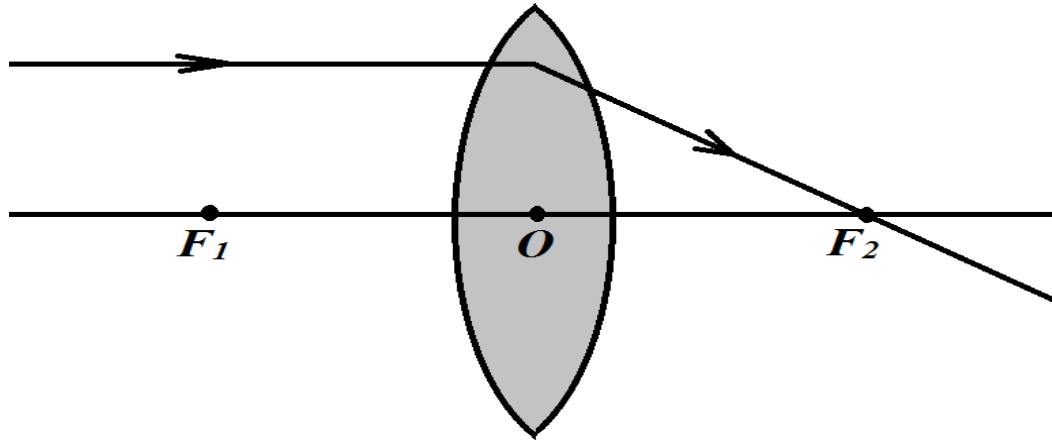
The following are the procedures used to locate image in a lens

- Choose a suitable scale
- Draw a principal axis and the lens
- Draw object in the position
- Measure the height and the distance of the image
- Convert the measurements into actual units using the chosen scale

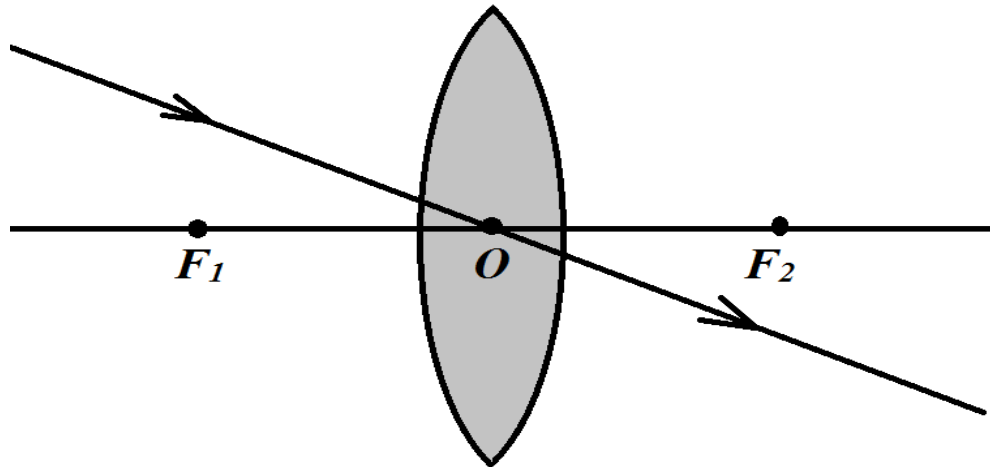
Drawing ray diagrams for converging lens

Three principal rays can be drawn to construct ray diagrams for convex lenses, as shown in **Figure** below.

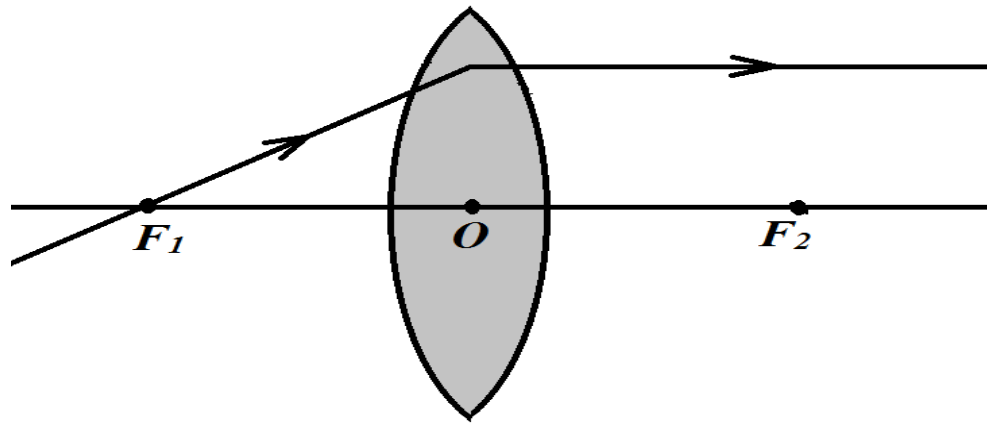
1. A ray parallel to the principal axis is refracted through the opposite focal point.



2. A ray passing through the optical center of the lens does not refract.



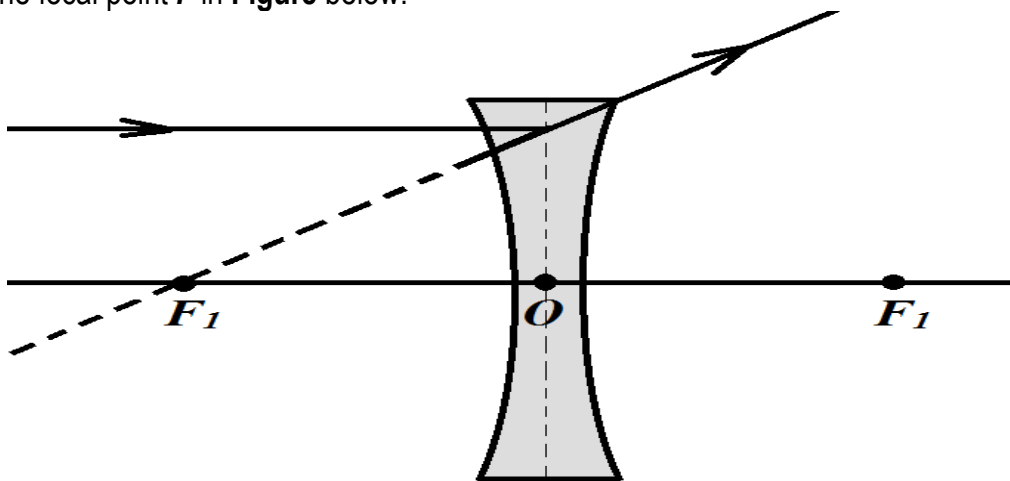
3. A ray passing through the focal point on the same side of the lens as the object is refracted parallel to the principal axis.



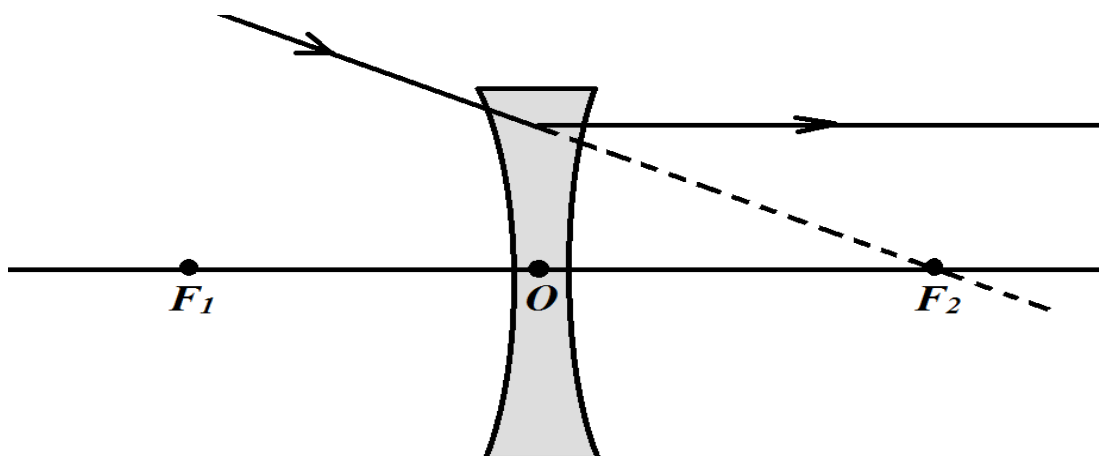
Drawing Ray diagrams for concave lens

Three principal rays can be drawn to construct ray diagrams for diverging lenses.

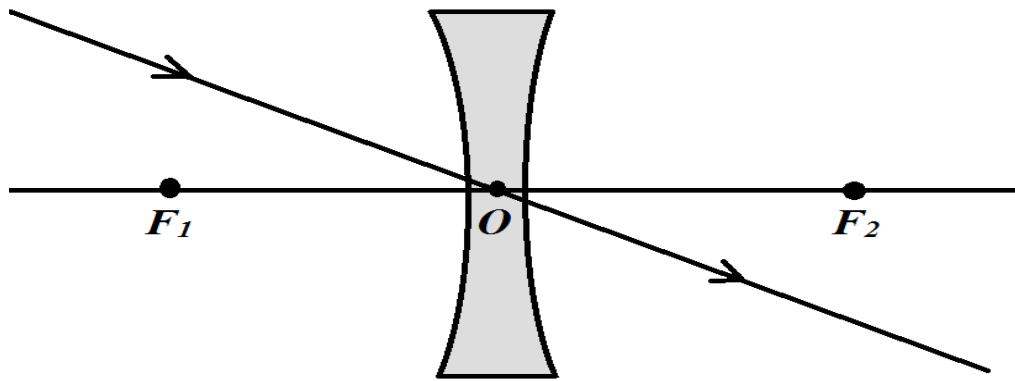
1. A ray parallel to the principal axis refracts so that its extension (the dashed line) passes through the focal point F in **Figure** below.



2. A ray heading towards focal point F refracts parallel to the principal axis, as in **Figure** below.



3. A ray passes through the center of the lens with no apparent refraction.

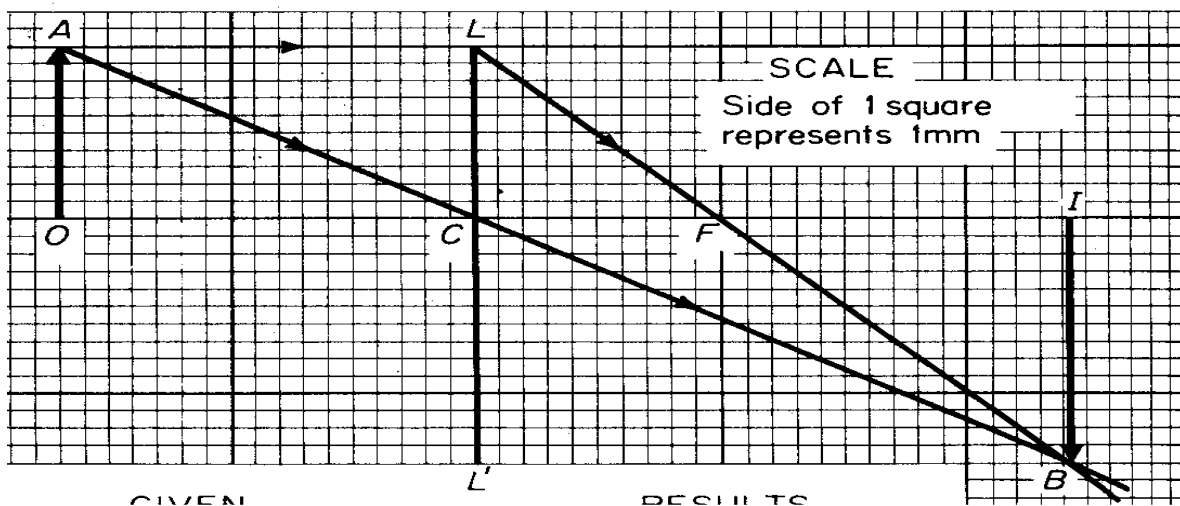


NB: The intersection of the rays for both convex and concave lenses shows where the image is formed.

Worked Examples;

1. An object 10 cm tall stands vertically on the principal axis of a convex lens of focal length 10 cm and at a distance of 17 cm from the lens. By means of accurate graphical construction find the position, size and nature of the image formed

Solution:



- (a) Position of the image = $24 \times 1 = 24$ cm from the lens
- (b) Size of the image = $(14 \times 1) = 14$ cm tall
- (c) Nature of the image: The image is (i) Real and (ii) inverted

Class Activity – 3:5

1. An object 10 cm tall stands vertically on the principal axis of a convex lens of focal length 10 cm and at a distance of 17 cm from the lens. By means of accurate graphical construction find the position, size and nature of the image formed (**ANS:** $v = 24$ cm from the lens, Size of the image = 14 cm tall, The image is Real and inverted)
2. An object 8 cm tall is placed 20 cm in front of a convex lens of focal length 16 cm. By means of accurate graphical construction, determine the position, size and nature of the image formed. (**ANS:** $V = 60$ cm, **Size:** = 24 cm, **Nature:** Real image)
3. An object of height 7.0 cm object is placed 35.0 cm from a diverging lens of focal length 14.0 cm. By means of accurate graphical construction

- (a) Find the image position [ANS; $v = 10$ cm, the same side as the object]
 (b) Find the size of the image formed [ANS; $h_i = 2$ cm, the image is smaller than the object]
 4. An object 0.05 m high is placed 0.15 m in front of a convex lens of focal length 0.1 m. Find, by construction, the nature, the position and the size of the image.
 (ANS: $H_o = 0.1$, $V = 0.3$ m, Real)

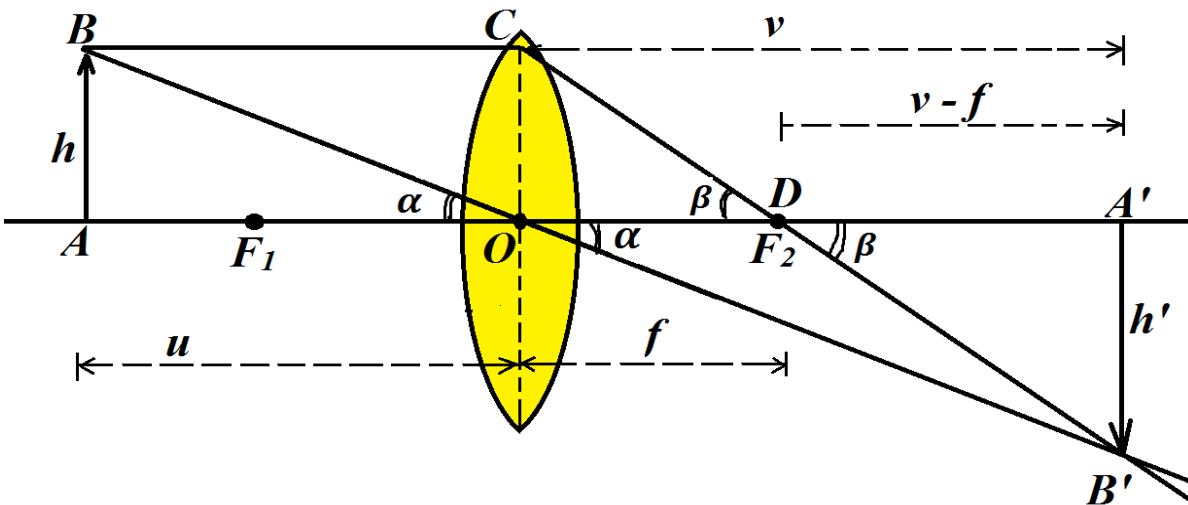
Linear Magnification (m)

- Magnification is a measure of the extent to which an optical system enlarges or reduces an image in relation to the object.

$$m = \frac{\text{height of image}(h_i)}{\text{height of object}(h_o)} = \frac{\text{image distance}(v)}{\text{object distance}(u)}$$

The Lens Formula

- Consider the figure below, an object h is placed in a convex mirror of focal length f , whereby u is the object distance and v is the image distance



- From the figure above, the following triangles are equiangular, ie., $\Delta AOB \sim \Delta A'OB'$, and $\Delta OCD \sim \Delta A'B'D$
- Now consider, $\tan\alpha = \frac{h}{u} = \frac{h'}{v} \rightarrow \frac{h}{h'} = \frac{u}{v} \dots \dots \dots (i)$,
- Also consider, $\tan\beta = \frac{h}{f} = \frac{h'}{v-f} \rightarrow \frac{h}{h'} = \frac{f}{v-f} \dots \dots \dots (ii)$,

Then compare equation (i) and (ii) $\frac{u}{v} = \frac{f}{v-f} \rightarrow u(v-f) = fv$

$$uv - uf = fv \rightarrow uv = fv + uf = (u+v)f$$

$$uv = (u+v)f \rightarrow \frac{uv}{f} = u+v \rightarrow \frac{1}{f} = \frac{u}{uv} + \frac{v}{uv} = \frac{1}{v} + \frac{1}{u}$$

\therefore the lens formula is given by: $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

Real-Is-Positive Convention

To calculate the values of u and v , a sign rule or convention is adopted. The rule is referred to as the **real-is-positive convention**.

- ❖ Sign for real object and image are $u = +$ and $v = +$
- ❖ Sign for virtual object and image are $v = -$ and $u = -$
- ❖ Sign of virtual object and image is negative. Because the principal focus of a concave lens is virtual

NB;

- ❖ For a convex lens, f is positive and for a concave lens, f is negative.
- ❖ If image is above the principal axis, object height will be **positive**. It means that the image formed is **upright and virtual**
- ❖ If image is below the principal axis, object height will be **negative**. It means that the image formed is **inverted and real**
- ❖ m is **positive (+)** for an image that is upright with respect to the object.
- ❖ m is **negative (-)** for an image that is inverted with respect to the object.

Works examples

1. An object 1.0 cm high is 8.0 cm to the left of a convex lens that has a 6.0 cm focal length. Find the image location and image height.

Solution; given; $h_o = 1\text{cm}$, $u = 8\text{cm}$, $f = 6\text{cm}$, $v = ?$, $h_i = ?$

$$\text{From; } \frac{1}{f} = \frac{1}{u} + \frac{1}{v} \rightarrow \frac{1}{v} = \frac{1}{f} - \frac{1}{u} \rightarrow v = \frac{fu}{u-f} = \frac{6 \times 8}{8-6} = 24\text{cm}$$

$$\text{But; } m = \frac{v}{u} = \frac{24}{8} = 3$$

$$\text{Thus; } m = \frac{h_i}{h_o} \rightarrow h_i = m \times h_o = 3 \times 1 = 3\text{cm}$$

Therefore; the image formed is 24 cm to the right of the lens, and 3cm high (3 times the object's size)

Class Activity – 3:5

1. An object is placed 12 cm from a convex lens of focal length 18 cm. using the lens formula. Find the position of the image.
(ANS: $v = -36$ cm, the image is virtual)
2. An object is placed 10 cm from a concave lens of focal length 15 cm. using the lens formula, determine the nature and the position of the image.
(ANS: Since, $v = -6$, The image is virtual and erect)
3. An object is placed to the left of a 25 cm focal length convex lens so that its image is the same size as the object. Determine the object and image locations.
ANS; The object and image are each 50 cm from the lens on opposite sides. Since the object is placed to the left, the image is on the right.
4. An object 2 cm high is placed 24 cm from a converging lens. An erect image which is 6 cm high is formed. Find focal length of the lens.
(ANS: $f = 36$ cm)
5. An object of height 3cm is placed at 10cm from a concave lens of focal length 15cm. Find the size of the image. [ANS; image size = 1.8 cm]
6. The focal length of a converging lens is 10 cm. How far should the lens be placed from an illuminated object to obtain an image which is five times the size of the object on a screen?
(ANS: $u = 12$ cm)

- An object is placed 15 cm in front of a concave lens with a focal length of 8 cm. Find the image distance. **[ANS; 5.2 cm in front of the lens]**
- An object is placed 3 cm in front of a concave lens with a focal length of 5 cm. Find the image distance. **[ANS; $v = 1.875$ cm in front of the lens]**
- A 2.00 cm tall object is placed 20.0 cm in front of a concave lens with a focal length of 5.00 cm. Find the image distance and the height of the image. **[ANS; 4cm, 0.4cm tall]**

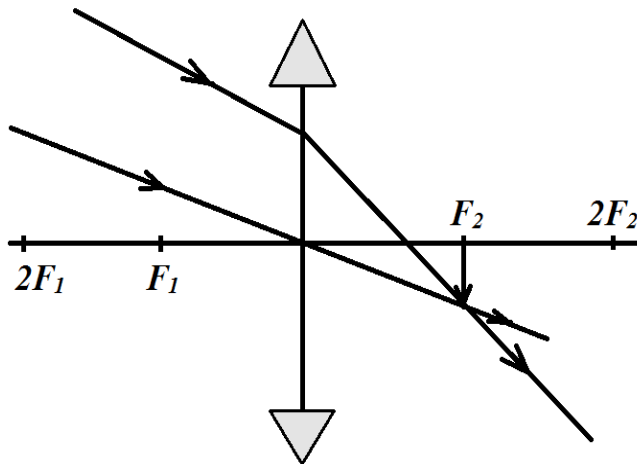
Images formed by thin Lenses

Characteristics of the Image Formed by a Convex Lens

- As with a curved mirror, the position and size of an image can be found by drawing a ray diagram.
- Any two of the following three rays are sufficient to fix the position and size of the image.
- The characteristics, position and size of the image formed by a convex lens depends on the object distance (u) relative to the focal length (f)

Images Formed By Convex Lens

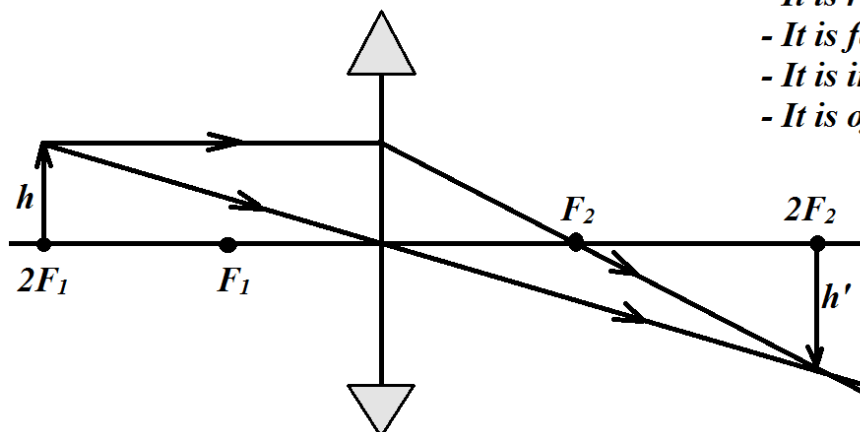
Object At Infinity



Properties of image formed

- It is real
- It is formed at F
- It is inverted (Up side down)
- It is diminished

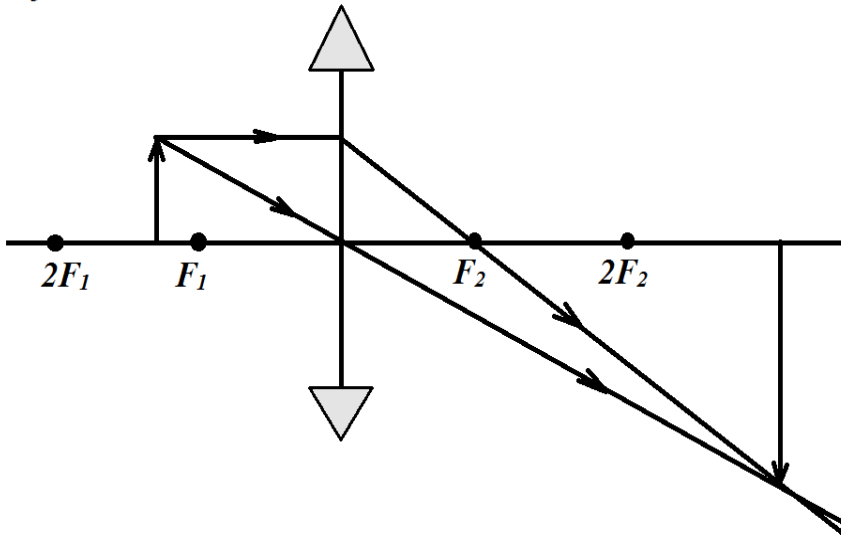
Object at C (2F)



Properties of image formed

- It is real
- It is formed at $2F$
- It is inverted
- It is of the same size as the object

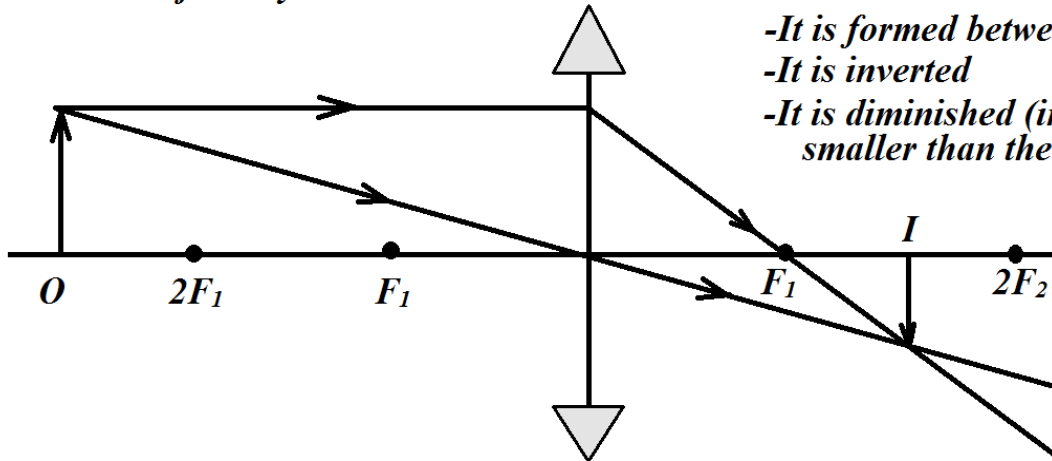
Object between C and F



Properties of Image formed

- It is real
- It is formed beyond $2F$
- It is inverted
- It is magnified (image size is larger than the object)

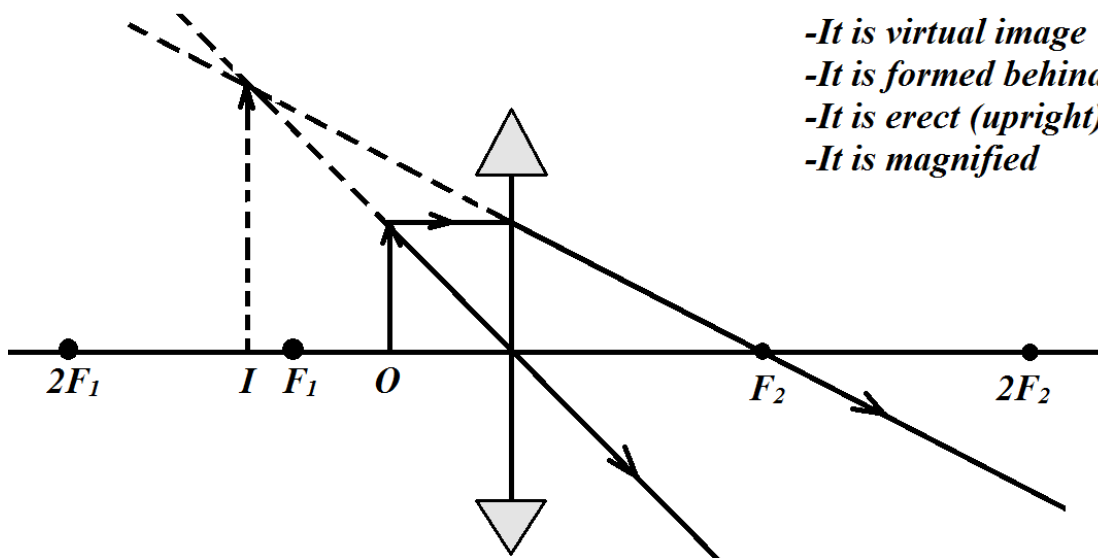
Object beyond $2F$



Properties of image formed

- It is real
- It is formed between F and $2F$
- It is inverted
- It is diminished (image size is smaller than the object)

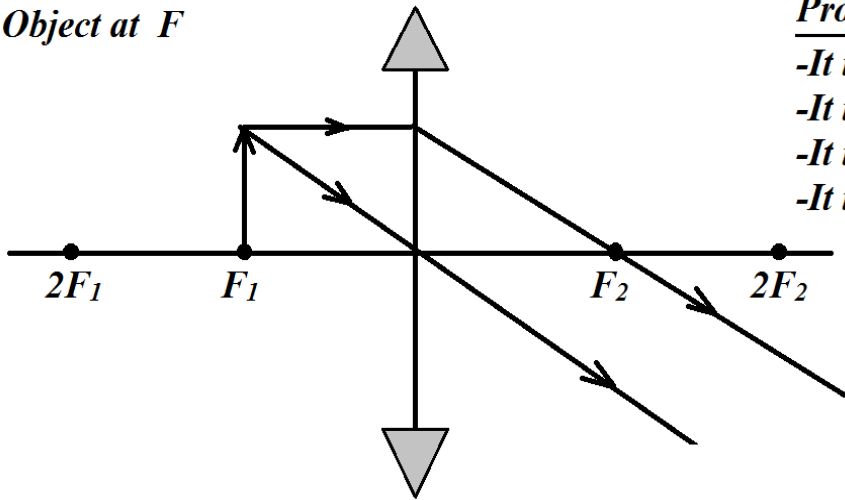
Object between F and lens



Properties of image formed

- It is virtual image
- It is formed behind the object
- It is erect (upright)
- It is magnified

Object at F

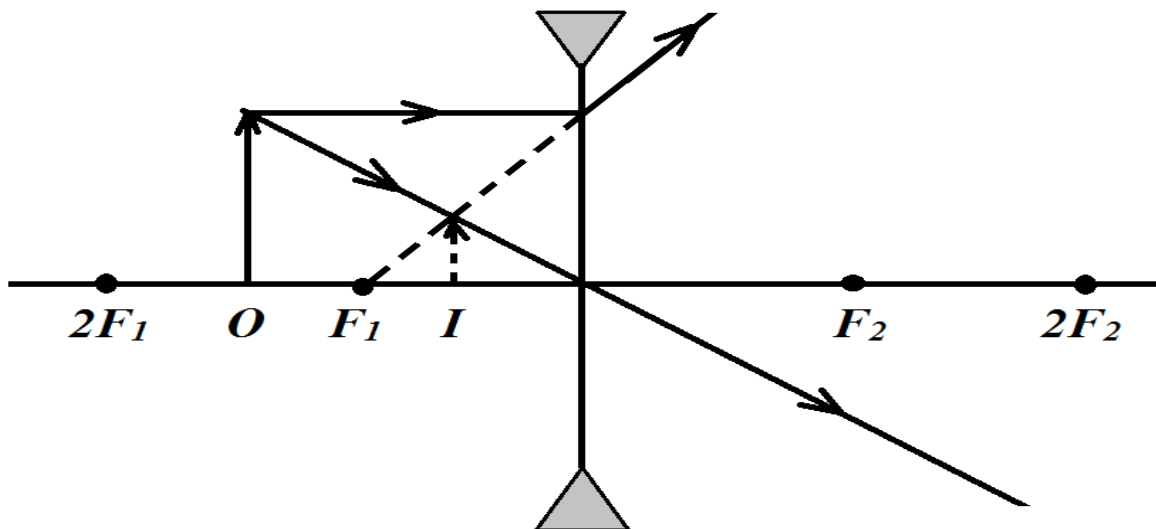


Properties of image formed

- It is real*
- It is formed at Infinity*
- It is larger than the object*
- It is inverted*

Image formed in Concave Lens

- Concave lenses create only virtual images. After the rays are refracted, they never converge and so there will be no real images.
- All concave lens images will be upright, virtual, and diminished, and can be found between the F and the lens for all object positions.



Properties of image formed

- Virtual
- Formed between the object and the lens
- Erect
- Diminished
- As U increase to infinity also V increase to F

Class Activity – 3:6

1. Describe how a convex lens could be used to make a magnifying lens.

ANS; A convex lens increases the size of the image if the object is inside $2F$. However, the image will be inverted if the object is between F and $2F$. Therefore, a convex lens will make a good magnifying lens if the objects being observed are inside the focal point.

2. What physical characteristic of a lens distinguishes a converging lens from a diverging lens?

ANS; A converging lens is a convex lens, while a diverging lens is a concave lens. Physically, this means that a converging lens has a wider center than edges, but a diverging lens has wider edges than the center.

3. What four points are marked on the principal axis of a lens?
4. Under what circumstances will a convex lens form a virtual image?
5. What are the characteristics of images from concave lenses?
6. **If you have a convex lens whose focal length is 10.0 cm, where would you place an object in order to produce an image that is virtue?**

ANS; To have a virtual image, the object must be between the lens and the focal point. Therefore, the object must be less than 10 cm from the lens.

Class Activity – 3

1. **The object is placed 20 cm from a converging lens for focal length 15cm. find the position, the magnification and the nature of the image**
ANS $V = 60 \text{ cm from the lens}$, $M = 3$ and the nature is a real image
2. Find the nature and position of the image of an object, placed 10cm from a diverging lens of focal length 15cm (concave). (**ANS: $V = -6 \text{ cm from the lens}$, $Real\ m = 0.6$**)
3. The apparent depth of a certain point at the bottom of water pond is 25cm. find the real depth of this point given that the refractive index is $\frac{4}{3}$ (**ANS: $H = 33. \text{ cm}$**)
4. An object stands vertically on the principle axis of a converging lens of focal length 10mm and at a distance of 17mm from the lens. Find the position, size and nature of the image. **ANS: ($\therefore V = 24.14 \text{ cm}$, $M = 2.9\text{mm}$, $Real$)**
5. Calculate the critical angle for air and water medium if the refractive index of water is $\frac{4}{3}$. (**ANS: The critical angle is $48^\circ 38'$**)
6. **Given that the refractive index of glass is 1.5, what is the value of the critical angle?** (**ANS: The critical angle is $41^\circ 49'$**)
7. Given that the refractive index of ethyl alcohol is 1.36. Find the apparent depth in the beaker if the real depth of the optical pin is 52cm. (**ANS: $h = 38.23 \text{ cm}$**)
8. A fish is located 10m deep in the liquid when viewed from the top. The depth of the fish is 8m. Find the refractive index of the liquid. (**ANS: 1.25**)
9. If the light has a velocity of 3×10^8 and has a velocity $1.97 \times 10^8 \text{ m/s}$ in the glass.
(a) What is the refractive index of the glass? (**ANS; 1.52**)
(b) Calculate the refractive index for light traveling from glass to air. (**ANS: 0.6458**)
10. The refractive for light traveling from air to water is 1.3. Find the refractive index of light travelling from water to air. **ANS: 0.7693**
11. A small pin 3cm high is placed 30 cm away from a concave mirror of focal length 12cm. By using the mirror formula, find the position, the height and the nature of the image formed. (**ANS : $V = 20\text{cm}$, $m = 2\text{cm}$ high and is It is real,**)
12. An object 2cm is high erected 8cm in front of a concave mirror of radius of curvature 10cm. by graphical method, find the position, size and nature of the image. (**ANS: $V = 13.3 \text{ cm}$, $H_i = 3.3 \text{ cm}$. The image is real).**)
13. A small spring is 4 cm long is kept at 10cm in front of the converging mirror of radius of curvature 12 cm. By scale drawing, determine the position, size and state the nature of the image formed. (**ANS; $V = 15 \text{ cm}$, $H_i = 3 \text{ cm}$ and it is real.**)
14. A convex mirror produces an image that is 22 cm behind the mirror when an object is placed 34 cm in front of the mirror .What is the focal length of the mirror

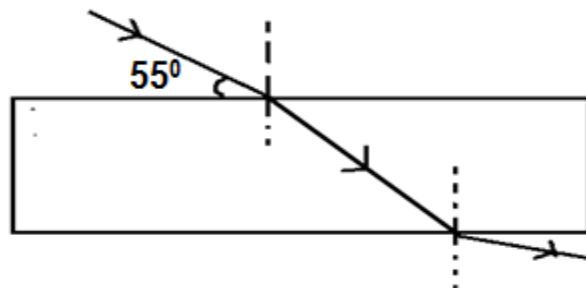
15. A concave mirror has a focal length of 40 cm .How far from the mirror must an object be placed to produce an image that is
- (a) Twice the size of the object (b) Half the size of an object
(c) 40 times the size of the object
16. Show that to obtain an image with a magnification of M using a concave mirror with a focal length f, the object distance ,u, is given by $u = \frac{M+1}{M} f$
17. .What happens to the image formed by
- (a) A Convex mirror
(b) A Concave mirror as the object distance is decreased?
18. Parallel light rays from a distant star are incident on a concave mirror with a radius of curvature of 120 cm .How far from the mirror will the star's image be formed ?
19. An object is placed 18 cm from a concave mirror. An image that is twice the size of the object is formed .Determine the image distance and the focal length of the mirror
20. A Converging lens forms an upright image that is four times the size of the object .Given that the focal length of the lens is 20 cm , determine the object distance
21. **The lens of a slide projector focuses an image of height 1.5 m on a screen placed 9.0 m from the projector. If the height of the picture on the slide is 6.5 cm , determine**
- (a) **The distance between the slide (picture) and the lens**
(b) **The focal length of the lens**
22. An object 2 cm high is placed 9 cm from a convex lens of focal length 6 cm .Determine the position and nature of the image formed
23. **A rectangular glass block 5 cm thick is placed on top of the page of a book .If the refractive index of the glass block is 1.53, calculate apparent depth of the letters on the book**
24. A ray of light is incident at an angle of 60° on a block of glass of refractive index 1.5 .Determine the angle of refraction of the ray
25. **A small coin was placed at the bottom of a tall glass containing some water and viewed from above .The real and apparent depths of the coin were then measured .By varying the depth of the water in the jar ,the following readings were obtained**

Real depth(cm)	8.1	12.0	16.0	20.0
Apparent depth (cm)	5.9	9.0	12.0	15.1

By plotting an appropriate graph from the results, determine the refractive index of the water

26. The refractive index of water is 1.33 and that of glass is 1.5 .Calculate the critical angle for:
- (a) a glass – air interface (b) a water – air interface
27. **A pin at the bottom of a basin full of water appears to be 6 cm from the surface .Given that the refractive index of the water is $\frac{4}{3}$, what is the actual distance of the pin from the surface?**
28. Given that the refractive index of water is $\frac{4}{3}$, what is the angle of refraction of the ray of light?
29. Paraffin has greater refractive index than water
- (a) What information does the above statement give with regard to the relative velocities of light in paraffin and in water
- (b) Draw a diagram to demonstrate the path of a ray of light when passing from water into a layer of paraffin oil floating on top of it

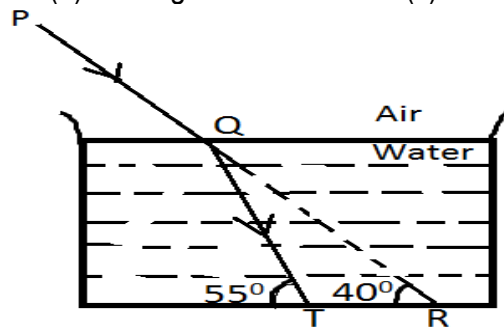
30. When an object is placed 25 cm from a convex lens , an inverted image which is twice as large as the object is formed .How far from the lens must the object be placed to obtain an image four times the size of the object ?
31. An object 5 cm high is placed 25 cm from a convex lens with a focal length of 20 cm .Using the lens formula, determine position, size and nature of the image formed
32. An object 20cm high is placed 40cm from a concave mirror of focal length 15 cm. determine the position, nature and size of the image formed by drawing a ray diagram
33. A ray of light strikes a rectangular glass block at an angle of 45° to the surface of the glass .Given that the refractive index of the glass with respect to air is 1.5. Determine the angle of refraction
34. A ray of light is shone through a rectangular glass prism at an angle of 55° to the air glass interface as shown in the figure below



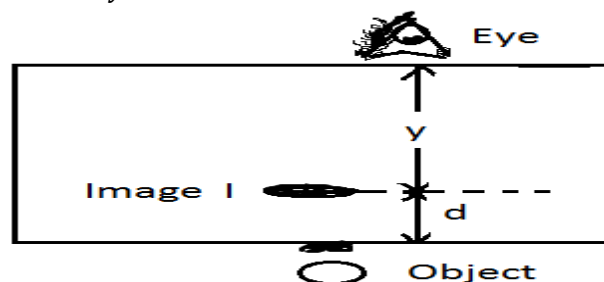
The glass block is 12 cm long and 10 cm wide .Calculate the distance the ray of light travels through the glass before emerging into the air ($n_{\text{glass}} = 1.5$)

35. Taking the refractive index of glass is $3/2$, What is the critical angle?
36. Define angle of incidence and angle of refraction .State the laws of refraction of light
37. What is meant by the refractive index of a substance? If the velocity of light in a vacuum is 3.0×10^8 m/s ,find the velocity of light in crown glass of refractive index 1.52
38. **Distinguish between**
- Converging and diverging lenses
 - Real and Virtual images. Draw two diagrams ,one showing a converging lens producing a real image and the other showing the same lens producing a virtual image
39. A 4.0 cm bulb tall light bulb is placed a distance of 8.3 cm from a concave mirror having a focal lens of 15.2 cm .Determine the image distance and the image size .What additional information do the answers give?
40. **An object is at a distance of 30 cm from a convex lens of focal length 10 cm .Find by graphical method the position and nature of the image formed**
41. An object is placed 20 cm from (a) convex lens (b) Concave lens of focal length 16 cm .Find the position, nature and linear magnification of the image produced
42. **List out the factors on which the refractive index of a medium depends**
43. What is meant by the refraction of light? Define incident, refracted and emergent rays of light
44. A glass prism has three sides of angle 60° .A ray of light falls on one of the faces and the angle of incidence is 48° .The ray is refracted and now travels parallel to the second face. When it reaches the third face it is again refracted and emerges from the prism .Find
- The refractive index of the glass prism
 - The angle between the ray entering the prism and the ray leaving the prism
45. A glass prism has two parallel sides which are 6 cm apart .A ray strikes one of the two parallel sides at an angle of incidence of 50° .Find by drawing the perpendicular distance between the ray entering the prism and the ray leaving the prism

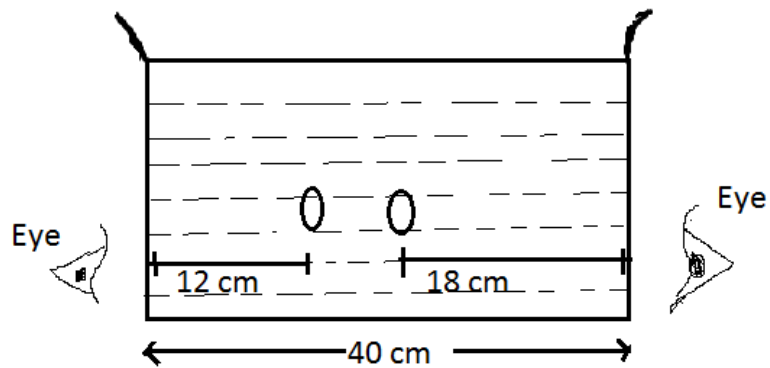
46. A Swimming pool is 2 m deep .Given that $\mu_{\text{water}} = 1.33$. How deep does it appear to be when
 (a) Completely filled with water (b) Filled halfway with water
47. Give scientific reasons for the following observations:
 (a) A pencil dipped obliquely into water appears to be bent at the point where it enters the water
 (b) A light ray passing from air to glass bends closer to the normal
 (c) The speed of light in diamond is less than the speed of light in ice
48. Explain the meaning of the following terms
 (a) Refraction of light (b) Angle of incidence
 (c) Angle of refraction (d) Refractive index
49. A ray of light is passing from air into water along PQ .The ray strikes the bottom surface at T instead of R as shown in the figure below calculate
 (a) the angle of incidence (b) the angle of refraction (c) the refractive index



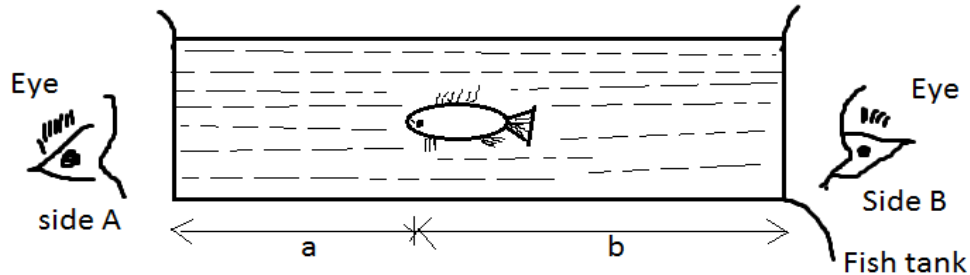
50. A ray of light passing from air into oil at an angle of incidence 30° .Calculate the angle of refraction in oil if the velocity of light in air is 3.0×10^8 m/s and that in a transparent oil is 2.2×10^8 m/s
51. The light ray passing from glass to air is monochromatic and has a frequency of 4×10^{14} Hz and a wavelength of 5×10^{-7} m in glass
 (a) What is meant by monochromatic?
 (b) Calculate the velocity of light in glass
 (c) Calculate the velocity of light in air (refractive index of glass is 1.50)
52. In an attempt to determine the refractive index of a glass block , a student finds the displacement produced due to refraction by glass as d and apparent thickness of the block as y as shown in the figure below. Show that the refractive index of glass may be expressed as $n = (1 + \frac{d}{y})$



53. A ray of light passes from a liquid to air. Calculate the critical angle for the liquid – air interface ,if the velocity of light in the liquid is 2.4×10^8 m/s, while in air is 3.0×10^8 m/s
54. In a transparent liquid container, an air bubble appears to be 12 cm when viewed from one side and 18 cm when viewed from the other side (see the figure below).Where exactly is the air bubble, if the length of the tank is 40 cm?



55. In a fish aquarium (as shown in the figure below) the image of a fish seems to be 30 cm when seen from side A and 42 cm when seen from side B. Calculate the length of the fish tank , if the refractive index of water is 1.33.



56. Calculate the critical angle of a material of refractive index 2

57. Sunlight making an angle of 60° with the horizontal enters a pool which is 50 cm deep .Determine the distance travelled by the sunlight in the water ($n_w = 1.33$)

58. An observer looks into a water tank half filled with water .If the height of the tank is 180 cm. A solid that is 80 cm beneath the water surface is seen to be 60 cm below the water surface. Determine

(a) The Refractive Index of water (b) the vertical displacement of the solid

59. A small coin was placed at the bottom of a tall glass containing some water and viewed from above. The real and apparent depths of the coin were then measured. By varying the depth of the water in the jar, the following readings were obtained

Real depth (cm)	8.1	12.0	16.0	20.0
Apparent depth (cm)	8.9	9.1	12.0	15.1

By plotting an appropriate graph from the results, determine the refractive index of the water.

60. The critical angle for a beam of light travelling between water and air is 49° .

(i) A beam strikes a water or air boundary and undergoes total internal reflection. Will the beam stay in the air or in the water?

(ii) Explain what happens when a beam of light from the air strikes the surface of a calm lake at an angle of 50° from the normal.

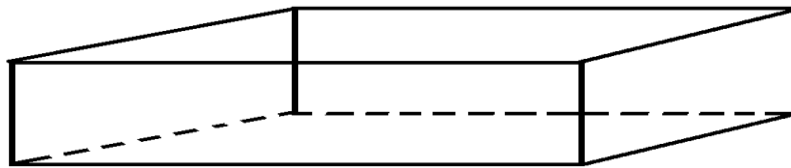
Chapter – 04:

Dispersion and Colors

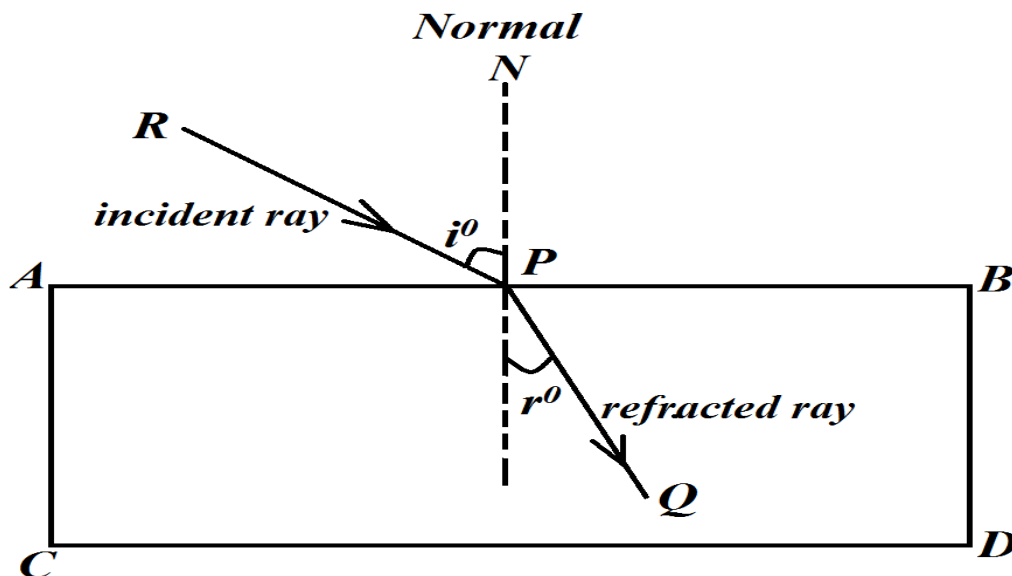
Refraction through Prism

- Prism is a solid piece of glass or transparent material that has at least two planes inclined toward each other through which light is refracted.

Refraction of Light by rectangular glass Prism (Glass block)



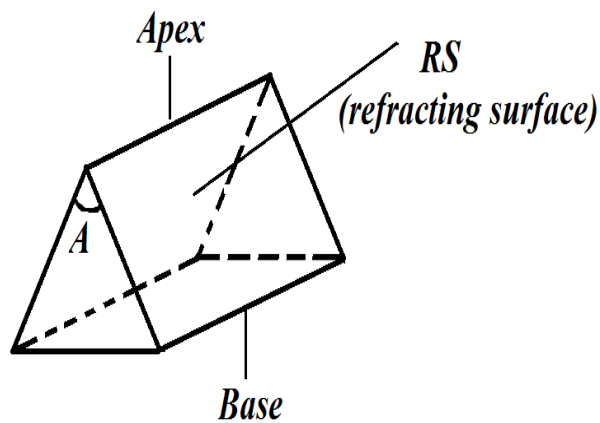
- Let us consider a rectangular glass slab **ABCD**. When a light ray **RQ** after travelling through air, is incident on the glass slab at a point **P** the light gets refracted. Since the glass slab is a denser medium compared to air, thus the ray gets bend towards the normal.



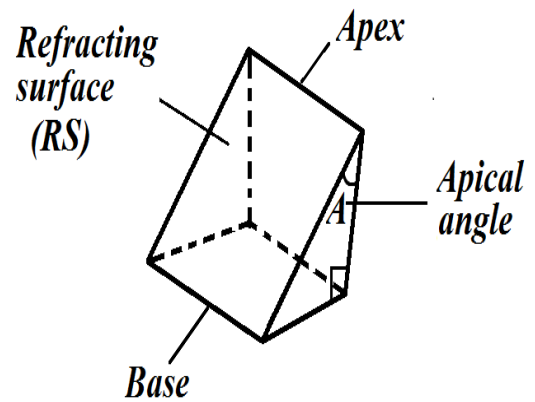
- The difference between glass slab and glass prism is that the opposite faces of a rectangular glass slab is parallel while the opposite faces of prism is not parallel to each other.

Triangular prism

- A transparent substance that can reflect light and has at least two lateral surfaces that are obliquely inclined to one another. It contains five surfaces, including three rectangle lateral surfaces and two triangle bases. The angle of the prism refers to the angle created by two lateral surfaces. **For a standard prism, the prism's angle is always 60°.** See figs below

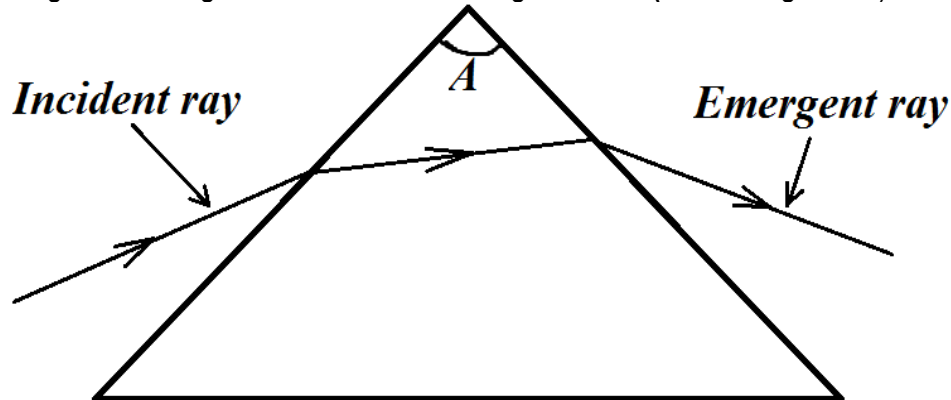


(i) Equilateral prism ($60^\circ - 60^\circ - 60^\circ$)



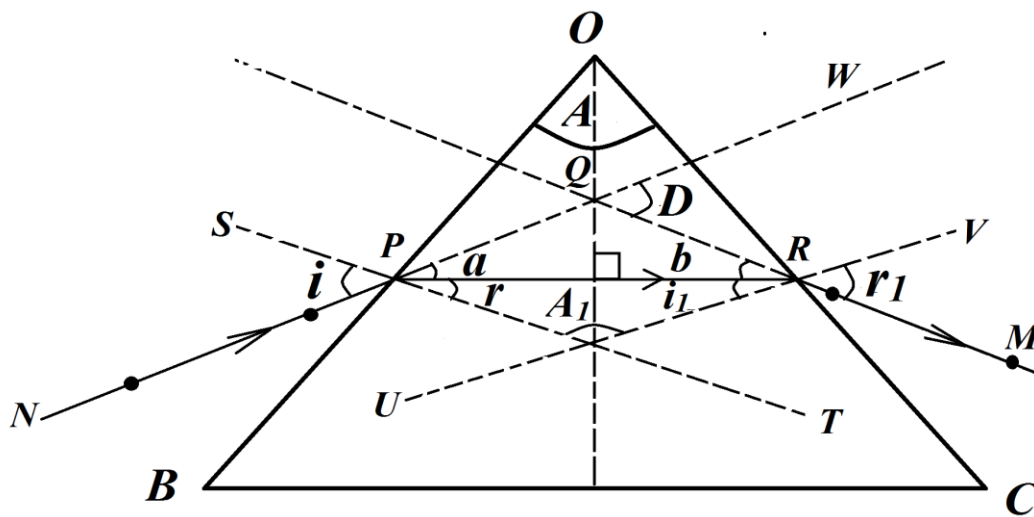
(ii) Right angle prism ($45^\circ - 90^\circ - 45^\circ$)

- Apex = The point at which two refracting surfaces of the prism meet is called the **refracting edge**
- Rays leaving the prism is called **emergent ray**.
- Rays entering the prism is called **incident ray**
- Apical angle is the angle between the refracting surfaces (See the fig below)



Refraction of light through triangular glass prism

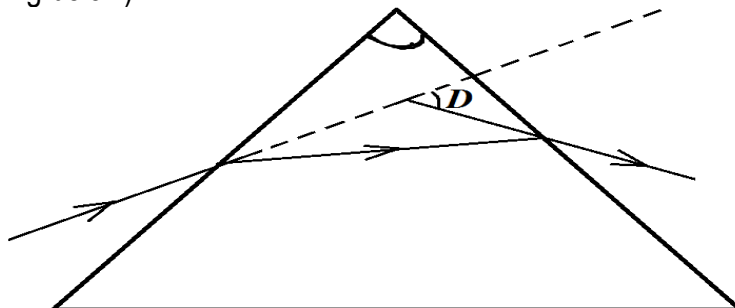
- When a light ray is allowed to pass through a glass prism, it gets refracted twice. First, at the entrance of the prism and second when the light exits from the prism. Here the two refracting surface i.e. **BO** and **CO** are not parallel to each other. **Thus the incident ray is not parallel to the emergent ray**. As it can be seen from the diagram below a glass prism **BOC** is made to stand on its base **BC**. Here the line **ST** and **UV** are normal to the surfaces **BO** and **CO** respectively. **NW** is the incident light that is made to fall on the surface of prism that is, **BO**. As the light ray is travelling from rarer medium to the denser medium the light ray bends towards the normal **ST** and follows the path **PR** inside the prism.
- **PR** is the refracted ray of light which bends towards the base of prism **BC**. Now the refracted ray **PR** after travelling through the prism, exits the prism at point **R**. At this point the ray is again refracted. Now since the refracted ray **PR** is travelling from denser to rarer medium (glass to air) it bends away from the normal **UV** and traverses the path **RM**.



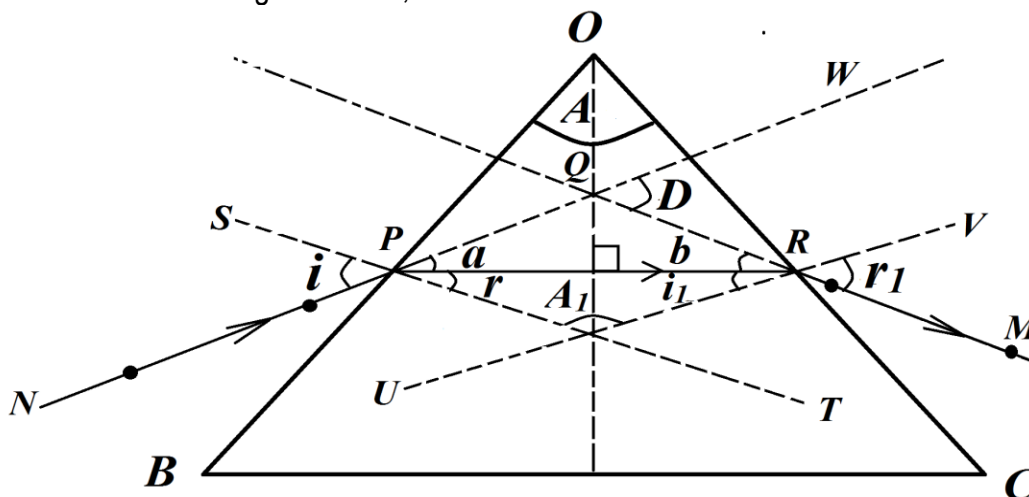
- This ray is known as **Emergent Ray**. In this case also we can see that the emergent ray bends towards the base of the prism.
- Thus we can conclude that when a ray of light travels through a prism, it always tends to bend towards the thicker part of the prism.
- As we already know that both the surfaces of the prism are not parallel to each other, therefore the emergent ray **RM** and incident ray **NP** are also not parallel to each other. Now let us extend the incident ray **NP** to a point **W**. This extended line is the original direction of the incident ray.

Angle of Deviation, D

- Is the angle formed by the intersection of the incident ray direction and the emergent ray direction. (see the fig below)



- Now consider the diagram below;



From Snell's Law

$$a\eta_g = \frac{\sin i}{\sin r} \dots\dots\dots 1 \quad \text{and} \quad g\eta_a = \frac{\sin i_1}{\sin r_1} \dots\dots\dots 2$$

From the figure above; $D = a + b$ (sum of two opposite interior angles)

but $\dots\dots i = a + r \rightarrow a = i - r$ and $r_1 = b + i_1 \rightarrow b = r_1 - i_1$

then, $D = a + b = (i - r) + (r_1 - i_1) \dots\dots\dots (i)$

Since: A and A_1 is supplementary to each other $\rightarrow A + A_1 = 180^\circ \dots\dots (ii)$

But: $r + i_1 + A_1 = 180^\circ \dots\dots\dots (iii)$ (sum of interior angles of a Δ)

- Compare equation (ii) and (iii)

$r + i_1 + A_1 = A + A_1 \rightarrow A = r + i_1 \rightarrow r = A - i_1 \dots\dots\dots (iv)$

- Then substitute, equation (iv) into (i)

$D = (i - r) + (r_1 - i_1) \rightarrow D = i - (A - i_1) + (r_1 - i_1)$

$D = i - A + i_1 + r_1 - i_1 = i + r_1 - A$

\therefore The angle of Deviation is given by: $D = i + r_1 - A$

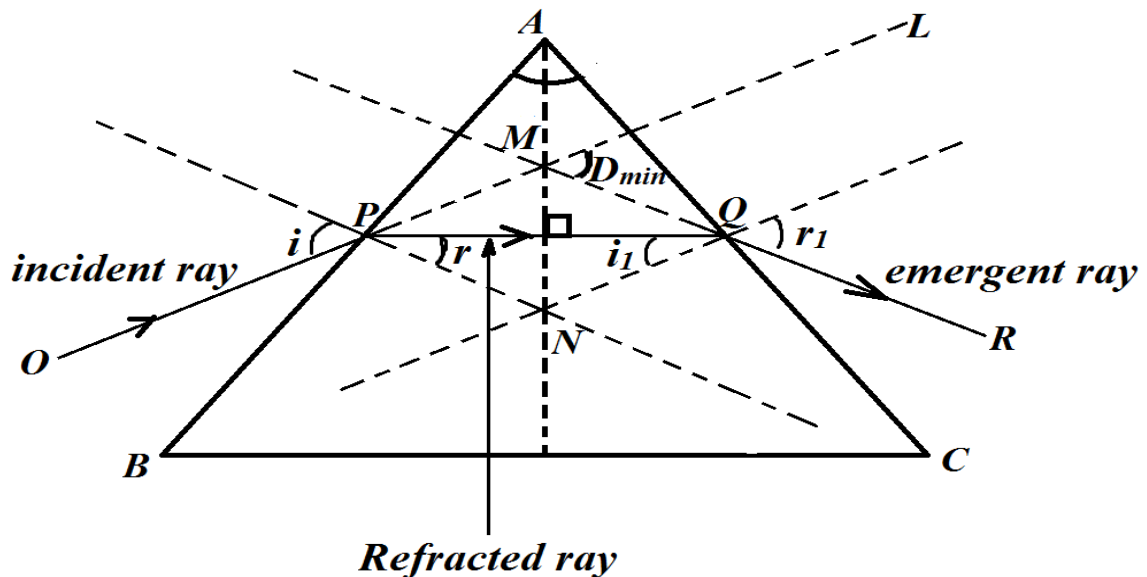
The angle of deviation depends on

- The apical angle of the prism, A
- The Angle of incidence, i
- The refractive index of the glass prism.

NB: Angle of deviation decreases with an increase in the angle of incidence and vice versa

Minimum Angle of Deviation, D_{min}

- Is the deviation angle occurs when the emergent ray is refracted at an angle equal to the angle of incidence.
- A prism is said to be in minimum deviation position when the angle of incidence at the **first surface** is equal to the angle of emergence at the **second surface**. In such a condition, the rays travel parallel to the base inside the prism as seen from the figure below



Since: $r = i_1$, $i = r_1$ and $D = D_{min}$

Substitute: $r = i_1$, $i = r_1$ and $D = D_{min}$ in the eqn $D = i + r_1 - A$

$$D = i + i - A = 2i - A, \quad \text{but } A = r + i_1 = 2r \rightarrow D = 2i - 2r$$

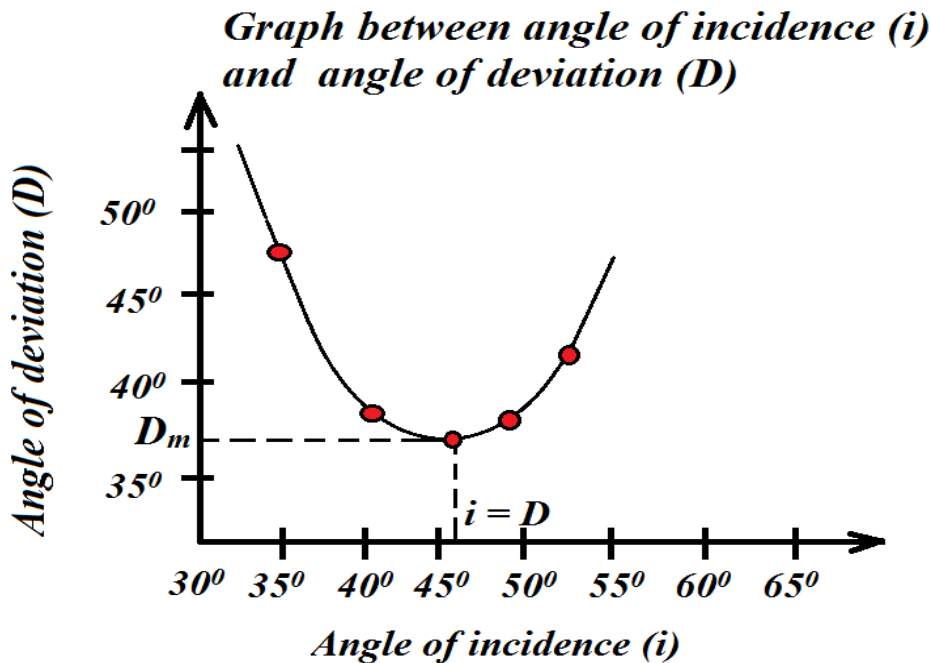
In making i the subject, then $i = \left(\frac{D_{min}}{2}\right) + r$ but $r = \frac{A}{2}$

Thus;
$$i = \left(\frac{D_{min}}{2}\right) + \frac{A}{2} = \frac{D_{min} + A}{2}$$

From: Snell's law

$$\text{ang} = \frac{\sin i}{\sin r} = \frac{\sin\left(\frac{D_{min} + A}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

- Consider the graph below shows the relationship between i and D , for which the angle of deviation is obtained



Worked examples;

- A monochromatic light is incident on an **equilateral glass prism** at an angle of 30° and emerges at an angle of 75° . What is the angle of deviation produced by the prism?

ANS; given; $i = 30^\circ$, $r_1 = 75^\circ$, $A = 60^\circ$ (for equilateral prism)

The angle of Deviation is given by: $D = i + r_1 - A = 30 + 75 - 60 = 45^\circ$

- The refractive index of the material of a prism with 60° angle for yellow light is $\sqrt{2}$. At the position of minimum deviation, calculate the angle of;
 - Minimum deviation
 - Incidence; and
 - Refraction

ANS;

$$a\eta_g = \sqrt{2}, A = 60^\circ$$

$$(a) \text{ From; } a\eta_g = \frac{\sin\left(\frac{D_{min}+A}{2}\right)}{\sin\left(\frac{A}{2}\right)} \rightarrow \sqrt{2} = \frac{\sin\left(\frac{D_{min}+60}{2}\right)}{\sin\left(\frac{60}{2}\right)}$$

$$\sqrt{2} = \frac{\sin\left(\frac{D_{min}+60}{2}\right)}{\sin\left(\frac{60}{2}\right)} \rightarrow \sqrt{2} \times 0.5 = \sin\left(\frac{D_{min}+60}{2}\right)$$

$$\frac{D_{min}+60}{2} = \sin^{-1}(\sqrt{2} \times 0.5) = 45^\circ \rightarrow D_{min} + 60 = 45 \times 2 = 90^\circ$$

$$\therefore D_{min} = 90 - 60 = 30^\circ$$

$$(b) \text{ From; } i = \left(\frac{D_{min}}{2}\right) + \frac{A}{2} = \frac{D_{min}+A}{2} = \frac{30+60}{2} = 45^\circ$$

$$(c) r = \frac{A}{2} = \frac{60}{2} = 30^\circ \text{ (alternatively; you may use; } a\eta_g = \frac{\sin i}{\sin r})$$

3. Light travelling through transparent oil enters into glass of refractive index 1.5. If the refractive index of glass with respect to the oil is 1.25, what is the refractive index of the oil?

ANS; $\eta_g = 1.5$, $\eta_{gO} = 1.25$, $\eta_o = ?$

From; Refractive index of glass with respect to oil, $\eta_{gO} = \frac{\eta_g}{\eta_o} \rightarrow \eta_g = \frac{\eta_g}{\eta_{gO}}$

$$\therefore \eta_o = \frac{\eta_g}{\eta_{gO}} = \frac{1.5}{1.25} = 1.2$$

Class Activity – 4:1

- How does the angle of minimum deviation produced by a prism change with increase in
 - the wavelength of incident light and
 - the refracting angle of the prism?

Answer

- The angle of minimum deviation produced by a prism decreases with increase in the wavelength of incident light.
 - The angle of minimum deviation produced by a prism increases with increase in the refracting angle of the prism.
- Write a relation for the angle of deviation (δ) for a ray of light passing through an equilateral prism in terms of angle of incidence (i), angle of emergence (r_1) and angle of prism (A).
 - A ray of light incident at an angle of incidence i passes through an equilateral glass prism such that the refracted ray inside the prism is parallel to its base and emerges at an angle of emergence r_1 .
 - How is the angle of emergence ' r_1 ' related to the angle of incidence ' i '
 - What can you say about the angle of deviation in such a situation?

Answer

- Then angle of incidence i is equal to the angle of emergence r_1 .
Hence, $i = r_1$
 - In this situation the **angle of deviation is minimum**
- Draw a ray diagram to show the refraction of a monochromatic ray through a prism when it suffers minimum deviation. How is the angle of emergence related to the angle of incidence in this position

5. What should be the angle of incidence for a ray of light which suffers minimum deviation of 36° through an equilateral prism? Given that $A = 60^\circ$ [ANS; $i = 48^\circ$ S]
6. How does the deviation produced by a prism depend on
 - (i) the refractive index of its material, and
 - (ii) the wavelength of incident light

Answer

- (i) For a given angle of incidence, the prism with a higher refractive index produces greater deviation than the prism which has a lower refractive index. For example — A flint glass prism produces more deviation than a crown glass prism for same refracting angle since $\mu_{\text{flint}} > \mu_{\text{crown}}$
- (ii) The refractive index of a given transparent medium is different for the light of different colours. It decreases with the increase in the wavelength of light. Thus, the refractive index of the material of a prism for visible light is maximum for the **violet colour** and minimum for the **red colour**.

Consequently, a given prism deviates violet light the most and red light the least. (i.e.

$\delta_{\text{violet}} > \delta_{\text{red}}$ since $\mu_{\text{violet}} > \mu_{\text{red}}$)

7. A light ray falls at normal incidence on the first face of an **equilateral prism** such that, the angle of incidence is 45° , and emerges from the second face by making the same angle with the normal. What is the refractive index of the material of the prism? [ANS; 1.4]

ANS; Given; $A = 60^\circ$, $i = 45^\circ$, $D = 45 + 45 - 60 = 30^\circ$

$$\eta = \frac{\sin\left(\frac{D+A}{2}\right)}{\sin\left(\frac{A}{2}\right)} = \frac{\sin\left(\frac{30+60}{2}\right)}{\sin\left(\frac{60}{2}\right)} = \frac{\sin 45}{\sin 30} = 1.4$$

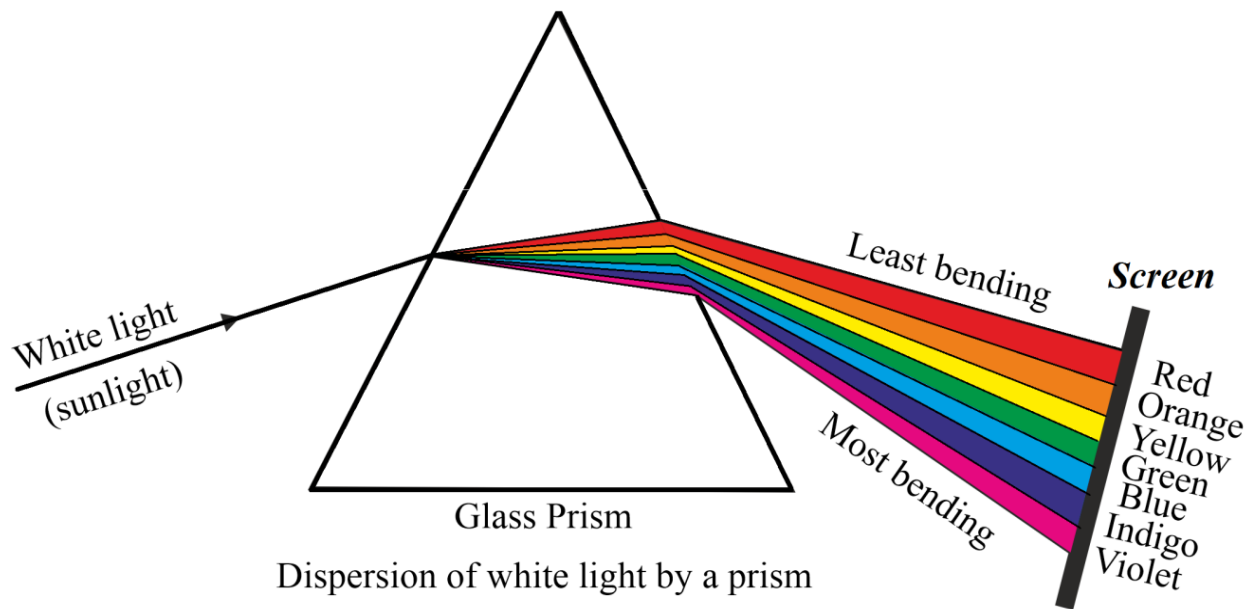
8. The angle of minimum deviation for a prism is 37° . If the angle of the prism is 60° , find the refractive index of the material of the prism. [ANS; 1.5]

Dispersion of white light

- When white light is passed through a glass prism it splits into its spectrum of colours (in order **violet, indigo, blue, green, yellow, orange and red**) and this process of white light splitting into its component colours is termed as **dispersion**. The simplest way to explain dispersion is through dispersion in the prism.

Activity

- We can perform an activity to show that white light consists of seven colours, as follows: Take a glass prism. Allow a narrow beam of sunlight through a small hole in the window of a dark room to fall on one face of the glass prism. Let the light coming out of the other face of the prism fall on a white sheet of paper or on a white wall (**which will act as screen**).
- A patch of seven colours will be formed on the sheet of white paper (or on white wall). This shows that sunlight consists of seven colours.
- Please note that bulb light is also white light. So, bulb light coming through a narrow slit can also be used to perform this activity.
- See the figure below:

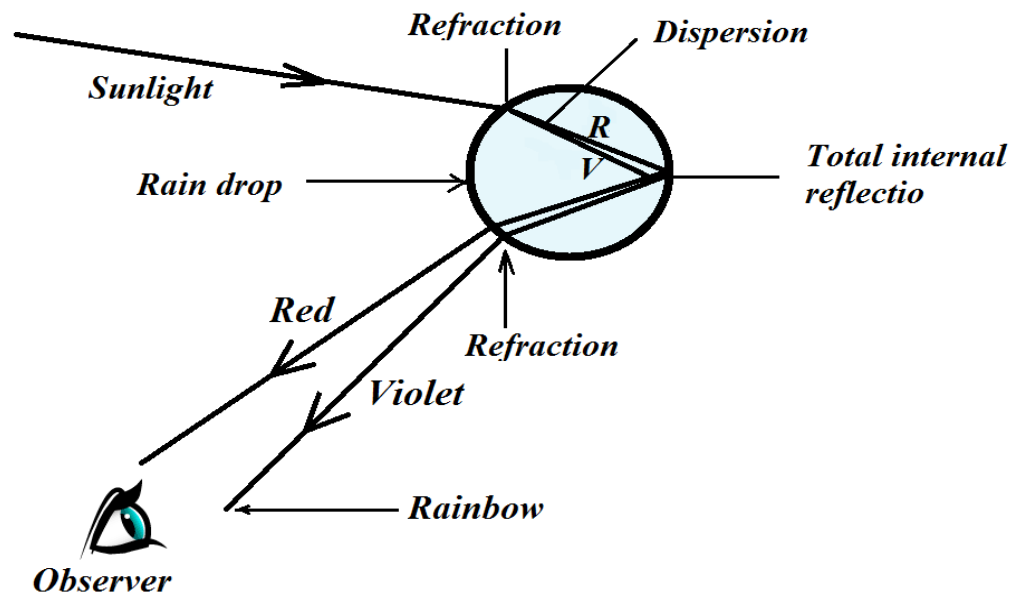


The Rainbow

- Any light that gives a spectrum similar to that of sunlight is usually referred to as white light. A rainbow is a natural spectrum appearing in the sky after a rain shower on a sunny day. It is caused by the dispersion of sunlight by tiny water droplets present in the atmosphere. A rainbow is always formed in a direction opposite to that of the Sun.
- The water droplets act like small prisms. They refract and disperse the incident sunlight, then reflect it internally, and finally refract it again when it comes out of the raindrop. Due to the dispersion of light and internal reflection, different colors reach the observer's eye in distinct bands. You can also see a rainbow on a sunny day when you look at the sky through a waterfall or through a water fountain, with the Sun behind you.

Formation of a Rainbow

- A rainbow is caused by dispersion of sunlight by tiny water droplets, present in the atmosphere. A rainbow is always formed in a direction opposite to that of the Sun. The water droplets act like small prisms. They refract and disperse the incident sunlight, then reflect it internally, and finally, refract it again when it comes out of the raindrop. Due to the dispersion of light and internal reflection, different colors reach the observer's eye. (See the fig below)



Types of white Colour

- Light is defined as part of the electromagnetic spectrum which is perceived by our naked eyes. The visible light which we see consists of different wavelengths of light in the range of 400-700 nm. The different wavelengths of light have different colours. Thus, we can see various colours around us. This type of light is known as Polychromatic light. Opposed to this is **Monochromatic light** which consists of only one wavelength and thus only one colour.

NB:

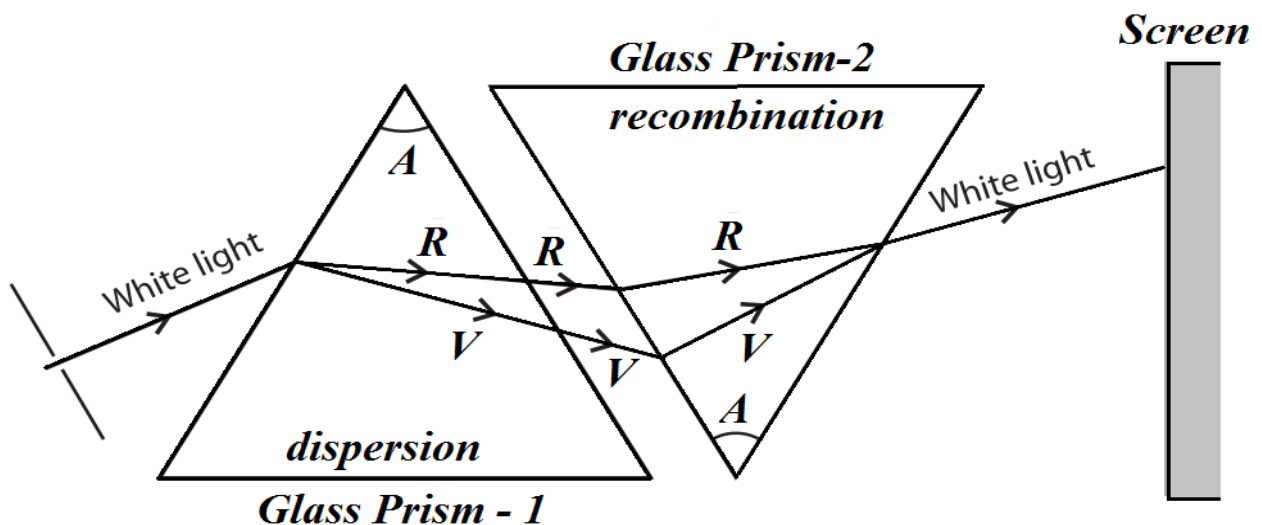
- ✓ Spectrum of colour (red, orange, yellow, green, blue, indigo and violet) is abbreviated as **ROYGBIV**. And each of these colours has a different wavelength.
- ✓ These coloured lights are refracted differently on passing through the prism
- ✓ The velocity of light in a medium (refractive index) depends on the wavelength of incident light. As a result different wavelengths are refracted by different amounts.
- ✓ White colour split due to difference in wave length. Shorter wavelengths have higher refractive indices and get bent more than longer wavelengths

Recombining Colours of white Light

- Spectrum comes from white light can recombine to form white colour.
- White light is a mixture of lights of seven colours, red, orange, yellow, green, blue, indigo and violet. The dispersion of white light occurs because colours of white light travel at different speeds through the glass prism. The amount of refraction depends on the speed of coloured light in glass.
- When white light consisting of seven colours falls on a glass prism, each colour in it is refracted by a different angle, with the result that seven colours are spread out to form a spectrum. **The red light bends the least, while violet light bends the most.**

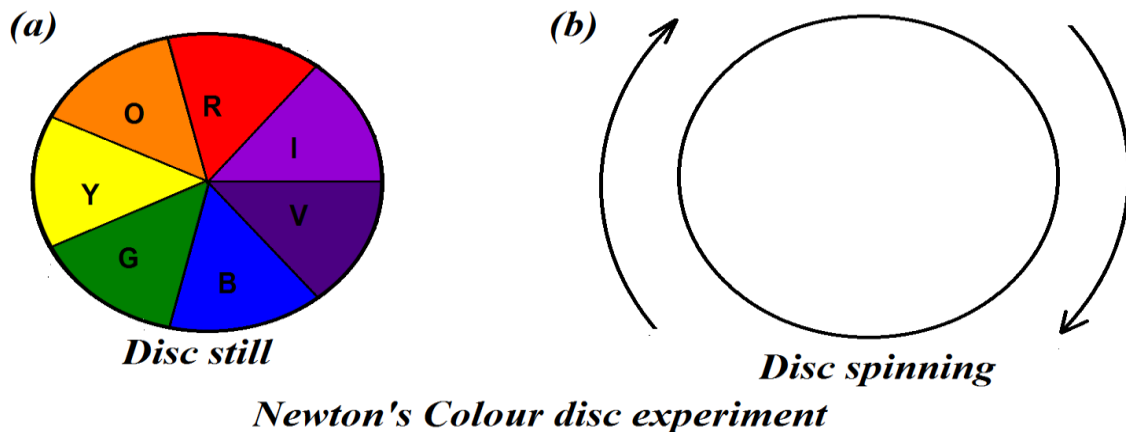
Newton's experiment with two identical prisms:

- When a beam of white light is passed through a glass prism, a band of seven colours is formed on a white screen. This band of seven colours is called spectrum of white light.
- Newton showed that the seven coloured light of the spectrum can be recombined to give back white light.
- First he tried to split the colours of the spectrum of white light using a prism.
- He then placed a second identical prism in an inverted position with respect to the first prism. This allowed all the colours of the spectrum to pass through the second prism. He found a beam of white light emerging from the other side of the second prism (see the figure below).



Alternatively; by using Newton's color disc

- Newton used a colour disc to recombine the colours.
- The disc consists of sectors painted with different colors of the spectrum
- When the disk is allowed to spin about its axis at very high speed, all the colors of the spectrum recombine to form white light. When it is slowed down, the individual colors of the spectrum are seen again



NB;

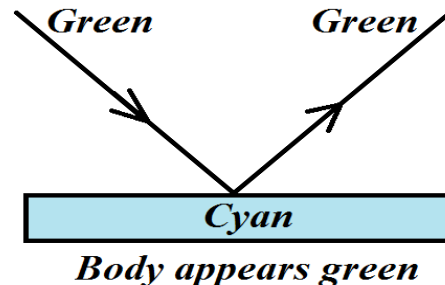
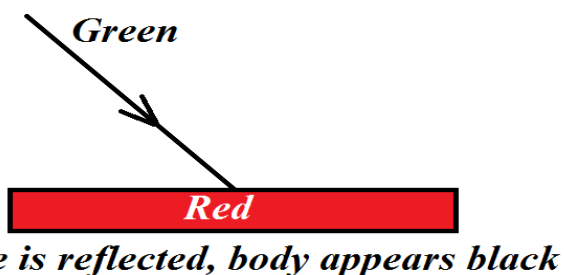
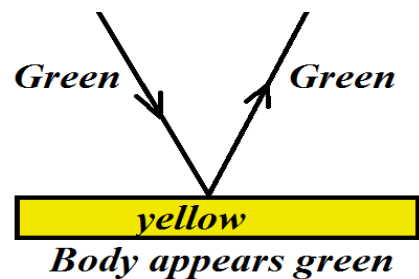
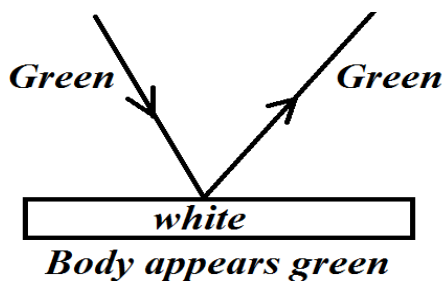
You may see that white light is slightly grey because paints are not pure colors

Colour

- Colour is the property of light that reaches our eyes.

Appearance of coloured objects under white light

- The object seems to have kind of colour due to the fact that it absorbs all colours and reflect the colour that the object has.
- Example; Green object absorbs the entire colour in white light except green.



Appearance of white objects under coloured light

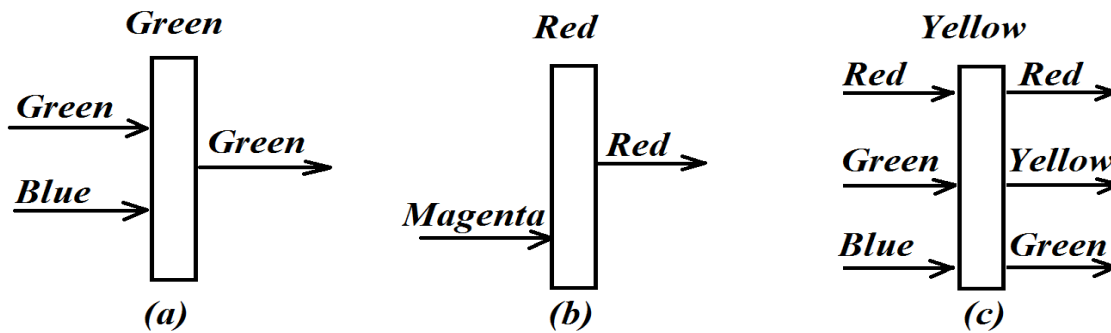
- When a coloured object is viewed under a coloured light, it takes the colour of that light.

Example:

- The object will appear blue in blue light and red in red light.
- A colour filter is working on this principle.

Colour filters

- A filter is a coloured sheet of plastic or glass material which allows light of its own type to pass through it and absorbs the rest of the coloured lights
- For example; a green filter transmits only green, a blue filter transmits only blue, a yellow filter transmits red, green and yellow lights as seen in the figure below



Appearance of a white object to coloured light

White object	coloured light	Colour of object
White object	Red filters	Red colour
White object	Yellow filters	Yellow colour
White object	Green filter	Green colour
White object	Blue filter	Blue colour

Colors of objects

- The color of an object depends on:
 - (i) The color of light falling on it
 - (ii) The color it transmits or reflects e.g
- An object appears blue because it reflects blue light into the eyes and absorbs the other colours of the spectrum
- Similarly, an object appears red because it reflects red light into the eyes and absorbs all other colors
- A white object reflects all the colors of the spectrum into the eyes and absorbs none.

Types of Colour

- Primary colour
- Secondary colour

Primary colours

- Primary colours are the colors that cannot be obtained by mixing two different colours of light.
- Example, red, blue and green

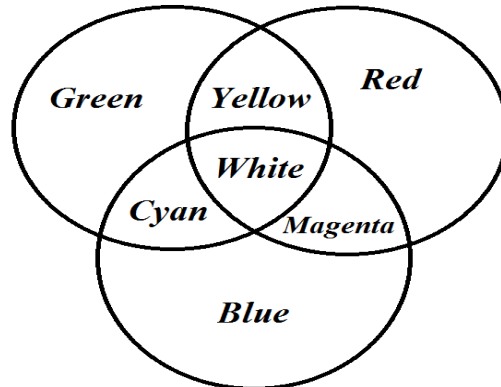
Secondary colours

- Secondary colours are the colours which obtained by adding two primary colors together
- Example of secondary colors are cyan (peacock blue), magenta and yellow
- The mixing of these two primary colors to produces a secondary color is given below;

- (a) Green + Blue = Cyan (GBC)
- (b) Green + Red = Yellow (GRY)
- (c) Red + Blue = Magenta (RBM)

Complementary Colour

- These are two different colors which when added produce white light.
- One of them is a secondary color and the other must be a primary color
- The pairs are given below
 - Red + Cyan = White light**
 - Green + Magenta = White light**
 - Blue + Yellow = White**
- The figure below shows the summary of coloured light



NB:

- From the figure above. When mixing the primary colors (red, blue and green), a white color is obtained. That is: **Red + Blue + Green = White**
- When mixing two of the secondary colors, white light is produced
That is: **Yellow + Magenta = Magenta + Cyan = Cyan + Yellow = White color**
- We are only concerned with colours of light and not with coloured substances (pigments)
- The complementary colour of white light is green, red and blue
- The complementary colour of yellow light is green and red.

Additive and subtractive mixing of colours

Additive mixing of Colours

- Is the mixing of coloured light
- The more colours you add, the closer the result draws to white. Therefore, mixing of coloured lights
- Additive mixing of colours deals with primary colour of light colour (**Red, Green and Blue**) not primary colour of pigments (paints and dyes)
- Adding different colours of light together increases the number of wavelengths present

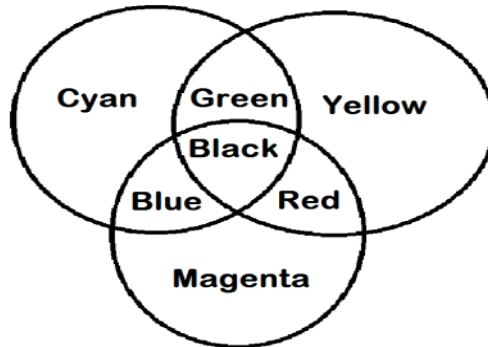
Subtractive mixing of pigments

- Is the mixing of colours of different paints (pigments)
- **Pigments:** These are substances which give color to paints by reflecting light of certain colors only and absorbing all other colors.
- Mixing different paints results in a darker colour because most of the light wavelengths are absorbed
- Mixing two primary pigments produces a secondary pigment .That is
 - a) cyan + magenta = blue (CMB)

b) magenta + yellow = red (MYR)

c) yellow + cyan = green (CYG)

- The pigments act as filters that subtract one or more colours from the visible spectrum
- Blue, Red and Green are, therefore, the **secondary pigments**.
- Subtractive complementary colours combine to produce **BLACK**.
That is **Blue + Green + Red = Black** (See the fig below)



N B:

- Each primary pigment absorbs one primary colour:
 - Yellow absorbs blue and reflects red and green
 - Magenta absorbs green and reflects blue and red
 - Cyan absorbs red and reflects green and blue.
- A secondary pigment absorbs two primary colours and reflects one:
 - Red absorbs green and blue and reflects red.
 - Green absorbs red and blue and reflects green.
 - Blue absorbs - red and green and reflects blue.
- The primary pigments are the secondary colours of light and the secondary pigments are the primary colours of light.
- If you mix a primary pigment with a secondary pigment you get total absorption (**black**).
ie **Blue + Yellow = Black**
- The primary pigments are the complementary colours of the three primary colours of light.

Worked example:

1. A plant with green leaves and red flowers is placed in:
 - (a) Green
 - (b) Red
 - (c) Blue light

ANS:

(a) In Green light	(b) In Red light	(c) In Blue light
❖ Green leaves will appear green	❖ In red light Green leaves will appear black	❖ In blue light Green leaves will appear black
❖ Red flowers will appear black	❖ Red flowers will appear red	❖ Red flowers will appear black

2. Why red light is used for danger signals?

ANS: Because red light is scattered the least by air molecules due to its highest wavelength so it is able to travel the longest distance through fog, rain and alike

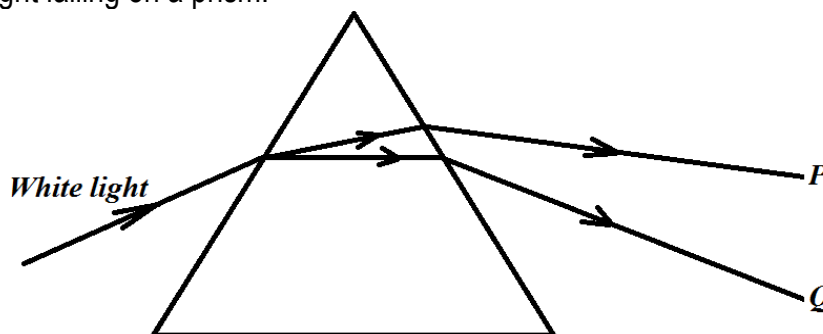
3. A red bus with blue letter on its stops in front of a yellow light at night. Describe the appearance of the bus

ANS: In yellow light

- the red bus will appear red because yellow is composed of green and red
- The blue letters will appear black

Class Activity – 4:2

1. With the help of a diagram, show the dispersion of white light of a prism.
2. Fig shows white light falling on a prism.



- (a) Name the colour at P and Q
 - (b) Explain why a prism disperses white light into its component colours
3. State the cause of dispersion of white light passing through a glass prism. How did Newton show that white light of Sun contains seven colours using two identical glass prisms. Draw a ray diagram to show the path of light when two identical glass prisms are arranged together in inverted position with respect to each other and a narrow beam of white light is allowed to fall
 4. Explain why an object which appears yellow in day light, appears red when illuminated with red light and black when illuminated with a blue light?
 5. A color printer needs just three colors of ink to print all of the colors that we can see. Which colors are they?

ANS: The three colors of ink in a color printer **are the three primary pigment colors which are cyan, magenta, and yellow.** These three colors can be combined in different ratios to produce all other colors, so they are the only colors needed for full-color printing.

6. What color do you see if an object absorbs all of the light that strikes it?
7. Describe and explain the appearance of a red tie with blue spots when observed in
 - (a) Red light
 - (b) Green light

ANS;

- (a) In red light; a red tie will appear red and blue spots will appear black because the red light reflects the red color while blue color is absorbed and by red light and appears black
- (b) In green light; the red tie and blue spots both will appear black. This is because both red and blue colors are absorbed in red light and non will be reflected

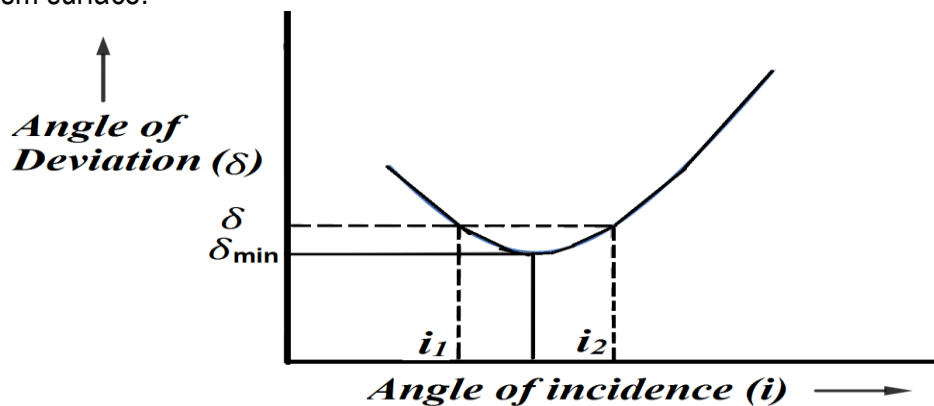
Self Assessment – 4

- 1 Explain what is meant by a spectrum .Describe with the aid of a diagram how would you obtain the spectrum of white light
- 2 Distinguish between a pure and an impure spectrum. Explain with the aid of a diagram how a pure spectrum can be produced in the laboratory .How are the colours of the spectrum recombined?
- 3 A beam of white light is allowed into a dark room through a hole .In the dark room ,the beam falls on a white screen .How will the screen appear **if:**
 - (a) a piece of red glass is placed in the path of the beam
 - (b) a piece of green glass is placed between the screen and the red glass?
- 4 Explain, giving examples, what is meant by:
 - (a) additive combination of colours
 - (b) subtractive combination of colours

- 5 Why does an object appear coloured when light falls onto it?
- 6 Explain, giving examples, what is meant by primary, secondary and complementary colours.
A flag has a green disc on a yellow background .How will the flag appear in:
(a) green light (b) red light (c) blue light
- 7 State , giving a reason for each answer, the colour that results when:
(a) a blue light and a yellow light are mixed (b) blue paint and yellow paint are mixed
- 8 **What color would be seen if white light is viewed through:
(a) a red filter? (b) a cyan filter? (c) an orange filter?**
- 9 **A book** which looks red in white light is viewed in magenta light. In what color does it appear?
- 10 **White light is viewed through a combination of a yellow filter and a red filter held in contact. What color is seen**
- 11 Explain the meaning of the term "dispersion" of white
- 12 A painter has a blue – green (cyan) paint which she wants to make pure green. What color pigment should she add to the paint? Explain your answer
- 13 Briefly explain, why the sky looks blue?
- 14 Under pure yellow light what will be the appearance of the blue piece of cloth?
- 15 Danger signs along the road as well as tail and brake lamps of motor vehicles rear are painted red. Briefly explain the reason behind.
- 16 Explain each of the following:-
(a) The appearance of a blue flag when viewed in day light through a sheet of red glass
(b) The appearance of a red flag with green stripes when viewed in day light through a sheet of green glass
(c) The appearance of a man wearing blue shirt and red trousers, holding a handkerchief of green color when viewed in the pure yellow light
- 17 (a) How does the angle of deviation produced by a prism change with increase in the angle of incidence. Draw a curve showing the variation in the angle of deviation with the angle of incidence at a prism surface.
(b) Using the curve in part (a) above, how would you infer that for a given prism, the angle of minimum deviation δ_{\min} is unique for light of a given wavelength.

Answer

- (a) It is experimentally observed that as the angle of incidence increases, the angle of deviation first decreases, reaches to a minimum value for a certain angle of incidence and then on further increasing the angle of incidence, the angle of deviation begins to increase. The below curve shows the variation in the angle of deviation with the angle of incidence at a prism surface:



- (b) For a given prism and given colour of light, δ_{\min} (angle of minimum deviation) is unique since only one horizontal line can be drawn parallel to i – axis at the lowest point of $i - \delta$ curve i.e only for one value of angle of incidence i , the refracted ray inside the prism is parallel to its base.

Chapter – 05:

Optical Instruments

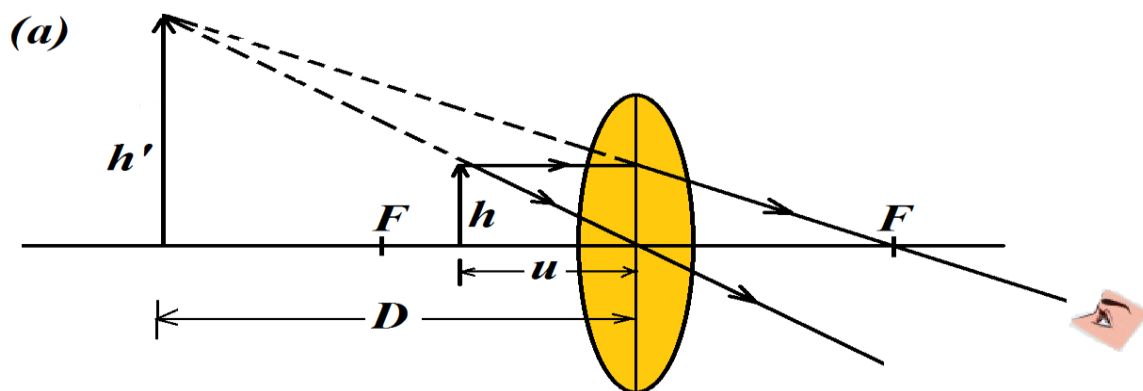
- **Optics** is the study of visible light and the ways it can be used to extend human vision and do other tasks. Knowledge of light was needed for the invention of optical instruments such as microscopes, telescopes, binoculars and cameras, in addition to optical fibers. These instruments **use mirrors and lenses** to reflect and refract light and form images.
- **Optical instruments** are devices which are used to help the human eye views small or distant objects more clearly.

Simple Microscope

- It consists of a biconvex lens which may be hand-held or placed in a simple frame. It is sometimes referred to as a **magnifying glass** of small focal length. The idea is to get an erect, magnified and virtual image of the object. For this the object is placed between **F and P** on one side of the lens and viewed from other side of the lens. There are two magnifications to be discussed for two kinds of focusing
 - (i) **Near point focusing** – The image is formed at near point, i.e. 25 cm for normal eye. This distance is also known as *least distance D* of distinct vision. In this position, the eye feels comfortable but there is little strain on the eye. This is shown in Figure (a) below
 - (ii) **Normal focusing** – The image is formed at infinity. In this position the eye is most relaxed to view the image. This is shown in Figure (b). below

Magnification in near point focusing

- The near point focusing is shown in Figure (a) below.



eye focused on near point

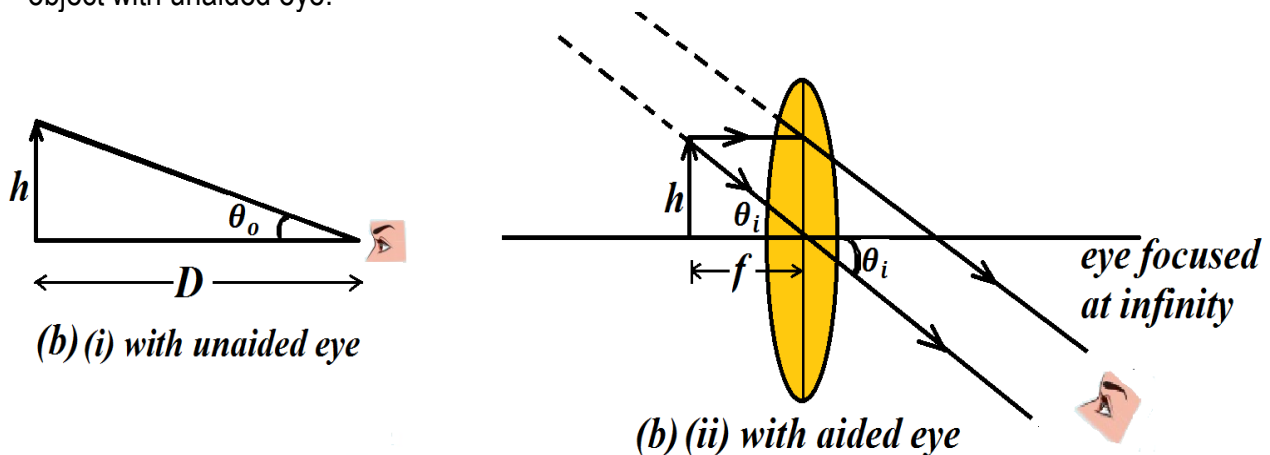
- Object distance u is less than f . The image distance is the near point D . The magnification m is given by the relation,
- With the help of lens equation, $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$, the magnification can further be written as

$$\frac{v}{f} = \frac{v}{u} + 1 \rightarrow \frac{v}{u} = 1 - \frac{v}{f}, \text{ but } \frac{v}{u} = m, \text{ therefore; } m = 1 - \frac{v}{f}$$

- Substituting for v with sign convention, ie., $v = -D$, we get $m = 1 + \frac{D}{f}$
- Therefore; $m = 1 + \frac{D}{f}$. This is the magnification for near point focusing (it is the maximum magnification).

Magnification in normal focusing (angular magnification)

- The normal focusing is shown in Figure (b). We will now find the magnification for the image formed at infinity. If we take the ratio of height of image to height of object ($m = \frac{h'}{h}$) to find the magnification, we will not get a practical relation, as the image will also be of infinite size when the image is formed at infinity.
- Hence, we can practically use the angular magnification. The angular magnification is defined as the ratio of angle θ_i subtended by the image with aided eye to the angle θ_o subtended by the object with unaided eye.



i.e. angular magnification, $m = \frac{\theta_i}{\theta_o}$

- For unaided eye shown in figure (b) (i) above, $\tan\theta_o \approx \theta_o = \frac{h}{D}$
- For aided eye shown in figure (b)(ii) above, $\tan\theta_i \approx \theta_i = \frac{h}{f}$
- The angular magnification, $m = \frac{\theta_i}{\theta_o} = \left(\frac{h}{f}\right) \div \left(\frac{h}{D}\right) = \frac{h}{f} \times \frac{D}{h} = \frac{D}{f}$
- Therefore angular magnification, $m = \frac{D}{f}$. This is the magnification for normal focusing. (It is the minimum magnification)

NB;

- The magnification for normal focusing is one less than that for near point focusing. But, the viewing is more comfortable in normal focusing than near point focusing. For large values of $\frac{D}{f}$, the difference in magnification is usually small.
- In the case of the simple microscope or magnifying glass, the size of the image formed depends on the focal length of the lens used. **The shorter the focal length the more powerful the microscope becomes.**

Worked examples

1. A diamond is viewed with a jeweler's loupe that has a focal length of 5.0 cm. Where must the diamond be placed to provide a virtual image at infinity? What will be the angular magnification of this simple magnifier?

ANS;

- The diamond should be placed at the focal point of the lens (the jeweler's loupe), that is, 5.0 cm from the lens to provide a virtual image at infinity.
 - The magnification is given by; $m = \frac{D}{f} = \frac{25}{f} = \frac{25}{5} = 5$
2. A man with a near point of 25 cm reads a book with small print using a magnifying glass, a convex lens of focal length 5 cm.
- (a) What is the closest and the farthest distance at which he should keep the lens from the page so that he can read the book when viewing through the magnifying glass?
- (b) What is the maximum and the minimum angular magnification (magnifying power) possible using the above simple microscope?

ANS; $D = 25 \text{ cm}$, $f = 5 \text{ cm}$

NB;

- For closest object distance, u ; the image distance, v is, -25 cm . (near point focusing)
- For farthest object distance, u' ; the corresponding image distance, v' is, $v = \infty$ (normal focusing)

(a) For closest and farthest distance

- For closest object distance; consider $\frac{1}{f} = \frac{1}{u} + \frac{1}{v} \rightarrow \frac{1}{u} = \frac{1}{f} - \frac{1}{v}$
- $\frac{1}{u} = \frac{1}{f} - \frac{1}{v} = \frac{1}{5} - \frac{-1}{25} = \frac{5+1}{25} = \frac{6}{25} \rightarrow v = \frac{25}{6} = 4.167 \text{ cm}$
- For furthest distance; consider $\frac{1}{f} = \frac{1}{u} + \frac{1}{v} \rightarrow \frac{1}{u} = \frac{1}{f} - \frac{1}{v}$
- $\frac{1}{u} = \frac{1}{f} - \frac{1}{\infty} = \frac{1}{5} - 0 = \frac{1}{5} \rightarrow v = \frac{5}{1} = 5 \text{ cm}$

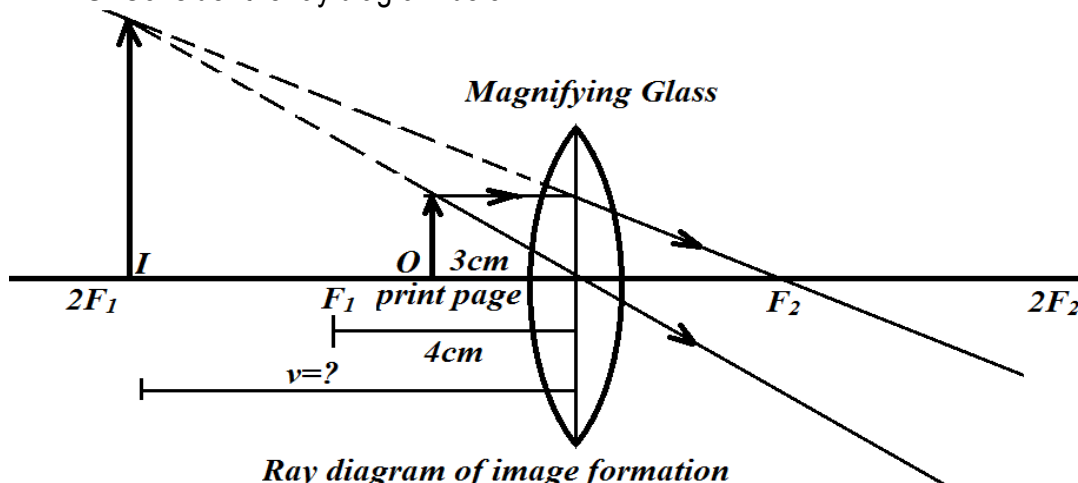
(b) Maximum and minimum Magnification;

- **Magnification in near point, $m = \frac{D}{f} + 1 = \frac{25}{f} + 1 = \frac{25}{5} + 1 = 6$**
- **Magnification at normal focusing, $m = \frac{D}{f} = \frac{25}{f} = \frac{25}{5} = 5$**

Class Activity – 5:1

1. A simple microscope with a focal length of 5 cm is used to read division of scale 1.5 mm in size. How large will the size of the divisions as seen through the simple microscope be?
(ANS: 9 mm when viewed through the simple microscope.)
2. A magnifying glass with a focal length of +4 cm is placed 3 cm above a page of print.
(i) At what distance from the lens is the image of the page?
(ii) What is the magnification of this image?

ANS: Consider the ray diagram below



$$(i) \text{ From; } \frac{1}{f} = \frac{1}{u} + \frac{1}{v} \rightarrow \frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{4} - \frac{1}{3} = -\frac{1}{12} \rightarrow v = -12\text{cm}$$

Therefore, the distance from the lens to the image of the page is 12 cm.

$$(ii) m = -\frac{v}{u} = -\frac{-12}{3} = 4$$

3. Least distance of distinct vision is 25 cm. What will be magnifying power of simple microscope of focal length 5 cm, if final image is formed at minimum distance of distinct vision

$$[\text{ANS; } m = \frac{D}{f} + 1 = \frac{25}{5} + 1 = 6]$$

4. Find the angular magnification produced by a simple microscope of focal length 5cm when used in normal adjustment. [ANS; 5]
5. Explain why angular magnification of a simple microscope is high for a lens of short focal length.
6. **Why the image formed by magnifying glass is free from chromatic aberration.**

ANS;

- We know that the chromatic aberration depends on the focal length of the lens. Thus, as the focal length of the lens increases the chromatic aberration will be reduced. Therefore; as the focal length of the lens in magnifying glass is very high, the images formed in magnifying glasses are free from chromatic aberration.
7. Explain why, when using a magnifying glass, the image distance should be approximately 25 cm from the eye for the object to be viewed clearly.
8. **Two magnifying glasses are for sale at a store. Magnifying glass 1 has a 4cm diameter with a long focal length, and glass 2 has a 1 cm diameter with a short focal length. Which one should you purchase if you wish to start a campfire using sunlight?**

ANS; The ability to start a fire with sunlight depends on how much light energy the lens can capture. A lens with a bigger area will capture more sunlight, hence the lens with the larger diameter is the better one for starting a campfire.

9. **Two magnifying glasses are for sale at a store. Magnifying glass 1 has a 6cm diameter with a long focal length, and glass 2 has a 1.5cm diameter with a short focal length. Which one should you purchase if you wish to examine tiny insects?**

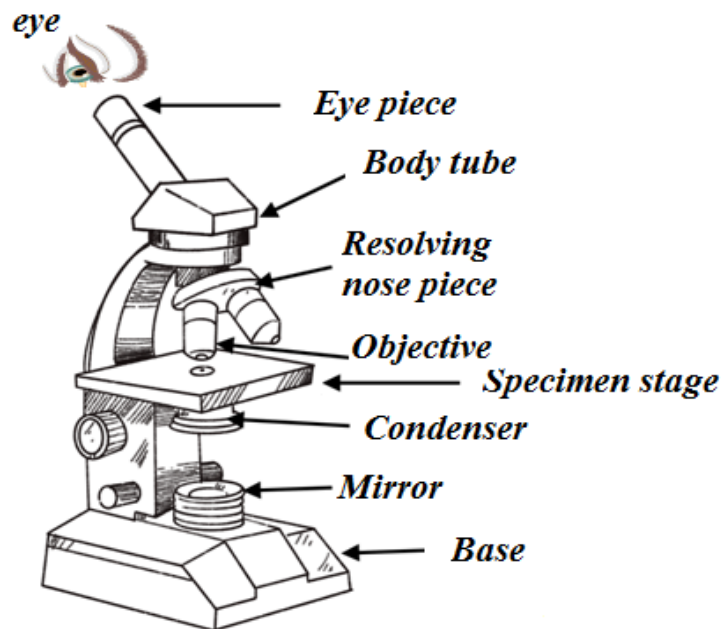
ANS; The angular magnification of a magnifying glass is inversely proportional to its focal length, so a shorter focal length means greater magnification and is better for examining tiny insects.

Uses

- It is used to view specimen in the laboratory
- It is used by watchmakers to obtain a magnified view of small parts of the watch.
- It is used by jewelers to get a magnified view of the fine parts of jewelry.
- A simple microscope is called the hand lens or the reading lens to obtain an enlarged view of hard-to-read small alphabets.
- It is used by a dermatologist (a skin specialist) to examine various skin diseases.
- It is used in studying soil particles (Pedology)

Compound Microscope

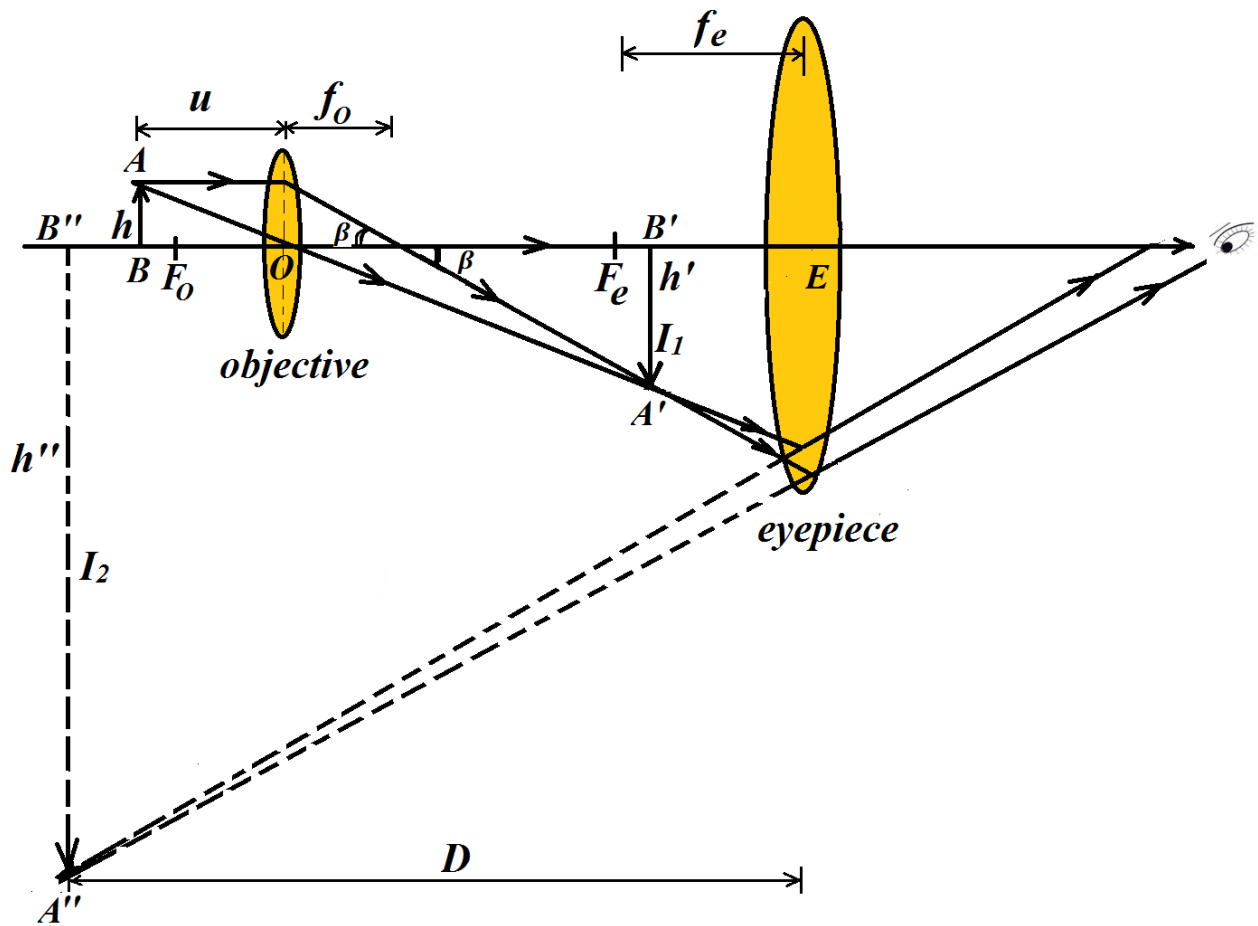
- A compound microscope is an optical instrument which uses two sets of lenses providing a high resolution and 2-dimensional image of the sample.
- Compound microscope is one type of optical microscope, the other type is a simple microscope. The difference between a simple and a compound microscope is that a simple microscope uses one lens whereas a compound microscope uses two or more lenses. The figure below shows a compound microscope with its various parts.



- A Compound microscope is composed of two convex lenses of short focal lengths placed in a tube. The two lenses are separated by a certain fixed distance.
- The diagram of a compound microscope is shown in Figure below. The lens near the object, called the **objective**, forms a real, inverted, magnified image of the object. This serves **as the object for the second lens** which is the **eyepiece**. **Eyepiece serves as a simple microscope** that produces finally an enlarged and virtual image. The first inverted image formed by the objective is to be adjusted close to, but within the focal plane of the eyepiece so that **the final image is formed nearly at infinity or at the near point**. The final image is inverted with respect to the original object. We can obtain the magnification for a compound microscope.

How compound microscope works to produce the final image

- Consider two lenses in succession as shown in figure below.
- The object is just beyond the focal length of the objective lens, producing a **real, inverted image** that is larger than the object. This first image serves as the object for the second lens, or eyepiece. The eyepiece is positioned so that the **first image is within its focal length** so that it can further magnify the image. In a sense, it acts as a magnifying glass that magnifies the intermediate image produced by the objective. The image produced by the eyepiece is a magnified virtual image. The final image remains inverted but is farther from the observer than the object, making it easy to view.
- The eye views the virtual image created by the eyepiece, which serves as the object for the lens in the eye. The virtual image formed by the eyepiece is well outside the focal length of the eye, so the eye forms a real image on the retina.



Magnification produced by a compound microscope

- From the Figure above, $\tan \beta = \frac{h}{f_o}$, also $\tan \beta \approx \frac{h'}{L}$,
 $\rightarrow \frac{h}{f_o} = \frac{h'}{L} \rightarrow \frac{h'}{h} = \frac{L}{f_o}$ Then, $m_o = \frac{h'}{h} = \frac{L}{f_o}$
- Here, the distance L is between the first focal point of the eyepiece to the second focal point of the objective. This is called the tube length L of the microscope as f_o and f_e are comparatively smaller than L
- If the final image is formed at P (near point focusing), the magnification m_e of the eyepiece is, $m_e = \frac{D}{f_e} + 1$
- Also here, f_o and f_e are the focal lengths of the Objective and the eyepiece, respectively. We assume that the final image is formed **at the near point of the eye**, providing the largest magnification. **Note that the angular magnification of the eyepiece** is the same as obtained earlier for the **simple magnifying glass**. This should not be surprising, because the eyepiece is essentially a magnifying glass, and the same physics applies here.
- Therefore, **the net magnification (m) in near point focusing** of the compound microscope is the product of the **linear magnification** of the objective and the **angular magnification** of the eyepiece

$$ie, \dots m = m_o m_e = \left(\frac{L}{f_o}\right) \left(\frac{D}{f_e} + 1\right) = \frac{L(D + f_e)}{f_o + f_e}$$

- If the final image is formed at infinity (normal focusing), the first inverted image will be at focal distance of the eyepiece,

From lens formula; $\frac{1}{f_e} = \frac{1}{u_e} + \frac{1}{v_e}$, whereby $v_e = \infty$
 $\frac{1}{f_e} = \frac{1}{u_e} + \frac{1}{\infty} \rightarrow \frac{1}{f_e} = \frac{1}{u_e}$ (thus; at infinity $f_e = u_e$)

From; $\frac{1}{f_e} = \frac{1}{u_e}$ multiply by **D** each side of the equation

$$\frac{D}{f_e} = \frac{D}{u_e}, \quad \text{but; } \frac{D}{u_e} = m_e \rightarrow m_e = \frac{D}{f_e}$$

Thus; the magnification m_e of the eyepiece at infinity is, $m_e = \frac{D}{f_e}$

- Therefore, the total magnification m in normal focusing (at infinity) is,

$$m_o m_e = \left(\frac{L}{f_o}\right) \left(\frac{D}{f_e}\right) = \frac{DL}{f_o f_e} = \frac{25L}{f_o f_e}$$

Worked Examples

1. A microscope has an objective and eyepiece of focal lengths 5 cm and 50 cm respectively with tube length 30 cm. Find the magnification of the microscope in the (i) near point and (ii) normal focusing.

ANS; given; $f_o = 5\text{cm}, f_e = 50\text{cm}, L = 30\text{cm}$

(iii) At near point, $m_o m_e = \left(\frac{L}{f_o}\right) \left(1 + \frac{D}{f_e}\right) = \left(\frac{30}{5}\right) \left(1 + \frac{25}{50}\right) = 9$

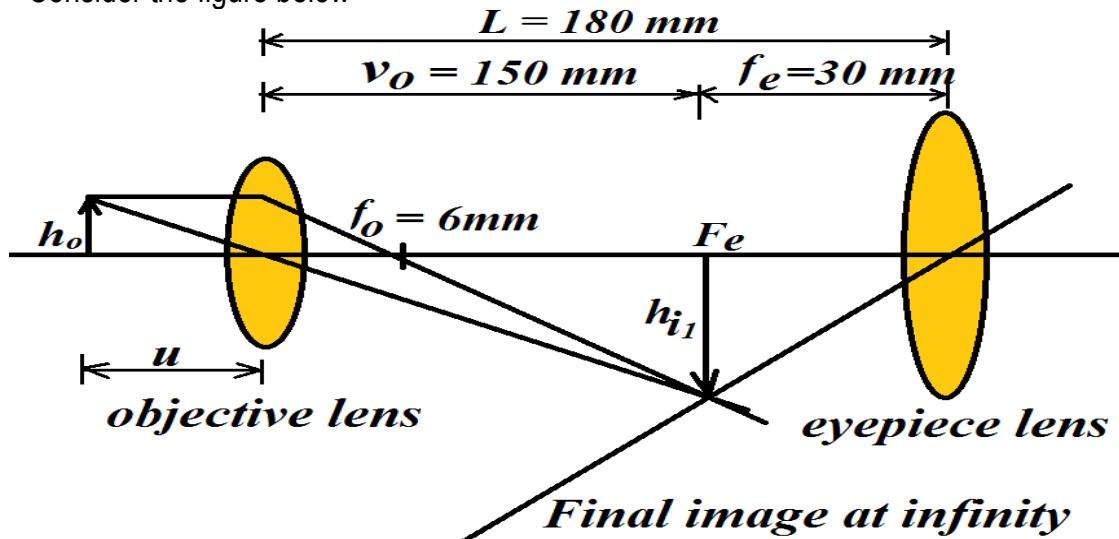
(iv) At normal focusing, $m_o m_e = \left(\frac{L}{f_o}\right) \left(\frac{D}{f_e}\right) = \left(\frac{30}{5}\right) \left(\frac{25}{50}\right) = 3$

2. A compound microscope has an objective and an eyepiece separated by a distance of 180 mm and having focal lengths of 6 mm and 30 mm respectively. Where must an object be placed so that the final image, as seen through the eyepiece, is formed at infinity?

ANS;

NB; when the final image is formed at infinity, the first image of the objective lens must be formed at the focus (focal point) of the eyepiece lens

Consider the figure below



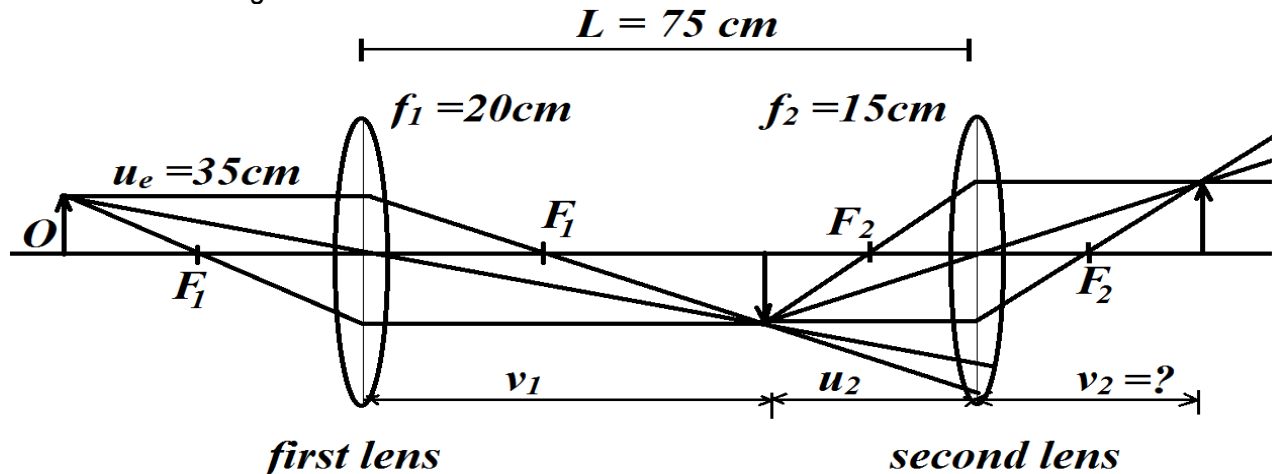
Then; $L = v_o + f_e \rightarrow v_o = L - f_e = 180 - 30 = 150\text{mm}$
 From; $\frac{1}{f_o} = \frac{1}{u_o} + \frac{1}{v_o} \rightarrow \frac{1}{u_o} = \frac{1}{f_o} - \frac{1}{v_o} = \frac{1}{6} - \frac{1}{150} = \frac{25-1}{150} = \frac{24}{150} = \frac{4}{25}$
 $\frac{1}{u_o} = \frac{4}{25} \rightarrow u_o = 6.25 \text{ mm}$

Therefore; the object must be placed 6.25 mm from the objective lens, so as the final image is formed at infinity

3. 5 An object sits 35 cm to the left of a converging lens with focal length of 20 cm. 75 cm to the right is a second converging lens with focal length of 15 cm. Locate and characterize the final image.

ANS;

Consider the figure below



From the first lens;

$$\frac{1}{f_1} = \frac{1}{u_1} + \frac{1}{v_1} \rightarrow \frac{1}{v_1} = \frac{1}{f_1} - \frac{1}{u_1} = \frac{1}{20} - \frac{1}{35} = \frac{35-20}{700} = \frac{15}{700} = \frac{3}{140}$$

$$\frac{1}{v_1} = \frac{3}{140} \rightarrow v_1 = \frac{140}{3} = 46.7\text{cm}$$

But; $L = v_1 + u_2 \rightarrow u_2 = L - v_1 = 75 - 46.7 = 28.3\text{cm}$

This image (which is real, inverted, and enlarged) becomes the object for the second lens then we again apply the Image Equation,

$$\frac{1}{f_2} = \frac{1}{u_2} + \frac{1}{v_2} \rightarrow \frac{1}{v_2} = \frac{1}{f_2} - \frac{1}{u_2} = \frac{1}{15} - \frac{1}{28.3} = \frac{28.3-15}{424.5} = \frac{13.3}{424.5}$$

$$\frac{1}{v_2} = \frac{13.3}{424.5} \rightarrow v_2 = \frac{424.5}{13.3} = 31.9 \text{ cm}$$

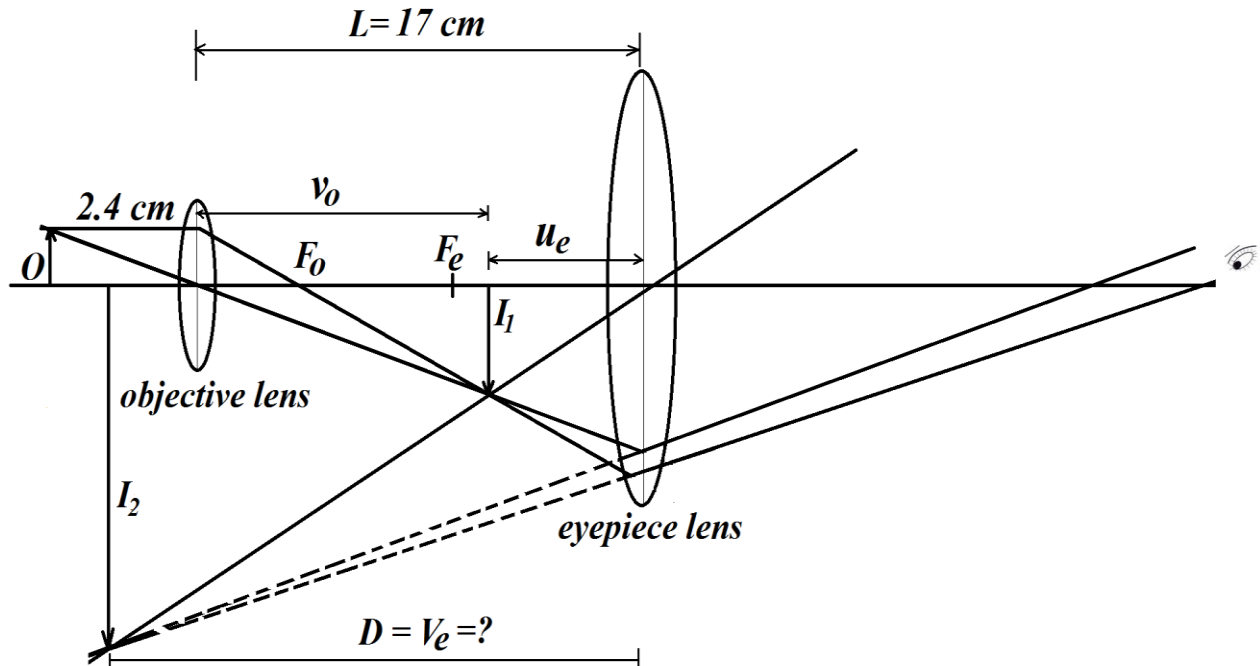
That is, the final image is located 31.9 cm to the right of the second lens. Since; v_2 is +ve, the image formed is **real**. It is inverted, compared to the object for lens 2. But that was an inverted image of the original object. Therefore, this final image is **upright**.

The total magnification is the product of the magnifications of the two lenses,

$$m = m_1 m_2 = \frac{v_1}{u_1} \times \frac{v_2}{u_2} = \frac{46.7}{35} \times \frac{31.9}{28.3} = 1.5$$

4. A compound microscope has an objective lens of focal length 2 cm and eye piece of focal length of 6 cm. An object is placed 2.4 cm from the objective lens. If the distance between the objective lens and the eyepiece lens is 17 cm find the distance of the final image from the eyepiece.

Soln: Given: $f_o = 2 \text{ cm}$, $f_e = 6 \text{ cm}$, $u_e = 2.4 \text{ cm}$, $L = 17 \text{ cm}$
 Consider the figure below:



Refraction from the objective lens:

$$\text{From: } \frac{1}{f_o} = \frac{1}{u} + \frac{1}{v} \rightarrow \frac{1}{2} = \frac{1}{2.4} + \frac{1}{v} \rightarrow \frac{1}{v} = \frac{1}{2} - \frac{1}{2.4} = \frac{2.4-2}{4.8} = \frac{0.4}{4.8} = \frac{1}{12}$$

$$v_o = 12 \text{ cm}, \text{ but } v_o + u_e = L \rightarrow u_e = L - v_o = 17 - 12 = 5 \text{ cm}$$

Also consider the refraction from the eyepiece lens

$$\text{From: } \frac{1}{f_e} = \frac{1}{u} + \frac{1}{v}, \text{ but } v_e = D \rightarrow \frac{1}{6} = \frac{1}{5} + \frac{1}{D} \rightarrow \frac{1}{D} = \frac{1}{6} - \frac{1}{5} = \frac{-1}{30}$$

$D = -30 \text{ cm}$, (According to real – is – positive the final image is virtual)

∴ The distance of the final image from the eyepiece is 30 cm

5. Calculate the magnification of an object placed 6.20 mm from a compound microscope that has a 6.00 mm focal length objective and a 50.0 mm focal length eyepiece. The objective and eyepiece are separated by 23.0 cm.

ANS;

Given; $u_o = 6.2 \text{ mm}$, $f_o = 6.0 \text{ mm}$, $f_e = 50.0 \text{ mm}$, $L = 230 \text{ mm}$

Refraction from the objective lens:

$$\text{From: } \frac{1}{f_o} = \frac{1}{u_o} + \frac{1}{v_o} \rightarrow \frac{1}{v_o} = \frac{1}{f_o} - \frac{1}{u_o} = \frac{1}{6} - \frac{1}{6.2} = \frac{6.2-6}{6 \times 6.2} = \frac{0.2}{37.2}$$

$$\frac{1}{v_o} = \frac{2}{372} \rightarrow v_o = 186 \text{ mm}$$

$$\text{But; } L = u_e + v_o \rightarrow u_e = L - v_o = 230 - 186 = 44 \text{ mm}$$

Refraction from the eyepiece lens:

$$\text{From: } \frac{1}{f_e} = \frac{1}{u_e} + \frac{1}{v_e} \rightarrow \frac{1}{v_e} = \frac{1}{f_e} - \frac{1}{u_e} = \frac{1}{50} - \frac{1}{44} = \frac{44-50}{50 \times 44} = \frac{-6}{2200}$$

$$\frac{1}{v_e} = \frac{-6}{2200} \rightarrow v_e = \frac{-2200}{6} = -366.7 \text{ mm}$$

NB; negative sign means, the final image is virtual and erect with respect to the first image

Therefore; total magnification is given by $m = m_o m_e = \left(\frac{v_o}{u_o}\right) \left(\frac{v_e}{u_e}\right)$

$$m = m_o m_e = \left(\frac{v_o}{u_o}\right) \left(\frac{v_e}{u_e}\right) = \frac{186}{6.2} \times \frac{366.7}{44} = 250$$

6. A compound microscope consists of two thin converging lenses. The focal length of the objective lens is 10 mm and that of the eyepiece lens is 20mm. If an object is placed 11mm from the objective lens, the instrument produces an image at infinity. Calculate the separation of the lenses and the magnifying power of the instrument

ANS: Given; $f_o = 10\text{mm}$, $f_e = 20\text{mm}$, $u_o = 11\text{mm}$, $L = ?$, $m = ?$

$$\text{From; } \frac{1}{f} = \frac{1}{u} + \frac{1}{v} \rightarrow \frac{1}{v_o} = \frac{1}{f_o} - \frac{1}{u_o} = \frac{1}{10} - \frac{1}{11} = \frac{1}{110} \rightarrow v_o = 110$$

But; when image at infinity; $u_e = f_e$, but; $v_o + u_e = L$

$$\therefore L = v_o + f_e = 110 + 20 = 130\text{mm}$$

7. The total magnification produced by a compound microscope is 20. The magnification produced by the eye piece alone is 5. The microscope is focused on a certain object. The distance between the objective lens and the eyepiece is observed to be 14 cm. If least distance of distinct vision is 20 cm, calculate the focal lengths of the objective and eyepiece lenses.

ANS;

Given: $m = 20$, $m_e = 5$, $L = 14\text{cm}$, $D = 20\text{cm}$, $f_o = ?$, $f_e = ?$

$$\text{From: } m = m_o m_e = \left(\frac{L}{f_o}\right) \left(1 + \frac{D}{f_e}\right) = \text{----- (i)}$$

$$\text{For distance of near vision, } m_e = \left(\frac{D}{f_e} + 1\right) \text{---(ii)}$$

$$\text{From eqn (i) } m = m_o m_e \rightarrow 20 = 5m_o \rightarrow m_o = \frac{20}{5} = 4,$$

$$\text{substitute } m_e = 5 \text{ in eqn (ii)} \rightarrow 5 = \left(\frac{D}{f_e} + 1\right) = \frac{20}{f} + 1 \rightarrow f_e = \frac{20}{5-1} = 5\text{cm}$$

$$m_o = \left(\frac{L}{f_o}\right) \rightarrow f_o = \frac{L}{m_o} = \frac{14}{4} = 3.5\text{cm} \quad \therefore f_o = 3.5 \text{ cm and } f_e = 5\text{cm}$$

8. The objective and eyepiece of a compound microscope are 24cm apart. Suppose the focal lengths of the objective and the eyepiece are 3cm and 9cm respectively.

(a) Where must a specimen be located to produce a final virtual image at infinity?

(b) What could be the total magnification of this microscope? (Assume a microscope is used by a person whose nearest distance for a distinct vision is 25cm).

ANSW: Given; $l = 24\text{cm}$, $f_o = 3\text{cm}$, $f_e = 9\text{cm}$, $D = 25\text{cm}$

$$\text{(a) } v_o = L - f_e = 24 - 9 = 15\text{cm, from; } \frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\therefore u_o = \frac{v_o f_o}{v_o - f_o} = \frac{15 \times 3}{15 - 3} = 3.75\text{cm}$$

$$\text{(b) } m = \frac{Dl}{f_o f_e} = \frac{25 \times 24}{3 \times 9} = 22.2$$

Class Activity – 5:2

1. A compound microscope consists of two lenses of focal length 12 cm and 6 cm for the objective lens and the eyepiece lens, respectively. The two lenses are separated by a distance of 30 cm. The microscope is focused so that the image is formed at infinity. Determine the position of the object. **ANS: $U = 24 \text{ cm}$**
2. A parallel beam of light falls on a converging lens arranged so that the axis lies along the direction of the light which is brought to focus 25 cm from the lens. **The** light then passes through a second converging lens of focal length 7.5 cm placed at 30 cm from the first lens. Calculate the position of the final image. Draw a ray diagram to show the final image formed. **(ANS: $D = 15 \text{ cm}$)**
3. When viewing through a compound microscope, our eyes should be positioned not on the eyepiece but a short distance away from it for best viewing. Why?
4. **In a particular compound microscope, the objective lens and eyepiece lens are 15cm apart. The focal length of the objective lens is 2.0cm, while that of the eyepiece lens is 6.25cm**
 - a. How far from the objective lens should an object be placed in order to obtain the final image of distant vision? **[ANS; 2.59cm]**
 - b. What is the total magnifying power of the microscope? **[ANS; 30]**
5. (a) Draw a labelled ray diagram of compound microscope, when final image forms at the least distance of distinct vision.
(b) Why is its objective of short focal length and of short aperture, compared to its eyepiece? Explain.
6. **The separation of the objective ($f_o=0.5 \text{ cm}$) and the eyepiece ($f_e=5 \text{ cm}$) in a compound microscope is $L= 7 \text{ cm}$. Where should a small object be placed so that the eye can see the image without strain and what is the magnification?**

Answer: The eye will see the image without strain if the final image is formed at infinity. In this case, the image formed by the objective should fall at the focus of the eyepiece,
 $\therefore u_o = 0.67 \text{ cm}$
7. A compound microscope has the objective of a focal length of 1.25 cm and an eyepiece with a focal length of 5 cm. A small object is kept at 2.5 cm from the objective. If the final image is formed at infinity, find the distance between the objective and the eyepiece.
ANS; $L = v_o + f_e = v_o + u_e = 2.5 + 5 = 7.5 \text{ cm}$
8. **Why does the objective lens in a compound microscope have the smallest focal length?**

ANS;

 - The objective lens in a compound microscope has the smallest focal length in order to provide a short focal distance, which allows for higher magnification and resolution.
 - This is essential for capturing detailed images of tiny specimens. By having a smaller focal length, the objective lens can gather more light and produce a clear, magnified image.
9. A compound microscope has an angular magnification equal to 10. If the object subtends an angle of 0.6° to the eye. What will be the angle subtended by the image to the eye?
ANS; $m = \frac{\theta_i}{\theta_o} \rightarrow \theta_i = m \times \theta_o = 6^\circ$
10. (a) Draw a ray diagram of compound microscope for the final image formed at least distance of distinct vision?
(b) An angular magnification of **30X** is desired using an objective of focal length 1.25 cm and an eye piece of focal length 5 cm. How will you set up the compound microscope for the final image formed at least distance of distinct vision?

ANS; when image is formed at least distance of distinct vision, $D = 25\text{cm}$, $m = 30\text{X}$
then; Angular magnification of eyepiece, $m_e = \frac{D}{f} + 1 = \frac{25}{5} + 1 = 6$

But, total angular magnification, $m = m_o m_e \rightarrow m_o = \frac{m}{m_e} = \frac{30}{6} = 5$

As the objective lens forms the real image, then $m_o = \frac{v_o}{u_o} \rightarrow v_o = m_o u_o = 5u_o$

Now; from; $\frac{1}{f} = \frac{1}{u} + \frac{1}{v} \rightarrow \frac{1}{f} = \frac{1}{u} + \frac{1}{5u} = \frac{6}{5u} \rightarrow \frac{1}{f} = \frac{6}{5u} \rightarrow u_o = \frac{6f}{5} = \frac{6 \times 1.25}{5} = 1.5\text{cm}$

Also, for eyepiece;

From lens formula; here, $D = v_e = -25\text{cm}$, $f_e = 5\text{cm}$, $u_e = ?$

$\frac{1}{f} = \frac{1}{u} + \frac{1}{v} \rightarrow \frac{1}{f} = \frac{1}{u} - \frac{1}{v} \rightarrow \frac{1}{u} = \frac{1}{f} + \frac{1}{v} \rightarrow u = \frac{fv}{f+v} = \frac{5 \times 25}{5+25} = \frac{125}{30} \approx 4.17\text{cm}$

Distance of separation, $L = v_o + u_e = 5u_o + u_e = 7.5 + 4.167 = 11.67\text{cm}$

Therefore; the object should be placed at 1.5 cm from the objective lens and separation between the two lenses should be adjusted to 11.67 cm for the final image to be formed at least distance of distinct vision

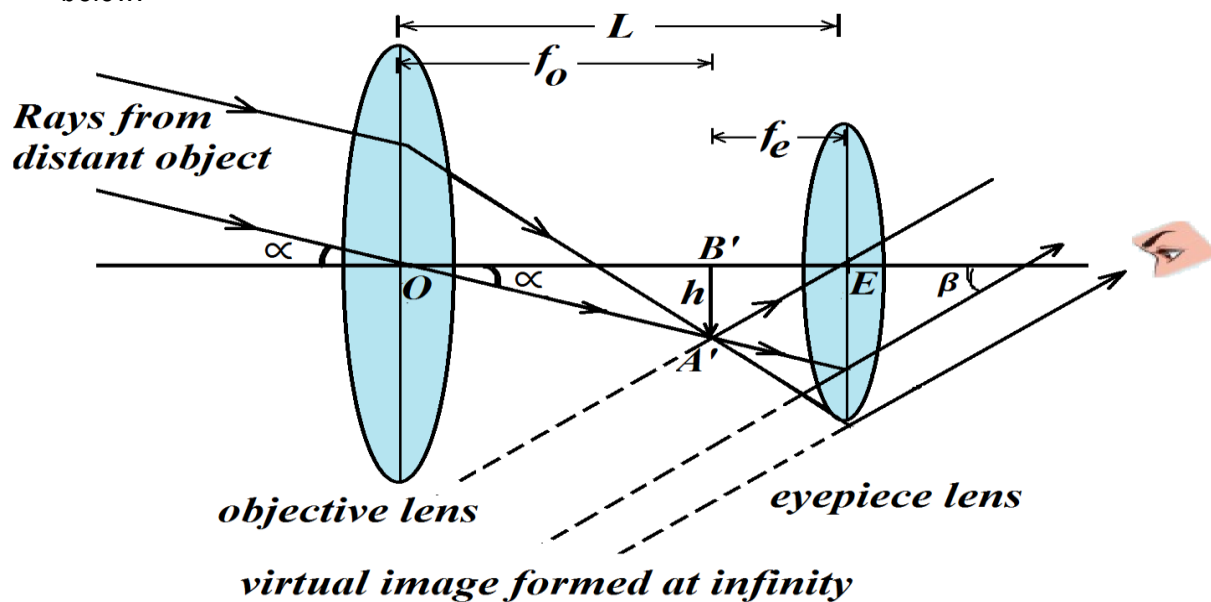
11. The tube of a microscope is of length 15 cm. Focal lengths of the eyepiece and objective are 3 cm and 2 cm respectively. Find the magnifying power of the microscope for the image formed at infinity. **[ANS; $m = \frac{DL}{f_o f_e} = \frac{25L}{f_o f_e} = 62.5$]**

Uses

- Observing Brownian motion in science
- Studying microorganisms and cells in biology
- Checking for infections caused by microorganisms in hospitals

Astronomical Telescope

- An astronomical telescope is used to get the magnification of distant astronomical objects like stars, planets, moon etc. The image formed by astronomical telescope will be inverted. It has an objective of long focal length and a much larger aperture than the eyepiece as shown in Figure below.



- Light from a distant object enters the objective and a real image is formed in the tube at its second focal point. The eyepiece magnifies this image producing a final inverted image.

Mode of action

- The objective lens forms a real, inverted and diminished image of a distant object at its focal point, f_o .
- This becomes the object for the eyepiece lens.
- The position of the eyepiece lens is adjusted until the object is at its focal point, f_e .
- This adjustment makes the final image to be formed at infinity.

Working Principle of Astronomical Telescope

An astronomical telescope works on the principle that when an object to be magnified is placed at a large distance from the objective lens of telescope, a virtual, inverted and magnified image of the object is formed at the least distance of distinct vision from the eye held close to the eye piece.

NB:

- ✓ The final image obtained in the astronomical telescope is small compared to the original object.
- ✓ The image looks larger because it is very much closer to the observer's eye.

Magnification produced by astronomical telescope

The magnification m is the ratio of the angle β subtended at the eye by the final image to the angle α which the object subtends at the lens or the eye, i.e.,

$$m = \frac{\text{angle subtended at the eye by the image}(\beta)}{\text{angle subtended at the eye by the object}(\alpha)} = \frac{\beta}{\alpha}$$

- Since both the object and the final image are at infinity, the angles they subtend at the eye are the same as those they subtend at the objective and at the eyepiece lens, respectively.
- Assuming that: angle α and β are very small, then $\tan \alpha = \alpha$ and $\tan \beta = \beta$
- Hence $\beta = \frac{A'B'}{EB'} = \frac{h}{f_e}$ and $\alpha = \frac{A'B'}{OB'} = \frac{h}{f_o}$
- Since: $m = \frac{\beta}{\alpha}$ substitute α and β as given above

$$m = \left(\frac{h}{f_e}\right) \div \left(\frac{h}{f_o}\right) = \left(\frac{h}{f_e}\right) \times \left(\frac{f_o}{h}\right) = \frac{f_o}{f_e}$$

- Therefore the magnification produced by an astronomical telescope is the ratio of the focal length of the objective lens to that of the eyepiece lens, i.e.

$$\therefore m = \frac{f_o}{f_e}$$

- The length of the telescope tube is approximately, $L = f_o + f_e$

Worked Examples:

1. A refracting telescope has an objective lens with a focal length of 5.0 cm and an eyepiece with a short focal length of 0.02 m; calculate the magnifying power of such a telescope in its normal adjustment

Soln:

Given: $f_o = 5.0$ cm, $f_e = 0.02$ m

From: $m = \frac{f_o}{f_e} \rightarrow m = \frac{5}{0.02} = 250$, Therefore the magnifying power is 250

2. A small telescope has an objective lens of focal length 125 cm and an eyepiece of focal length 2 cm. What is the magnification of the telescope? What is the separation between the objective and the eyepiece?

$$m = \frac{f_o}{f_e} = \frac{125}{2} = 62.5, \quad L = f_o + f_e = 125 + 2 = 127\text{cm}$$

Class Activity –5:3

1. What is the angular magnification of a telescope that has a 100 cm focal length objective and a 2.5 cm focal length eyepiece? (**ANS: 40**)
2. How would the magnification of an astronomical telescope be affected if the focal length of the eyepiece and the objective lens are increased
3. Find the distance between the objective and eyepiece lenses in the telescope in the above problem needed to produce a final image very far (at infinity) from the observer, (**A: 102.5 cm**)
4. An astronomer wants to order a large concave mirror for a telescope that is to produce high quality images. With explanation, advise whether the astronomer should order a spherical mirror or a parabolic mirror?
5. A large reflecting telescope has an objective mirror with a 10.0 m radius of curvature. What angular magnification does it produce when a 3.0 m focal length eyepiece is used?
6. A small telescope has a concave mirror with a 2.0 m radius of curvature for its objective. Its eyepiece is a 4.0 cm focal length. What is the telescope's angular magnification? What angle is subtended by a 25,000 km diameter sunspot?

Uses

- An astronomical telescope is used to view distant objects like stars and other objects in space.
- They are used in military bases to see enemies

Differences between Compound microscope and Astronomical telescope

Compound microscope	Astronomical telescope
Objective lens has a smaller focal length than the eyepiece lens	Objective lens has a larger focal length than the eyepiece lens
Distance between the objective lens and the eye piece lens is greater than $f_o + f_e$	Distance between the objective lens and eye piece lens is equal to $f_o + f_e$
It is used to see very small objects	It is used to see distant astronomical objects
It uses artificial light	It uses natural light at the focal point
It is very small in shape as compared to telescope	Very large in size as compared to microscope

Similarities

- Both Consists of two convex lenses.
- Objective lens forms a real and inverted image which acts as the object for the eyepiece.
- The eyepiece acts as a magnifying glass.
- Final image formed is virtual, inverted with respect to the object and magnified.

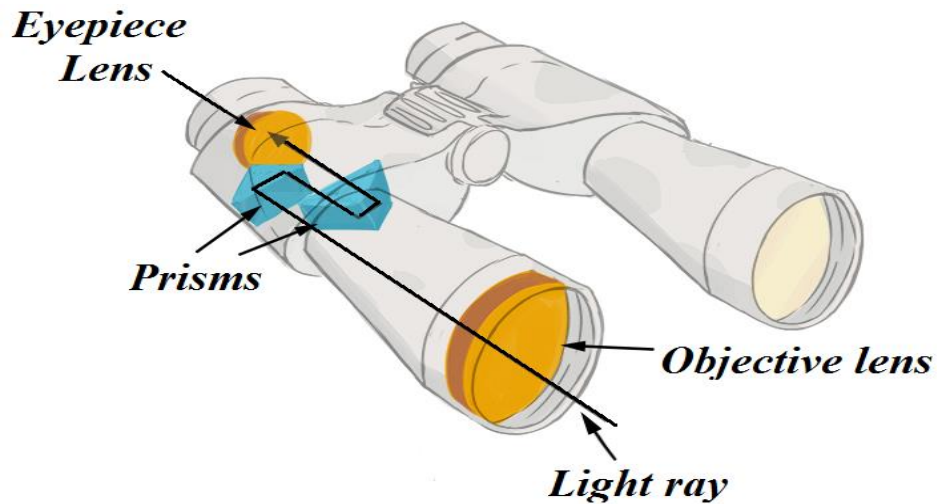
Binoculars

Binoculars are a type of telescope that allows a user to view far away objects using both eyes. This requires the use of two separate telescopes, one for each eye, thus allowing **binocular** vision.

Structure of binoculars

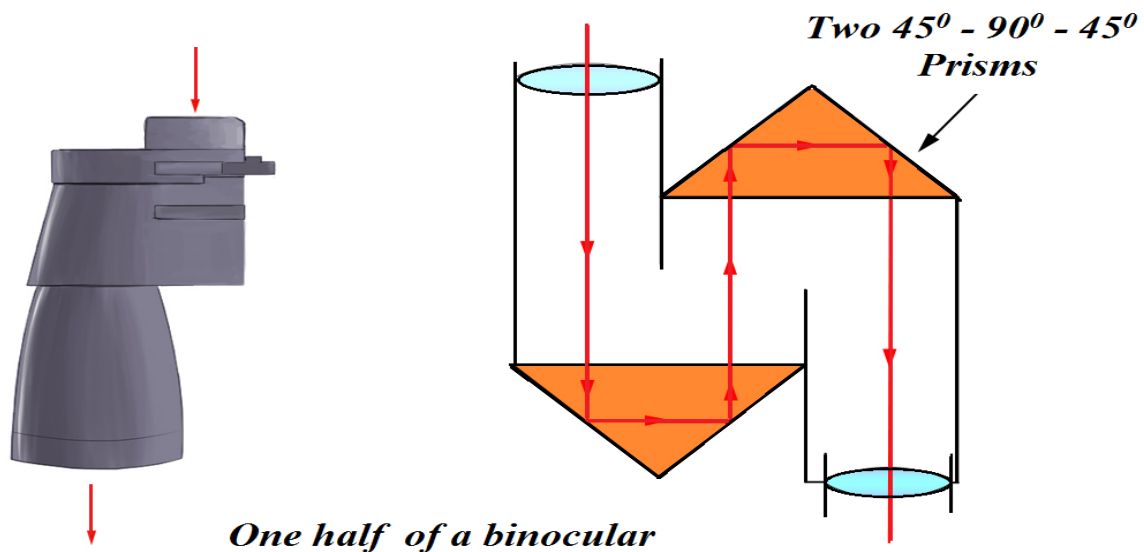
Modern binoculars consist of **two barrel chambers with an objective lens, eyepiece lens, and a pair of prisms inside** whereby;

- **Objective lens** is used to collect light from the distant object that the user is looking at and bringing the collected light into focus in the **Eyeiece lens**, which creates a visible and magnified image.
- **A pair of prisms** which is used to invert the image so that it can be seen properly upright by the eyes (*ie.*, without prisms in the binoculars, the observer would see **an inverted image**). The **prisms** are made of crown glass and **serve as correcting mirrors**. As the light beam passes through the prisms, multiple reflections flip the upside-down and inverted image projected by the objective lens, so the observer can see a normal-looking image.



Mode of action of binoculars

- When rays of light passing through the objective lens form an inverted image of the distant object, the arrangement of the prisms rotates it by 180° and the eyepiece, in turn, produces a magnified image of the initial image, enabling the observer to easily view that object with clarity. This happens for both the left and right tubes of the binoculars

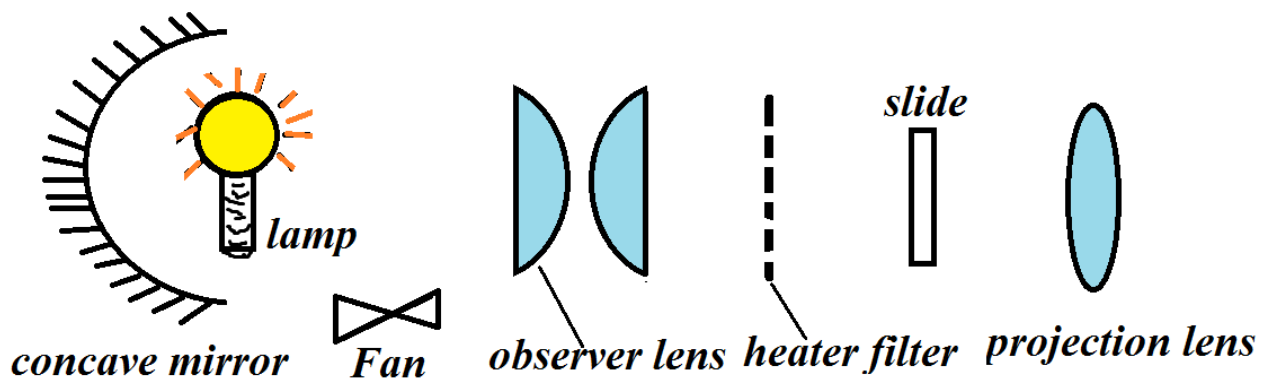


Magnification of binoculars

- **Since** binoculars are made of two identical refracting telescopes, the magnifying power of the binoculars can be calculated as in the case of the astronomical telescope. That is;

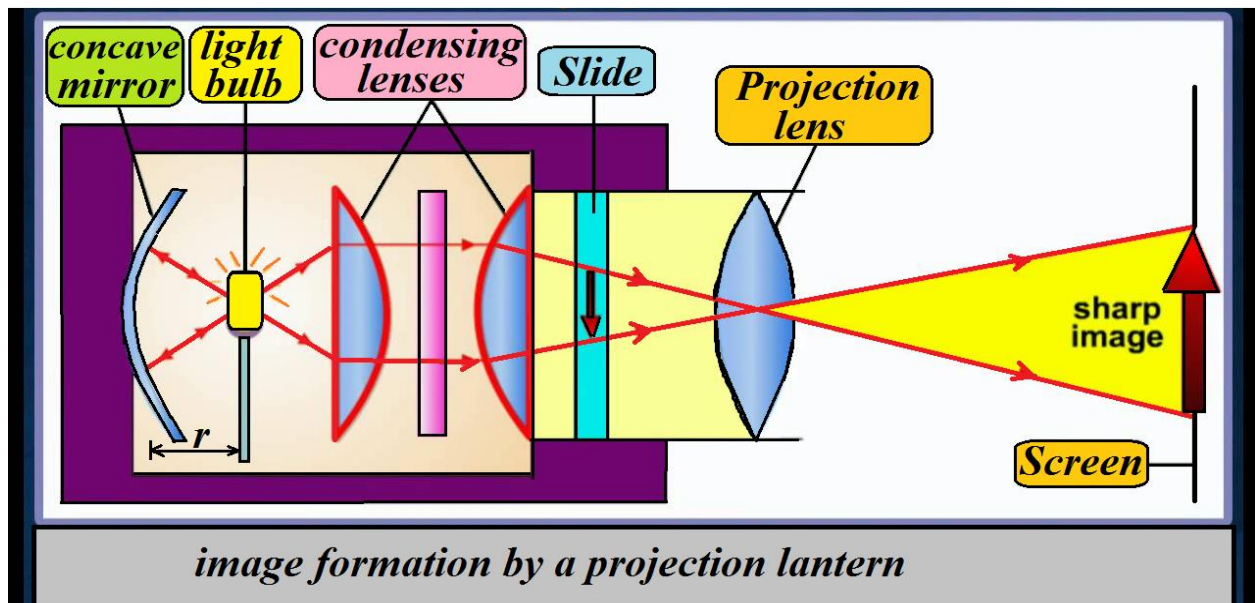
$$m = \frac{f_o}{f_e}$$

- **Whereby,** f_o is the focal length of the objective lens and f_e is the focal length of the eyepiece



Mode of action of a projection lantern

- When light from the projection lamp passes through the slide and projector lens, the resulting image is enlarged and projected onto a perpendicular flat screen where the audience can view the reflection of the slide.
- The slide being the **object**, is placed between f and $2f$ from the projector lens, where f is the focal length of the projector lens. The image formed on the screen is real, inverted and magnified.
- For an image of higher magnification to be formed on the screen, the slide is placed closer to f and the screen is placed further.



Magnification

- It is given by
$$m = \frac{v}{u} = \frac{HI}{HO}$$

Worked Examples:

1. A projection lantern is used to give the image of a slide on a screen. If the image is 24 times as large as the slide and the screen is 72 m from the projecting lens, what is the position of the slide from the lens

Soln:

Given: $m = 24$, $v = 72$ m, $u = ?$

$$\text{From: } m = \frac{v}{u} \quad \rightarrow \quad m = \frac{v}{u} \quad \rightarrow \quad 24 = \frac{72}{u} \quad \rightarrow \quad u = \frac{72}{24} = 3\text{m}$$

\therefore the slide is 3m from the lens

2. A lantern projector using a slide of 2cm x 2cm projects a picture 1m x 1m onto a screen 12m from the projection lens. How far from the lens must the slide be? Find the approximate focal length of the projection lens.

Soln:

Given: $H_o = 2 \text{ cm}$, $H_i = 1 \text{ m} = 100 \text{ cm}$, $v = 12 \text{ m} = 1200 \text{ cm}$, $u = ?$, $f = ?$

$$\text{From: } m = \frac{v}{u} = \frac{H_i}{H_o}$$

$$m = \frac{v}{u} = \frac{H_i}{H_o} \rightarrow \frac{v}{u} = \frac{H_i}{H_o} \rightarrow \frac{1200}{u} = \frac{100}{2} \rightarrow u = 24 \text{ cm}$$

$$\text{Also: } \frac{1}{f} = \frac{1}{u} + \frac{1}{v} \rightarrow \frac{1}{f} = \frac{1}{24} + \frac{1}{1200} = \frac{50+1}{1200} = \frac{51}{1200} = \frac{17}{400}$$

$$\frac{1}{f} = \frac{17}{400} \rightarrow f = \frac{400}{17} = 23.5 \text{ cm}$$

\therefore the slide must be 24 cm from the lens and the approximate focal length is 23.5 cm

Class Activity – 5:5

- A certain slide projector has a 100 mm-focal length lens.
 - How far away is the screen if a slide is placed 103 mm from the lens and produces a sharp image?
 - If the slide is 24.0 by 36.0 mm, what are the dimensions of the image?
- A projection lantern is used to project a slide measuring 3 cm x 3 cm onto a screen 12 m from the projection lens. If the size of the screen is 1.5 m x 1.5 m, how far from the lens must the slide be for the image to fill the entire screen? **ANS: $u = 24 \text{ cm}$.**

Uses of Projection Lantern

- ✓ Projection of films, slides and transparencies.
- ✓ Projection of opaque objects, i.e. episcopic projection.
- ✓ In searchlights and headlights.
- ✓ In physical experiments such as projection of the spectrum, polarisation experiments and interference experiments.
- ✓ Projection of minute objects, i.e. the projection microscope.

Lens Camera

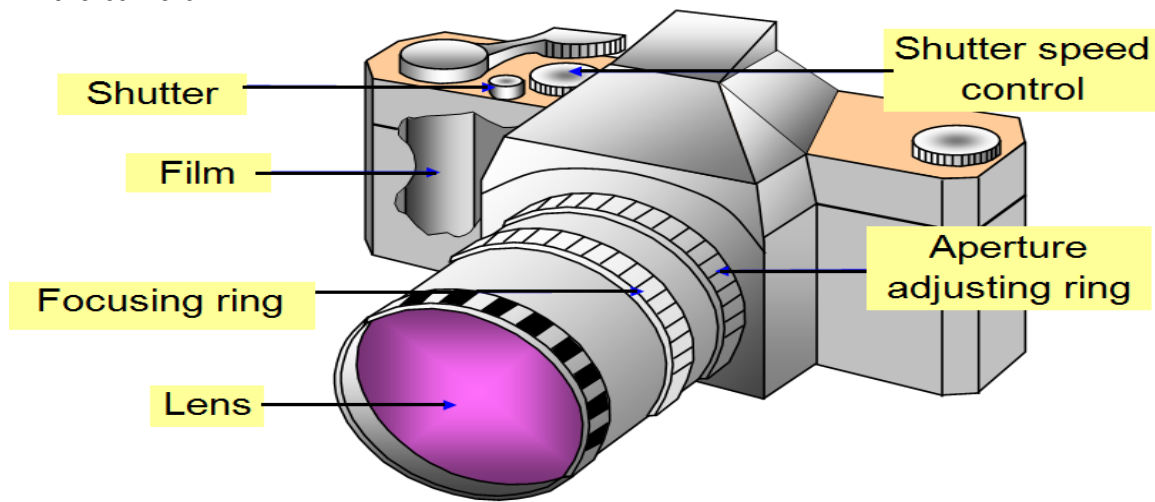
- The lens is the image-forming device on a camera.

Structure of a lens camera

The important parts of the camera are:

- The converging lens that forms** a real image of the object to be photographed. This image is diminished and inverted. The lens can be moved back and forward with the help of the focusing ring so that objects at different distances can be brought to focus. A focused image is located on the film or plate
- The shutter controls** whether light is allowed to reach the film or not. The speed of the opening and closing of the shutter is used to control the exposure time when taking a photograph.
- The diaphragm** which controls the amount of light falling on to the **film** per second by using more or less of the aperture of the lens. The aperture of the camera lens is usually described by the f-number or relative aperture. The larger the number the smaller the aperture. Aperture is used to controls the amount of light that will reach the camera's image sensor

(iv) **Film** which is a light – sensitive surface of the camera. It is normally rolled to the back of the camera



Parts of a lens Camera

Basic types of Lenses

- Normal (standard lens)
- wide angle lens
- telephoto (long-focus lens)
- Interchangeable lens

Normal (standard) Lens

- The viewing is much wider-about 50 degrees.
- The objects appear normal in size and shape, relative to the picture background

Wide Angle Lens

- The viewing is much wider-about 90 degrees.
- Used to make smaller objects look larger or to photograph large objects from close up.

Telephoto Lens

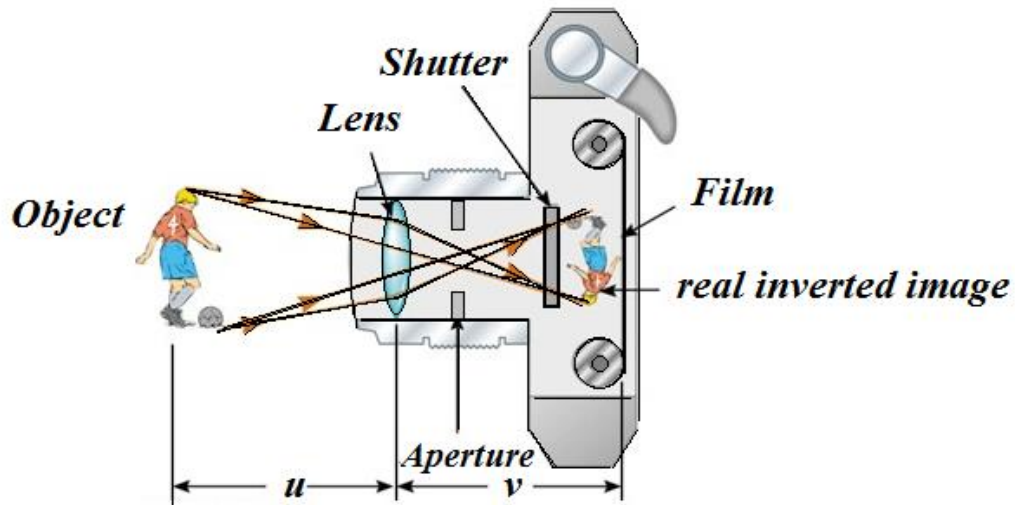
- It has wider fields of view than normal lenses.
- They show an enlarged detail of the image over the same film area.

Interchangeable lens

- It helps to offer the photography opportunity to select a focal length that is best for a given situation. In recent years variable focal lengths or “zoom” lenses have become very popular.

Mode of Action

- The image of the object must be sharply focused on the film by adjusting the distance of the lens from the film
- After focusing and correctly setting the aperture size and shutter time ,the click button is pressed
- The shutter opens to allow light to enter and expose the film to form an image of the object being photographed. The film is then developed to produce a photograph of the object



Magnification

Since magnification is given by

$$m = \frac{\text{image distance } (v)}{\text{object distance } (u)} = \frac{\text{image height } (h)'}{\text{object height } (h)'} ,$$

From; $m = \frac{v}{u}$, then $v = mu$, then substitute $v = mu$ into the lens formula; $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$, it follows that $\frac{1}{f} = \frac{1}{u} + \frac{1}{mu} = \frac{m+1}{mu}$ (reciprocate the two sides and make m the subject)

$$f = \frac{mu}{m+1} \rightarrow fm + f = mu \rightarrow f = mu - fm = m(u - f)$$

$$m(u - f) = f \rightarrow m = \frac{f}{u-f}$$

$$\text{hence; } m = \frac{f}{u-f}$$

therefore; the magnification produced by the lens camera depends on the focal length f of the convex lens used and the object distance u

Worked Examples

1. A lens camera of focal length 15 cm is used to take a picture of a man of height 1.8 m. If the man is standing 10 m ahead of the camera. Determine the:
 - (a) Magnification of the image
 - (b) size of the image

SOLN:

Given: $f = 15 \text{ cm} = 0.15 \text{ m}$, $H_o = 1.8 \text{ m}$, $u = 10 \text{ m}$, $m = ?$, $H_i = ?$

(a) From: $m = \frac{f}{u-f} \rightarrow m = \frac{0.15}{10-0.15} = \mathbf{0.0152}$

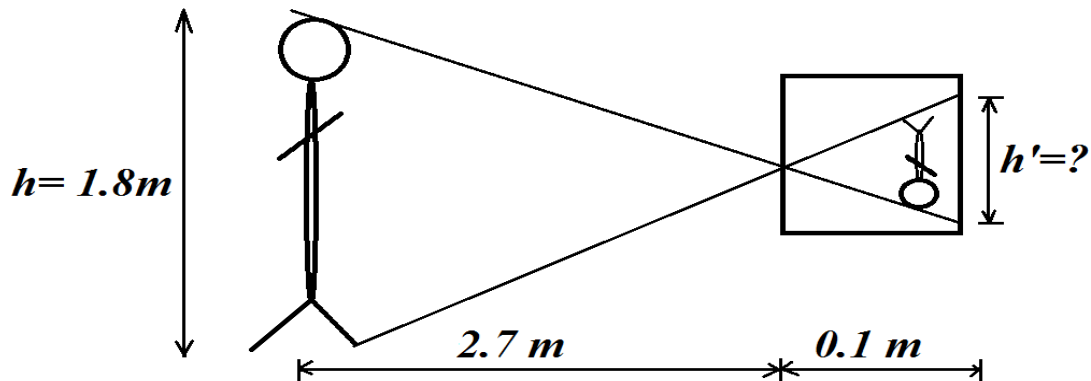
(b) Also; $m = \frac{H_i}{H_o} \rightarrow 0.152 = \frac{H_i}{1.8} \rightarrow H_i = 1.8 \times 0.0152 = 0.027\text{cm}$

\therefore height of the image formed = 2.7 m

2. A pin-hole camera is used to take a photograph of a student who is 1.8 m tall. The student stands 2.7 m in front of the camera. Film is placed 10 cm behind the pin-hole. Determine the height of the image produced.

ANS;

Consider a free body diagram below;



$$\text{From; } m = \frac{v}{u} = \frac{h'}{h} \rightarrow \frac{0.1}{2.7} = \frac{h'}{1.8} \rightarrow h' = \frac{1}{27} \times 1.8 = 0.067\text{m}$$

Therefore, the height of image produced is 6.7 cm

Class Activity – 5:6

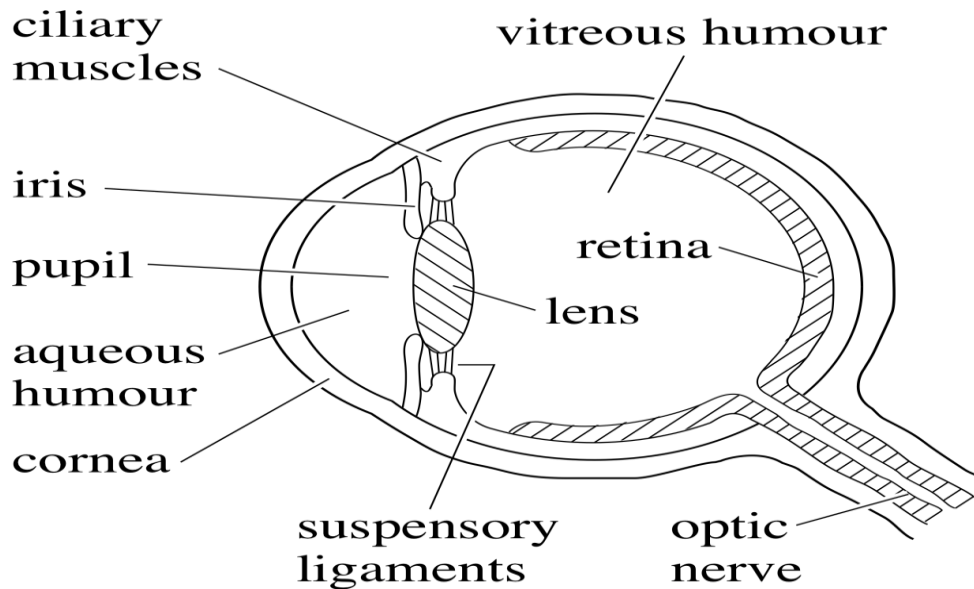
1. A lens camera of focal length 10 cm is used to take the picture of a girl 1.5 m tall. Determine the magnification of the image if the girl is 11m from the camera. ($m = 0.009$)
2. A lens camera has a lens of focal length 15 cm and a film (screen) of height 0.35 cm. How far would a boy of height 1.8 m stand from the camera so that his image just fits the film?
3. How far from the lens must the film in a camera be, if the lens has a 35.0-mm focal length and is being used to photograph a flower 75.0 cm away?
4. A camera lens used for taking close-up photographs has a focal length of 22.0 mm. The farthest it can be placed from the film is 33.0 mm.
 - (a) What is the closest object that can be photographed?
 - (b) What is the magnification of this closest object?
5. Suppose your 50.0 mm-focal length camera lens is 51.0 mm away from the film in the camera.
 - (a) How far away is an object that is in focus?
 - (b) What is the height of the object if its image is 2.00 cm high?
6. If you take a photograph of your image in a plane mirror, how many meters away should you set your focus if you are 2m in front of the mirror?
7. A professional photographer has a camera of focal length 2.5 cm. He uses it to take a photo of a tree of height 60 m. The distance between the lens of the camera and its film is 2.5 cm. Determine:
 - (a) Distance between the lens and the tree
 - (b) The height of the image
 - (c) The magnification of the camera

Uses of Lens Camera

- The sine or video camera is used to take motion pictures.
- High-speed cameras used to record movement of particles.
- Closed-circuit television cameras are used for surveillance in high-security
- Digital cameras are used to capture images

The Human Eye

- Is an optical device able to respond to an enormous range of light brightness
- It is able to focus on objects from billions of kilometers away to those a few centimeters away. It can also detect colour (See the fig. below)



Parts of the human Eye

1. Cornea

- The cornea is the transparent outer **'window'** of the eye, which covers the **iris and the pupil** at the front of the eye. The cornea, together with the lens, refracts or changes the direction of light to focus it on the retina.

2. Iris

- The iris is the coloured part of the eye which surrounds the pupil.
- It controls the amount of light going into the eye by changing the size of the pupil using tiny muscles to contract and dilate it. (It acts like diaphragm of the camera)

3. Pupil

- Your pupil is the black circle in the center of your iris. It regulates how much light enters your eye. **Interestingly**, the pupil appears black because most of the light entering the pupil is absorbed by the tissues inside the eye. When it is very light the pupil is small and when it is dark the pupil gets bigger to allow more light into the eye.
- The pupil functions in the same way as the aperture of a camera. The size of the pupil determines the amount of light entering the eye.

4. Lens.

- The lens is a transparent part of the eye behind the iris. The purpose of the **lens is to focus light onto the retina** at the back of the eye by bending light rays so that they form a clear image on the retina. The lens is elastic, so it is able to change shape to focus on to the retina. Its shape gets fatter to focus close objects and thinner to focus distant objects on the retina.
- In persons under 40 years of age, the lens is soft and flexible, allowing for fine focusing from a wide variety of distance

5. Retina

- The retina is a thin layer of nerve cells which contains millions of photoreceptors that form the light-sensitive inner lining at the back of the eye. Rays of light enter the eye and are focused

on the retina by the cornea and lens. The retina converts light into electrical impulses to produce an image which is sent along the optic nerve to the brain to interpret.

- There are two types of photoreceptors: **rods** work in low light and are responsible for night vision and **cones** work best in bright light and provide perceptions of colour and fine detail.
- The retina compares to the film in a lens camera.

6. Optic nerve

- This nerve carries electrical impulses from the rods and cones in the retina to the visual cortex in your brain. Without the optic nerve, your other eye components cannot send images to your brain and produce your sense of sight.

7. Sclera

- The sclera is more commonly known as the whites of your eyes. This fibrous layer contains collagen and protects the inner components of your eye from damage.

8. Vitreous humour

- The vitreous humour is a jelly-like substance that fills the body of the eye. It is normally clear
- Vitreous humour helps to maintain eye pressure and also helps in focusing light rays

9. The ciliary muscles:

- These control the thickness of the lens during focusing. By contracting or squeezing the lens, they make it thicker and vice versa. Because the power of the lens is directly related to its thickness, the ciliary muscles change the power of the lens by their movement.

10. Aqueous Humour:

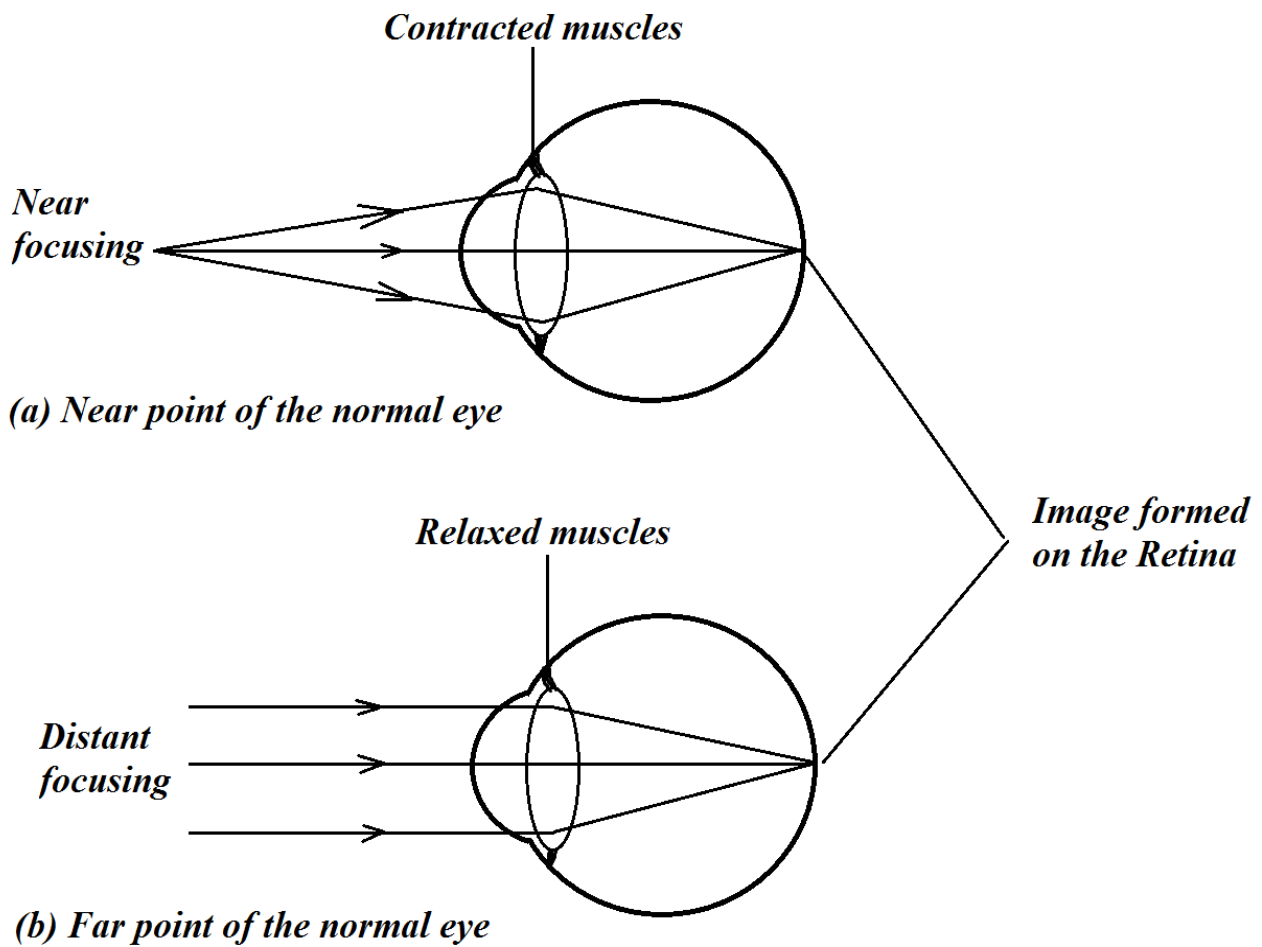
- The aqueous humour is a water-like fluid which fills the front of the eye between the lens and cornea and provides the cornea and lens with oxygen and nutrients.

How Human Eyes works (Mechanism of Vision)

- Light enters the eye by passing through the transparent **cornea** and **aqueous humor**. Some of this light enters the eye through an opening called the **pupil**. The iris controls the size of the **pupil**, which is the opening that allows light to enter the lens. Light is focused by the **lens** and goes through the **vitreous humor to the retina**.
- **Photoreceptors (Rods and cones)** in the **retina** translate the light into an electrical signal that travels from the **optic nerve** to the brain. Then the brain turns the signals into the images you see.

Accommodation of human eye

- The ciliary muscles adjust the shape of the eye lens for focusing on the nearby and far objects. By changing the shape of the eye lens the eye can change the focal length of the lens. This mechanism of the eye is called the **accomodation**.
- When the ciliary muscles are relaxed the lens is thin and has a longer focal length so that the **distant objects** are properly focused at the retina.
- **For a nearby object**, the eye needs a smaller focal length lens, which is achieved by contracting the ciliary muscles and thereby thickening the lens.
- **For a healthy human eyes**, visible range is **from 25 cm to infinity**.
- The distant of the closest object that a human eye can see is called the **near-point**. **For a normal eye**, the near point is located **25cm from the eye**.
- The distant of the farthest object that a human eye can see is called the **far-point**. **For a normal eye**, the far point is located at **infinity**.

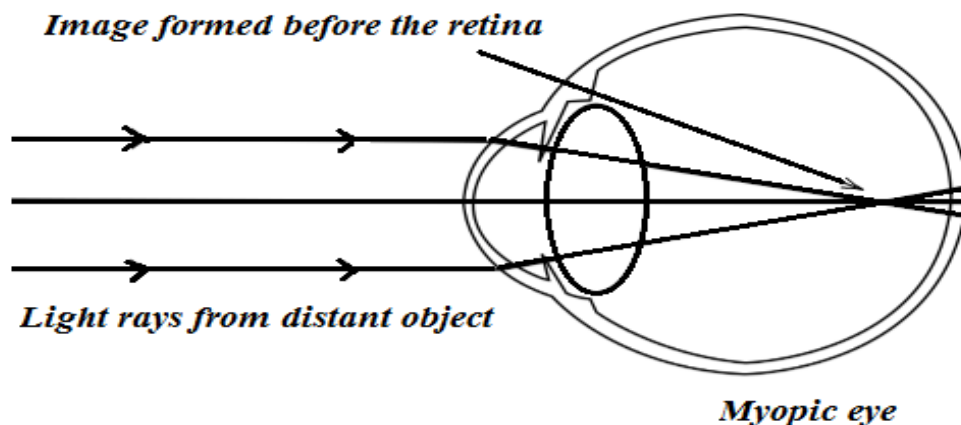


Eye Defects

- Many people have problems with their **vision**, or ability to see. Often, the problem is due to the shape of the eyes and how they focus light.
- Two of the most common vision problems are **nearsightedness (Myopia)** and **farsightedness (Hypermetropia/Hyperopia)**
- Therefore; Eye defect is the phenomenon where by eye defeat to see clearly.

Myopia

- Is a condition which occurs when a person can see near objects clearly but cannot see distant objects clearly (ie,. distant objects appear blurry).

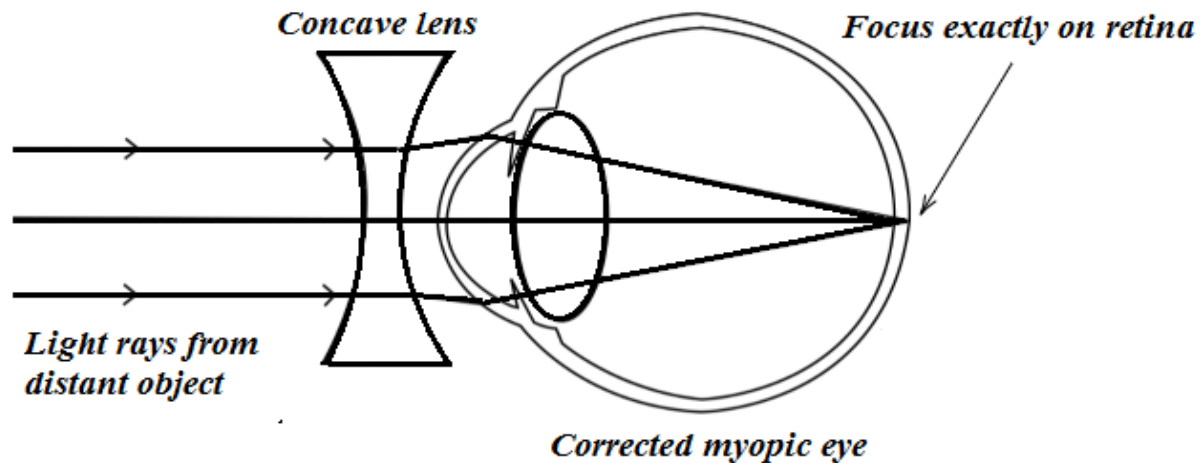


Causes

- When the eyeball is too long
- When the refractive power of the lens is too strong

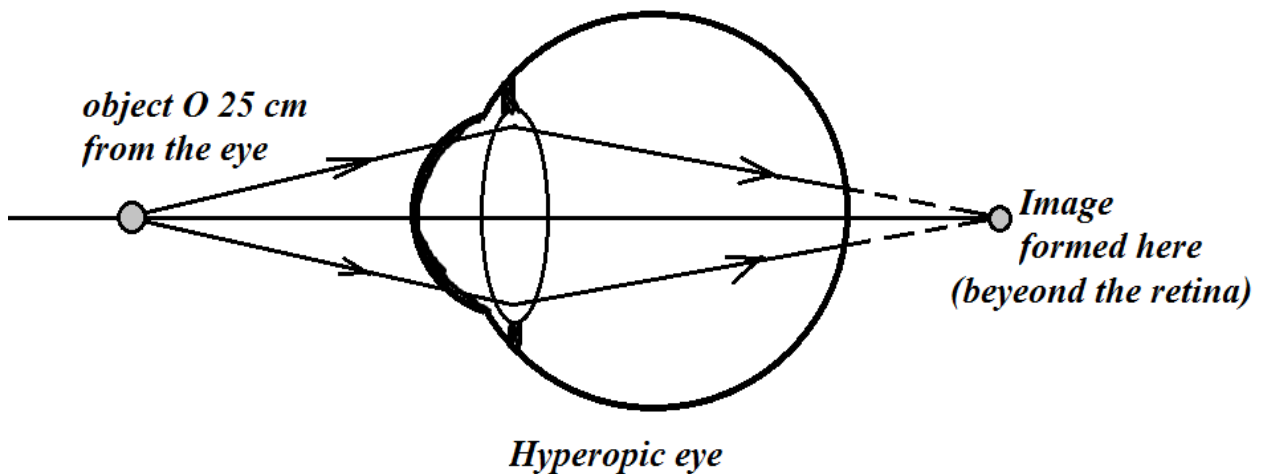
Correction

- To wear suitable concave lenses to diverge the rays from distant objects before they reach the eye.
- See the figure below



Hypermetropia/Hyperopia (Long – sightedness)

- Is a condition which occurs when a person cannot see near objects clearly (ie., nearby objects appear blurry) but can see distant objects clearly,

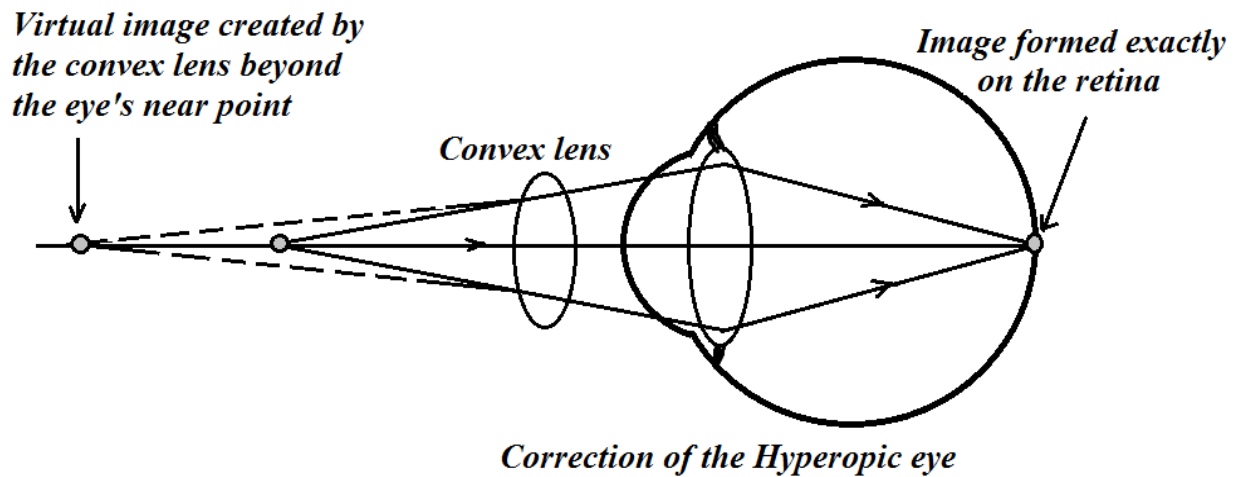


Causes

- When the eye ball is too short
- When ciliary muscles are weak such that unable to change the shape of the eye lens in order to focus the image (occurs when the refractive power of the eye lens is too **weak**)

Correction

- The defect can be corrected by wearing suitable convex lenses so that the rays from the near object are made to converge and focus on the retina. See the fig below



Other eye defects can be categorized into:-

Astigmatism

- Astigmatism is a condition caused by a refractive error, in which the eye does not focus light evenly on the retina. This results in **distorted or blurred vision at any distance**.
- Astigmatism is a common vision problem caused by a fault in the shape of the cornea, resulting in an irregular curve. This can change the way light passes through the cornea and refracts onto the retina. People with this condition have blurry, fuzzy, or distorted vision.

Presbyopia

- The eyes lose their power of accommodation with aging. As people grow old, the gradual weakening of the ciliary muscles and diminishing flexibility of the eye lens results in hardening of the eye lens, making it more difficult for the eye to focus on close objects. This causes the near point to gradually recede away in older people. These people may find it difficult to see nearby objects distinctly without corrective eyeglasses. This defect of farsightedness caused by the loss of elasticity of the lens of the eye is called **Presbyopia**.
- Sometimes, a person may suffer from both farsightedness and short-sightedness. Such people often require **bi-focal lenses**. A common type of bi-focal lens consists of both concave and convex lenses. The upper portion consists of a concave lens to assist distant vision. The lower part is a convex lens to assist near vision.

NB.

- **Optometrists** and ophthalmologists usually prescribe lenses measured in “**diopters**”
- The power, **P**, of a lens in diopters equals the inverse of the focal length in metres

$$\text{That is : } \text{Power} = \frac{1}{\text{Focal length (m)}} = \frac{1}{f}$$

Similarities of Human Eye and Lens Camera

- Both have a convex lens system to focus the image
- Both save images
- Both form a real, reduced and inverted image
- The amount of light entering is controlled by a variable aperture
- They both have surfaces on which the image is formed
- The retina behaves like the photographic screen of a camera

Differences between Lens Camera and Human Eye

Camera	Eye
Lens is hard glass	Lens is soft and elastic
Only the lens refracts the light	Aqueous and vitreous humour refracts the light
Cameras do not have blind spot	Human eyes have blind spot
Diaphragm controls the amount of light reaching the film	Iris adjusts the amount of light entering the eye through the pupil.
Focal length of the lens is fixed	Focal length of the lens can be changed
Focuses by changing the distance between the lens and film	Focuses by changing the thickness of the eye lens

Worked Examples

- The far point of a myopic person is 40 cm. What should be the power of the lens that he must use to see clearly?

Soln:

Given: $f = 40 \text{ cm} = 0.4 \text{ m}$, Power = ?

From: $P = \frac{1}{\text{Focal length}}$

$$\text{Power} = \frac{1}{\text{Focal length}} = \frac{1}{0.4} = \frac{10}{4} = 2.5$$

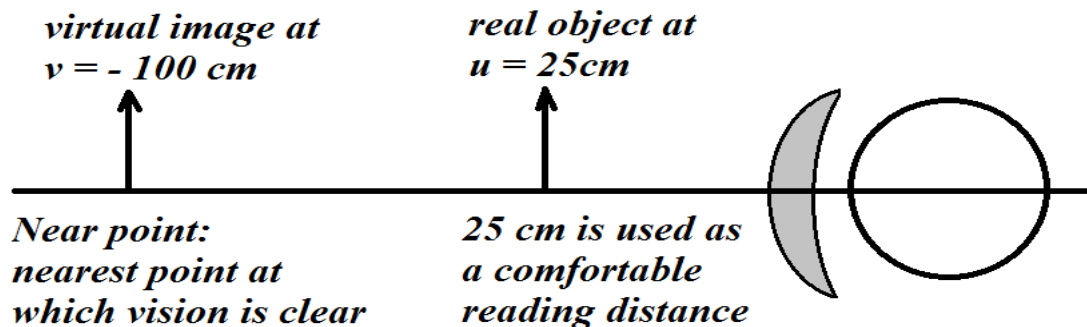
- A patient's eye can focus only on objects beyond 100 cm. What word characterizes this type of vision problem? What are the focal length and power of the contact lens needed to correct this problem?

ANS;

This patient is farsighted or hyperopic. This patient has trouble reading a book at a comfortable distance such as 25 cm, so we must find a lens that will take an object at 25 cm

(i. e, $u = 25 \text{ cm}$) and create an image (a virtual image) at 100 cm (ie, $v = 100 \text{ cm}$).

Consider the figure below;



- The image formed is a virtual image. Since It is on the left side of the lens as seen in the diagram above. **This means the image distance is negative.** Once we know u and v then it is easy to find the focal length of the lens from the lens formula,

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v} \rightarrow f = \frac{uv}{u-v} = \frac{25 \times -100}{25-100} = \frac{-2500}{-75} = 33.3 \text{ cm}$$

- The power of lens is given by; Power = $\frac{1}{\text{focal length (m)}}$

$$\therefore \text{Power} = \frac{1}{f} = \frac{1}{0.33} = 3 \text{ diopters}$$

3. A certain eye can focus only on objects closer than 50.0 cm. What word characterizes this type of vision problem? What sort of contact lens (described both in focal length and power) will correct this problem?

ANS; This eye is myopic or nearsighted. We need to find a lens that will take an object at infinity ($u = \infty$) and produce a virtual image at 50.0 cm ($v = -50\text{cm} = -0.5\text{m}$).

On solving using the lens formula; $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

$$\frac{1}{f} = \frac{1}{\infty} + \frac{1}{-50} \rightarrow \frac{1}{f} = \frac{1}{-50} \rightarrow f = v = -50\text{cm} = -0.5\text{m}, P = \frac{1}{f} = \frac{-1}{0.5} = -2 \text{ diopter}$$

Class Activity – 5:7

1. John has hyperopia. When is he more likely to need his glasses: when he reads a book or when he watches TV?

ANS:

With hyperopia, John is farsighted. He can probably see the TV more clearly than the words in a book because the TV is farther away. Therefore, he is more likely to need his glasses when he reads than when he watches TV.

2. Explain how lenses can be used to correct vision problems.
3. By mistake, an eye surgeon puts a concave lens in place of the lens in the eye after a cataract operation. Will the patient be able to see clearly any objects placed at any distance?
4. A student has difficulty reading the blackboard while sitting in the last row. What could be the defect the child is suffering from? How can it be corrected?
5. Which vision problem occurs with aging due to loss of elasticity of the eye lens?
6. **Sometimes squinting the eyes can help someone see more clearly. Why do you think this works?**

ANS: Squinting may improve focus by slightly changing the shape of the eyes. When you squint, you tighten muscles around the eyes, putting pressure on the eyeballs.

Self Assessment – 5

1. Given that the focal length of the simple microscope is 12cm. Find the magnification of the image of the object distances. (**ANS: $m = 8.1$**)
2. **If the focal distance of the converging lens is 5 cm the object distance is 4 cm. Find the magnification of the image** (**ANS: $m = 6 \text{ cm}$**)
3. **Given that an object 2m high is placed 2010cm in front of the lens camera of focal length 10cm. calculate the minimum size of the film frame.** (**ANS: $M = 0.0005$**)
4. The lantern projector uses a slide of 2 cm by 2 cm, 2x2 to produce a picture 1 m by 1m on a screen 12cm from the projection lens. How far from the lens must the slide be?
5. An astronomer telescope has its 2 lens 78 cm apart. If the objective lens has a focal length of 75.5 cm, what is the magnification produced by the telescope under normal vision
ANS: The magnification is 30.2
6. A compound microscope has an objective lens of focal length 2 cm and eye piece lens of focal length 6cm. An object is placed 2.4 cm from an objective lens. If the distance between the objective lens and eye piece lens is 19 cm. Find
(a) The distance of the final image from the eyepiece lens
(b) Compound magnification
7. A simple microscope has a focal length of 15 cm
(a) What is the maximum magnification of the lens (**ANS: 2.667**)
(b) What is the magnification of this lens when the eye is relaxed (**A: 1.667**)

8. The near point of a longsighted patient is 90 cm
 - (a) Determine the focal length of a lens that can be used to enable the patient clearly see objects that are 25 cm from the eye
 - (b) What is the power of the lens
 - (c) What is the magnification of the lens?
9. A short sighted person is unable to clearly see objects that are beyond 150 cm from the eye .Determine the focal length ,power and magnification of the lens that should be used to detect the eye defect
10. A patient requires a lens of -5 diopters in order to see far away objects clearly .Determine the
 - (a) Focal length of the lens used
 - (b) Far point of the patient 's eye
11. Define the term "accommodation" as used in the human eye
12. Give two similarities and 2 differences between the human eye and the camera
13. State one advantage of human eye over a lens camera
14. The near point of an eye is 50 cm
 - (a) What focal length lens should be used so that the eye can clearly see an object 25 cm away?
 - (b) What is the power of this lens? (ANS: (a) $f = 50\text{cm}$ (b) $P = 2$ diopters)
15. (a)State the causes of the short – sightedness and long – sightedness
 - (b) Use a ray diagram to show how
 - i. Short – sightedness in a human eye can be corrected
 - ii. (ii) Long – sightedness can be corrected
16. A farsighted man has a near point of 100 cm .Wearing his glasses, he can see objects that are 25 cm away .What is the focal length of the lens in his glasses?
17. In a compound microscope ,the focal length of the objective lens is 4.0 cm and that of the eyepiece is 3.3 cm and they are placed at a distance of 15.0 cm .A real object of size 2 mm is placed 6 cm from the objective lens .By using the lens formula , Calculate
 - (a) Position of the final image
 - (b) The size of the final image viewed by the eye
 - (c) Magnification produced by the arrangement of the lenses
18. Small object is placed 3cm from the lens of a simple microscope .If the focal length of the lens is 5 cm ,Find the linear magnification produced by the simple microscope .How far from the lens would you place the object in order to obtain maximum magnification of the image ?
19. A magnifying glass of focal length of 15 cm is used to view an object so as to obtain maximum magnification
 - (a) Where should the object be placed? [ANS;
 - (b) What is the magnification of the magnifying lens? [ANS; 2.67]
20. A compound microscope has an objective lens of focal length 25 cm and an eyepiece with focal length 14 cm, If it has a tube of length 36 cm determine the magnification of the microscope
21. A lens of focal lens 10 cm is positioned from an object so as to obtain maximum magnification of the image .Determine the:
 - (a) Object distance
 - (b) Image distance
 - (c) Magnification
22. A lens camera is to be used to take a picture of a man 2 m tall .If the lens of the camera has a focal length of 10 cm , Calculate the minimum size of the film frame required ,given that the man is 20.1 m from the camera
23. A certain microscope consists of two converging lenses of focal lengths 10 cm and 4 cm for the objective and eye piece respectively .The two lenses are separated by a distance of 30 cm .The instrument is focused so that the final image is at infinity .Calculate the position of the object and the magnification of the objective lens (ANS $u = 16.25\text{ cm}$, $m = 1.6$)
24. Draw a clearly labeled diagram of a lens camera and explain briefly how the image of an object is focused on the film .A camera with a lens of focal length 15 cm is used to take a

- photograph of a man standing 4.5 m from the lens .Find the length of the image formed if the man is 1.75 m tall**
25. Describe with the aid of a diagram the optical system of the projection lantern .A projection lantern is to be used for the projection of slides measuring 7.5 cm by 7.5 cm onto a screen measuring 2.1 m by 2.1 m .If the distance between the projection lens and the screen is 6 m, Find the focal length of the lens
 26. Describe how would you construct an astronomical telescope. What will be the maximum distance between the objective lens and the eyepiece lens?. Draw a ray diagram to illustrate the paths of two rays from a point on an object which is not on the axis of the telescope
 27. Define and explain the terms near point and far point as applied to the human eye .Draw diagrams to illustrate the defects of long and short sight .How can each defect be corrected by using lenses ?
 28. **A man whose least distance of distinct vision of 75 cm wants spectacles to allow him to read a book held at a distance of 25 cm from his eyes. Find the focal length of the lens he needs .Discuss briefly whether a short – sighted person can use a telescope without wearing any spectacles**
 29. Draw a clearly labelled diagram of the human eye and explain how it can focus near and distant objects. How does the human eye resemble the lens camera?
 30. A telescope of 5.0 m diameter reflector of focal length 18.0 m is used to focus the image of the sun. Using the distance of the sun from the earth and diameter of the sun as 1.5×10^{11} m and 1.4×10^9 m respectively, calculate the:
 - (a) Position of the image of the sun ($v = f = 18$ cm, since the object is at infinity)
 - (b) Diameter of the image of the sun(image size) ($m = \frac{v}{u} = \frac{h_i}{h_o} \rightarrow d_i = 16.8$ cm)
 31. **A person whose sight is normal wishes to view objects which are 5 cm from his eyes. Find the focal length of the lenses needed for his spectacles**
 32. When is a person said to be suffering from long sight? Draw a diagram of the eye to show how this defect may be corrected by the use of a suitable type of lens.
 33. Mention two ways in which a photographic camera is similar to the human eye and one way in which it is different
 34. A converging lens has a focal length of 5 cm (**ANS: P = 20 D**)
 - (i) What is the power of the lens?
 - (ii) If this lens were used in an astronomical refracting telescope, for which part of the telescope would it be most suitable?
 - (iii)What would be the distance between the two lenses if the telescope were in normal adjustment (i.e with the final image at infinity)?

(ANS :($l = f_o + f_e$) \rightarrow sum of focal lengths)
 35. Explain the terms real image and virtual image
 36. Describe the construction of a photographic camera. If the focal length of the camera lens is 20 cm, how far away from the film must the lens be set in order to photograph an object 100 cm from the lens? (**ANS: v = 25 cm**)
 37. A slide projector using a slide 5 cm x 5 cm produces a picture 3 m x 3 m on a screen placed at a distance of 24 m from the projection lens. How far from the lens must the slide be?
(**ANS: 40 cm x 40 cm**)
 38. In a compound microscope, why should the objective lens form an image at a point around the focal point of the eyepiece?
 39. Given two converging lenses of focal lengths 50cm and 5 cm, indicate (on a ray diagram) how the lenses would be arranged, with other accessories, to obtain a magnified image of a distant object which is not on the axis of the lens system

Chapter: 06

Thermal Expansions

Most substances expand when heated and contract when cooled. When a body's temperature changes, its dimensions change as well. **The term “thermal expansion”** refers to the expansion of a body's size as its temperature rise.

Terms used

- ✓ **Expansion** is the process whereby object increases its volume due to increase in temperature
- ✓ **Contraction** is the process whereby object decreases its volume due to decrease in temperature

Sources of thermal Energy

The sun (the sun generates its energy by nuclear fusion)

- Solar thermal radiation is a process that emits thermal energy from the Sun, in the form of electromagnetic waves that can travel freely through space, without any intervening medium required for conduction of energy from one point to another
- The source of solar thermal energy itself is nuclear hydrogen radioisotopes, which are abundant in the Sun. These isotopes are constantly undergoing fusion reactions that lead to the formation of helium molecules with release of tremendous amount of radiant heat.

Fossil fuels

- Fossil fuels emit thermal energy through the process of combustion, whereby a fuel is ignited and burnt in the presence of oxygen, with release of significant amounts of heat.
- Examples of fossil fuels that produce thermal energy are; natural gas, petroleum and coal.

Nuclear energy (is energy generated from nuclear reactions)

- Nuclear energy originates from the splitting of uranium atoms – a process called fission. This generates heat to produce steam, which is used by a turbine generator to generate electricity.

Geothermal energy (the heat energy delivered from the earth core)

- The process of **geothermal energy** production commences with nuclear fission as radioisotopes in the Earth's core undergo radioactive decay.
- Geothermal processes include nuclear fission, thermal conduction and fluid convection.
- The two **types of thermal energy transfer** that occur in geothermal systems, as indicated above, are conduction and convection. These mechanisms help to circulate geothermal heat in the subsurface, and are also responsible for the extraction of this heat and its use by humans.
- Thermal energy from *geothermal* processes and systems can be utilized to **generate electricity**, as well as for **domestic heating and cooling purposes**.

Why Substance expands?

- Substance expands when heated because its particles vibrate more rapidly. As a result they collide and push each other further apart

- All states of matter (solids, liquids and gases) expand when heated.

Explain what happen when solids (liquids or gases) are heated?

- When a solid is heated, its molecules gain kinetic energy and vibrate more vigorously. As the vibration become larger, the molecules are pushed further apart and the solid expands slightly in all directions.

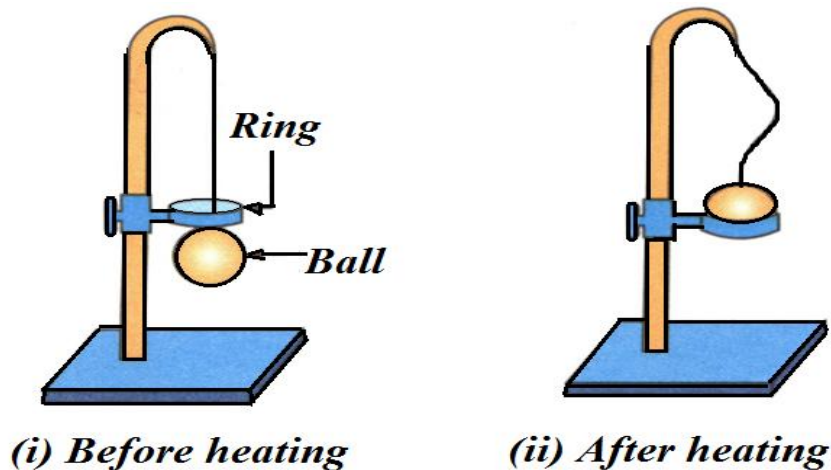
Explain what happen when solids (liquids or gases) are cooled?

- When solid is cooled, its molecules lose kinetic energy and have less vibration. As the vibration become lower, the molecules are pulled closer and the solid contracts slightly in all directions

Thermal Expansion of Solids

- The expansion of solid substance is so small such that it is difficult to observe its changes
- As the temperature of a solid increases, the atoms vibrate with large amplitudes and the average separation between them increases. As a result, the entire solid occupies a large volume as the temperature increases.

This can be seen in the ball and ring experiment as shown in the fig. below



Observation and Explanations

- The metal ball can just pass through the ring at room temperature
- On heating, the metal ball expands. There is an increase in volume and the ball cannot pass through the ring
- On cooling , contraction occurs and the original volume is regained .The ball can now pass through the ring

Linear expansivity (Coefficient of linear expansion)

- Is the ratio of increase in length to its original length per degree rise in temperature
- **OR** – Is the increase in length per unit length of the substance when its temperature rises by 1°C or 1 K.
- The SI unit for linear expansivity is K⁻¹
- Mathematically: **Linear expansivity** = $\frac{\text{increase in length (expansion)}}{\text{original length} \times \text{rise in temperature}}$ $\rightarrow \alpha = \frac{\Delta l}{l_1 \times \Delta \theta}$

Where:

α = Linear expansivity

$\Delta \theta = (\theta_2 - \theta_1)$ = rise in temperature

θ_2 = initial temperature, θ_1 = final temperature

$\Delta L = (L_2 - L_1) = \text{increase in length}$
 $L_1 = \text{original length, } L_2 = \text{new length}$

Linear expansivities of different substances.

Substance	Linear expansivity (K^{-1}) $\times 10^{-6}$	Substance	Linear expansivity (K^{-1}) $\times 10^{-6}$
Aluminium	26	Steel	11
Brass	19	Glass	8.5
Copper	17	Pyrex glass	3.0
Iron	10.2	Invar	0.9
Diamond	1	Silica	0.42

- **The knowledge of linear expansivity** is used in designing various materials to ensure that they are able to operate well under varying thermal conditions. For instance ordinary glass has a higher linear expansivity than a pyrex glass. When hot water is put in an ordinary glass, it breaks but when a pyrex glass is used it does not crack. The pyrex glass has lower linear expansivity and cannot suffer very large forces of expansion while the ordinary glass does as it undergoes temperature changes.
- In building and construction, concrete is always reinforced using steel because both have the same linear expansivity.

Worked examples

1. The main span of San Francisco's Golden Gate Bridge is 1275 m long at its coldest. The bridge is exposed to temperatures ranging from -15°C to 40°C . What is its change in length between these temperatures? Assume that the bridge is made entirely of steel.

ANS; $L_1 = 1275\text{m}$, $\alpha_{\text{steel}} = 11 \times 10^{-6} K^{-1}$, $\Delta T = 40 + 15 = 55\text{K}$

From;

$$\text{Linear expansivity} = \frac{\text{increase in length}}{\text{original length} \times \text{rise in temperature}} \rightarrow \alpha = \frac{\Delta l}{l_1 \times \Delta T}$$

$$\alpha_{\text{steel}} = \frac{\Delta l}{l_1 \times \Delta T} \rightarrow \Delta L = \alpha_{\text{steel}} \times L_1 \times \Delta T$$

$$\therefore \Delta L = \alpha_{\text{steel}} \times L_1 \times \Delta T = 11 \times 10^{-6} \times 1275 \times 55 = 0.77\text{m}$$

2. A rod is heated to 50°C to increase its length from 20 m to 30 m. Calculate the expansion coefficient if the room temperature is 20°C .

ANS;

$$\text{Linear expansivity} = \frac{\text{increase in length}}{\text{original length} \times \text{rise in temperature}} \rightarrow \alpha = \frac{\Delta l}{l_1 \times \Delta T}$$

$$\text{Therefore; } \alpha_{\text{Rod}} = \frac{\Delta l}{l_1 \times \Delta T} = \frac{30-20}{20 \times (50-20)} = \frac{10}{600} = 1.67 \times 10^{-2} K^{-1}$$

3. A rod is heated to 30°C to increase its length by 15 m. Calculate the initial length if the expansion coefficient is $0.02 K^{-1}$ for a room temperature of 10°C .

ANS; $l_1 = ?$, $\Delta l = 15\text{m}$, $\alpha = 0.02 K^{-1}$, $\Delta T = 30 - 10 = 20^{\circ}$

$$\text{From; Linear expansivity} = \frac{\text{increase in length}}{\text{original length} \times \text{rise in temperature}} \rightarrow \alpha = \frac{\Delta l}{l_1 \times \Delta T}$$

$$0.02 = \frac{15}{l_1 \times 20} \rightarrow \therefore l_1 = \frac{15}{0.02 \times 20} = \frac{15}{0.4} = 35\text{m}$$

4. An iron rod A and B are of equal length at 0°C . If at 100°C they differ by 1mm find their lengths at 0°C . Given that; $\alpha_A = 8 \times 10^{-6}$, $\alpha_B = 12 \times 10^{-6} K^{-1}$

ANS: L = 2.5 m

At 0°C ; $l_{1A} = l_{1B} = x\text{mm}$

At 100°C ; $l_{2B} - l_{2A} = 1\text{mm}$ ----- (i)

But; $\alpha_A = \frac{\Delta l}{l_1 \times \Delta T} \rightarrow l_{2A} = \alpha_A \times l_1 \times \Delta T + l_1 = (\alpha_A \times \Delta T + 1)l_1$ ---- (ii)

$\alpha_B = \frac{\Delta l}{l_1 \times \Delta T} \rightarrow l_{2B} = \alpha_B \times l_1 \times \Delta T + l_1 = (\alpha_B \times \Delta T + 1)l_1$ ---- (iii)

Substitute the values of l_{2A} and l_{2B} into $l_{2B} - l_{2A} = 1$

$(\alpha_B \times \Delta T + 1)l_1 - (\alpha_A \times \Delta T + 1)l_1 = 1$

$(12 \times 10^{-6} \times 100 + 1)l_1 - (8 \times 10^{-6} \times 100 + 1)l_1 = 1$

$1.000012l_1 - 1.000008l_1 = 1 \rightarrow 0.000004l_1 = 1$

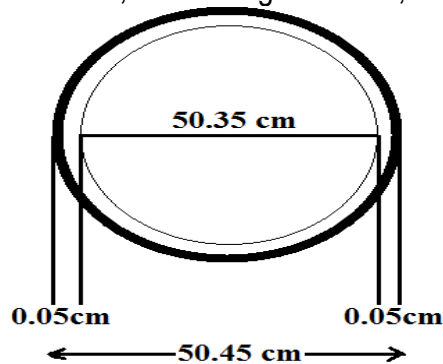
$l_1 = \frac{1}{0.000004} = 2500\text{mm}$; Therefore; their lengths at 0°C is 2500mm (2.5m)

5. An iron tyre of diameter 50 cm at 15°C is to be shrunk on to a wheel of diameter 50.35cm. To what temperature must the tyre be heated so that it will slip over the wheel with radial gap of 0.5 mm? (Linear expansivity of iron is $1.2 \times 10^{-5}\text{K}^{-1}$)

ANS;

The iron tyre has a diameter, D_1 of 50 cm at $T_1=15^{\circ}\text{C}$, and that diameter must increase to 50.35 cm plus 2 radial clearances of 0.5mm (or 0.05 cm) each, for a new diameter,

$D_2 = 50.35\text{ cm} + 2 \times 0.05\text{cm} = 50.45\text{ cm}$, see the figure below;



From; $\alpha = \frac{\Delta D}{D_1 \times \Delta T} \rightarrow \Delta T = \frac{\Delta D}{D_1 \times \alpha} = \frac{50.45 - 50}{50 \times 1.2 \times 10^{-5}} = 750^{\circ}\text{C}$

But; $T_2 = T_1 + \Delta T = 15 + 750 = 765^{\circ}\text{C}$

Therefore, 765°C is the temperature at which the tyre must be heated so that it will slip over the wheel of radial gap 0.5mm

Class Activity – 6:1

1. A rod is heated to 30°C to increase its length from 10 m to 25 m. Calculate the expansion coefficient if the room temperature is 10°C . **ANS; 0.075K^{-1}**
2. Calculate the expansion of a 15m copper pipe, when it is heated from 5°C to 60°C , if the linear expansivity of copper = 0.000012K^{-1} **[ANS; $\Delta L = 0.0099\text{m}$]**
3. A block of concrete 5.0 m long expands to 5.00412 m when heated from 25°C to 100°C . Determine the linear expansivity of concrete. (**ANS: $\alpha = 1.1 \times 10^{-5}\text{ }^{\circ}\text{C} = 1.1 \times 10^{-5}\text{ K}^{-1}$**)
4. The length of a wire at a temperature of 30°C is 1.002m. If the temperature of the wire is raised to 105°C and the linear expansivity of the wire is $1.89 \times 10^{-5}\text{K}^{-1}$, find the increase in the length of the wire. **[ANS; $\Delta L = 1.42 \times 10^{-3}\text{m}$]**
5. The difference in length between a brass and an iron rod is 14 cm at 10°C . What must be the length of the iron for this difference to remain at 14 cm when both rods are heated to 100°C ? Given that the linear expansivity of brass = $19 \times 10^{-6}/\text{K}$ and iron = $12 \times 10^{-6}/\text{K}$. (**A: L = 38 cm**)
6. A metal rod has a length of 99.4cm at 200°C . At what temperature will its length be 100cm, if the linear expansivity of the metal is 0.000021K^{-1} **[ANS; 485.6°C]**

7. A part of a steel tape used by a surveyor is 20.00m at 12^o. What is the overall length measured by using this part of the tape one hundred times on a warmer day corresponding to 22^o? [$\alpha_{\text{Steel}} = 1.1 \times 10^{-5}$]
ANS; 2000.22m]
8. A metal rod has a length of 100 cm at 200^o C. At what temperature will its length be 99.4 cm if the linear expansivity of the material of the rod is 0.00002/K (**ANS: $T_2 = T_1 + \Delta T = - 100^{\circ} \text{ C}$**)
9. A metal pipe which of 1m long at 40^oC increases in length by 0.3% when carrying a steam at 100^oC. Find the Coefficient of Linear Expansion (**ANS: $\alpha = 5 \times 10^{-5} \text{ K}$**)
10. A brick (30 cm x 18 cm x 10 cm) is at 20^oC, If the brick heated to a temperature of 150^oC, what will be its new dimensions? (The coefficient of linear expansion of concrete is $1.2 \times 10^{-5} \text{ K}^{-1}$) (**ANS: 30.05 cm x 18.03 cm x 10.02cm**)
11. An iron plate at 20^oC has a hole of radius of 8.92 mm in the centre, an iron rivet with radius of 8.95 mm at 20^oC, inserted into the hole. To what temperature the plate heated for the rivet to fit into the hole. (Linear expansivity of iron is $1.24 \times 10^{-5} \text{ K}^{-1}$). **ANS: 291^oC**
12. Which is heavier, 1 dm³ of glass at 4^o C or 1 dm³ of glass at 10^o C? Explain your answer.
13. A concrete railroad tie has a length 2.45m on a hot sunny day having a temperature of 35^oC. What is the length of the railroad tie in the winter when the temperature dips to -25^oC. Take the coefficient of linear expansion of railroad as $12 \times 10^{-5} / ^{\circ} \text{C}$
[ANS; $\alpha = \frac{\Delta l}{l_1 \times \Delta T}$ $\Delta l = \alpha \times l_1 \times \Delta T = -0.0018\text{m} \rightarrow l_2 = l_1 + \Delta l = 2.4482\text{m}$]
14. **Does it really help to run hot water over a tight metal lid on a glass jar before trying to open it? Explain your answer**
ANS; Yes. You can try to open a tight metal lid of a glass jar just by pouring hot water over it. The thermal expansion helps us to loosen the metal lid. Because it is the process in which the metal lid expands in the presence of heat therefore makes it easier for us to open it.
15. How much gap should be left between two 20m tracks made of steel if it is laid at 22^oC and is to operate up to a temperature of 47^o C?
ANS; Each track will expand half of the gap on each side. Hence the total expansion will be equal to the gap, ie., $\Delta l = \alpha \times l_1 \times \Delta T = 5.9 \times 10^{-3} \text{ m}$
16. Eiffel tower is made up of iron and its height is roughly 300 m. During winter season (January) in France the temperature is 2^oC and in hot summer its average temperature 25^oC. Calculate the change in height of Eiffel tower between summer and winter. The linear thermal expansion coefficient for iron $\alpha = 10 \times 10^{-6}$ per ^oC **[ANS; 69cm]**
17. A rod is found to be 0.04 cm longer at 30^o C than it is at 10^o C. Calculate its length at 0^o C if coefficient of linear expansion, $\alpha = 2 \times 10^{-5} / ^{\circ} \text{C}$. **[ANS; 1m or 100cm]**
18. A brass rod and an iron rod are each 1m in length at 10^o C. Find the difference in their lengths at 110^o C. α for brass is $19 \times 10^{-6} / ^{\circ} \text{C}$ and α for iron is $10 \times 10^{-6} / ^{\circ} \text{C}$
[A; 0.9mm or 0.0009m] .

Superficial expansion of solids (Areal expansion)

- is the ratio of increase in area to its original area for every degree increase in temperature
- If the surface area of a **solid increases on being heated**, it is called **Area or superficial expansion**. If the area of a solid is A_1 at 0^oC, then the area of that solid at T^oC is given by;
 $\Delta A = \beta A_1 \Delta \theta$

Area expansion on temperature change



- Whereby β is the coefficient of areal expansion
- Therefore; $\beta = \frac{A_2 - A_1}{A_1 \times \Delta\theta}$

Relationship between coefficient of linear expansion and Areal expansion for a solid

- For isotropic materials, and for small expansions, the linear thermal expansion coefficient is one half of the area coefficient. To derive the relationship, let's take a square of steel that has sides of length L . The original area will be $A = L^2$, and the new area, after a temperature increase, will be

$$\begin{aligned}
 A + \Delta A &= (L + \Delta L)^2 = L^2 + 2L\Delta L + \Delta L^2 \\
 &= L^2 + \frac{2L^2\Delta L}{L} + \Delta L^2 \\
 &\approx L^2 + \frac{2L^2\Delta L}{L}, \quad \text{as } \Delta L^2 = (\alpha \Delta\theta L)^2 \approx 0 \\
 &= L^2 + \frac{2L^2\Delta L}{L} = A + 2A \frac{\Delta L}{L} \\
 &= A + 2A \frac{\Delta L}{L} = A + 2A \frac{\alpha L \Delta\theta}{L} = A + 2A \alpha \Delta\theta
 \end{aligned}$$

$$A + \Delta A = A + 2A \alpha \Delta\theta \rightarrow \Delta A = 2A \alpha \Delta\theta$$

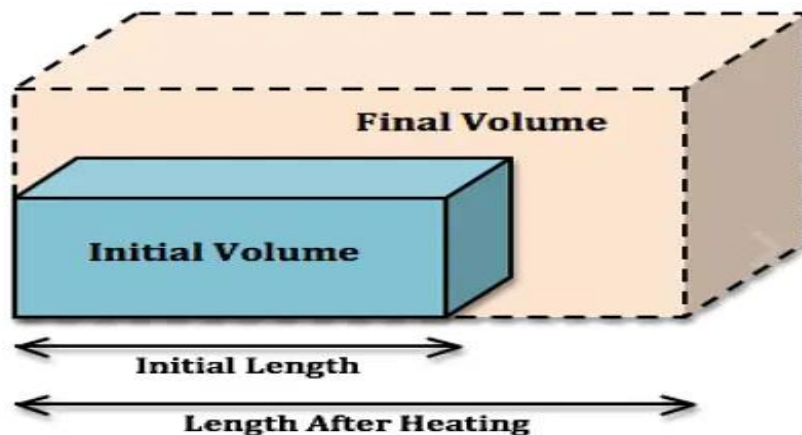
$$\text{But; } \Delta A = \beta A \Delta\theta \rightarrow \beta(A \Delta\theta) = 2 \alpha (A \Delta\theta) \rightarrow \beta = 2 \alpha$$

- Therefore, coefficient of areal expansion is twice the coefficient of linear expansion
ie., $\beta = 2 \alpha$

Volume expansion and expansivity of solids

- When a solid **increases in volume**, on being heated, it is called **volume expansion**. For an initial volume V , at $\Delta\theta$ change in temperature, volume expansion is given by $\Delta V = \gamma V_1 \Delta\theta$

- Whereby γ is the coefficient of volume expansion, Therefore; $\gamma = \frac{V_2 - V_1}{V_1 \times \Delta\theta}$



Volume expansion on temperature increase

Relationship between coefficient of linear expansion and volume expansion for a solid

- For isotropic material, and for small expansions, the linear thermal expansion coefficient is one third the volumetric coefficient. To derive the relationship, let's take a cube of steel that has sides of length L . The original volume will be $V=L^3$, and the new volume, after a temperature increase, will be:

- $$\begin{aligned}V + \Delta V &= (L + \Delta L)^3 = L^3 + 3L^2\Delta L + 3L(\Delta L)^2 + (\Delta L)^3 \\&= L^3 + \frac{3L^3\Delta L}{L} + 3L(\Delta L)^2 + (\Delta L)^3 \\&\approx L^3 + \frac{3L^3\Delta L}{L}, \quad \text{as; } 3L(\Delta L)^2 \approx 0, \Delta L^3 \approx 0 \\&= L^3 + 3L^2\Delta L = V + 3V\frac{\Delta L}{L} \\&= V + 3V\frac{\Delta L}{L} = V + 3V\frac{\alpha L\Delta\theta}{L} = V + 3V\alpha\Delta\theta\end{aligned}$$

$$V + \Delta V = V + 3V\alpha\Delta\theta \rightarrow \Delta V = 3V\alpha\Delta\theta$$

$$\text{But; } \Delta V = \gamma V\Delta\theta \rightarrow \gamma(V\Delta\theta) = 3\alpha(V\Delta\theta) \rightarrow \gamma = 3\alpha$$

- Therefore; the coefficient of cubical expansion is three times the coefficient of linear expansion *ie*, $\gamma = 3\alpha$

Class Activity – 6:2

- A rod is heated to 40°C to increase its area from 50 sq. m to 100 sq. m . Calculate the expansion coefficient if the room temperature is 25°C . **[ANS; 0.067K^{-1}]**
- The coefficient of linear expansivity of a metal is $2.7 \times 10^{-5}\text{K}^{-1}$. If its original area is 600mm^2 , what will be the change in its area if its temperature changes from 60°C to 80°C ?
ANS; $\beta = 2\alpha = 2 \times 2.7 \times 10^{-5} = 5.4 \times 10^{-5}$, $\Delta A = 0.64\text{mm}^2$
- A rod is heated to 30°C to increase its area by 40 sq. m . Calculate the initial area if the expansion coefficient is 0.05 K^{-1} for a room temperature of 10°C . **[ANS; 40sq]**
- An engineer has one piece of metal plate only to be used in fixing a gap on a metal bridge. If the steel plate has area of 0.8m^2 at 25°C , by how much must the engineer raise the temperature of the steel plate for it to fit a gap of 0.804m^2 ? use coefficient of linear expansion of steel as $12 \times 10^{-5}\text{K}^{-1}$ **[ANS; $\Delta T = 208^\circ\text{C}$]**
- A rod is heated to 40°C to increase its volume from 200 cu.m to 300 cu.m . Calculate the expansion coefficient if the room temperature is 20°C . **[ANS; 0.025K^{-1}]**
- By how much will a steel rod 1 m long expand when heated from 25°C to 55°C ? The coefficient of volume expansion of steel is $3 \times 10^{-5}/^\circ\text{C}$.

$$[\text{A; } \gamma = 3\alpha \rightarrow \alpha = 1 \times 10^{-5}\text{K}^{-1}, \rightarrow \alpha = \frac{\Delta l}{l_1 \times \Delta\theta} \therefore \Delta l = 0.0003\text{m} (0.3\text{mm})]$$

- The diameter of an aluminium sphere at 20°C is 2.5cm . If its diameter increases to 2.51cm when heated, what is the final temperature? the coefficient of linear expansion of aluminium is $2.6 \times 10^{-5}\text{K}^{-1}$

$$\text{ANS; } \gamma = 3\alpha = 7.8 \times 10^{-5}\text{K}^{-1}. \Delta V = V_2 - V_1 = \frac{4}{3}\pi(R_2^3 - R_1^3) = 0.1\text{cm}^3$$

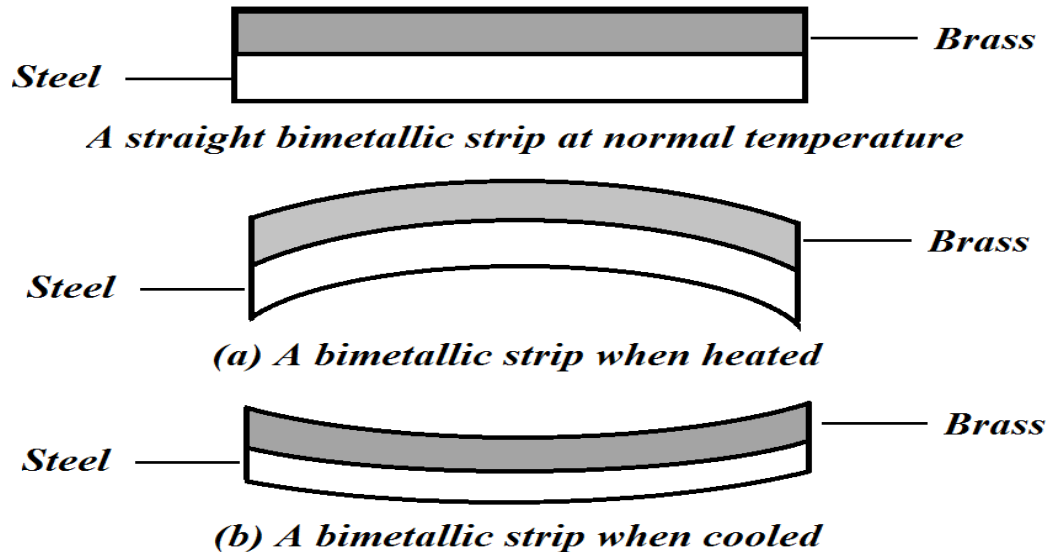
$$V_1 = \frac{4}{3}\pi R_1^3 = 8.18\text{cm}^3 \quad \Delta\theta = \frac{\Delta V}{\gamma V_1} = 156.7^\circ\text{C}, \therefore \theta_2 = \theta_1 + \Delta\theta = 176.7^\circ\text{C}$$

- The coefficient of linear expansion of iron is 0.000012K^{-1}

- Explain the meaning of this statement.
- Calculate the superficial expansion of the iron.
- Determine the cubical expansion of the iron.

The Bimetallic Strip

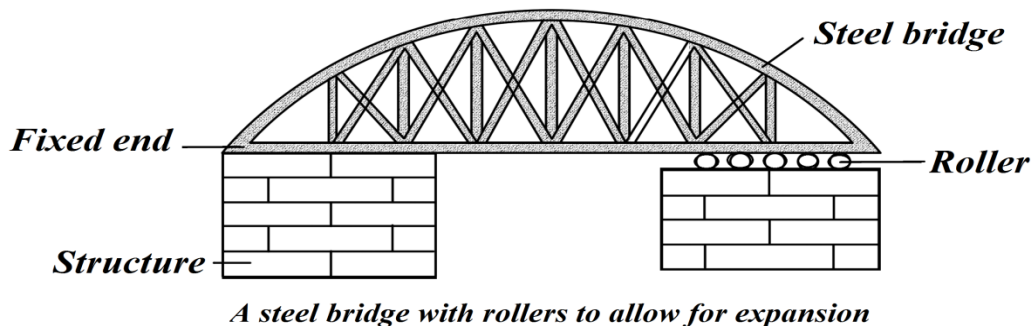
- The bimetallic strip consists of two different metals that expand at different rates when heated through the same temperature change.
- The bending of the bimetallic strip is due to the difference in the coefficient of linear expansion of two different metals used in the bimetallic strip.
- A bimetallic strip consists two strips of different metals fixed to each other lengthwise. An increase in temperature causes bending of the strip in such a way that the metal of greater linear coefficient of expansion lies on the outer side.
- A lowering of temperature again bends the strip, but with the metal of smaller linear coefficient of expansion on the outer side. Such strips can be used in **electric iron, electric oven, refrigerator** to control the temperature.



Applications of expansion of Solids

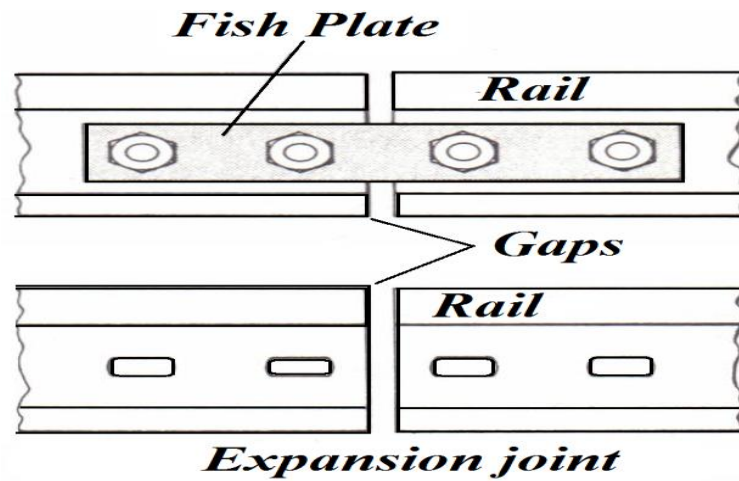
1. Applied in the construction of bridges and house roofs

- Iron or steel girders are used in the construction of a bridge. One end of the girder is rigidly fixed with bricks and concrete. The other end is not fixed. Instead, the end is set on a roller over the support, as shown in Fig. When there is rise or fall in temperature due to seasonal changes, girders may expand or contract, without developing any thermal stress.



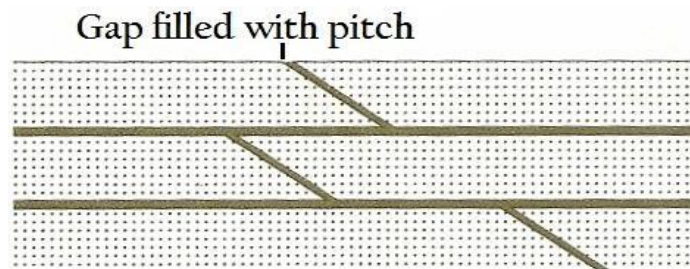
2. Railway lines construction

- The rails of a railway expand in summer and contract in winter. Therefore during construction gaps are left between successive rails to allow for their expansion. If there are no gaps, the increase in temperature will cause the rails to expand and they will overlap one another or dislodge from the position. This will be dangerous to the trains and may result in a severe accident.



3. Construction of roads and pavements

- While constructing cement roads using concrete slabs, a small gap is left between two slabs. The concrete slabs undergo thermal expansion during summer. The gap allows space for this expansion. If these gaps were not left, the concrete slabs would crack during summer due to thermal expansion. (See the fig below)



4. The use of bimetallic strip

- Bimetallic strips are used as temperature controlling devices.
- The bimetallic strip is used in thermostats, thermometers and valves.

(a) **In Thermostat:** Is a device used for maintaining a steady temperature.

Thermostat is used in many appliances such as electric irons, heaters, refrigerators, air conditioners, fire alarms and Valves. Thermostat is an automatic switch that turns on at a definite temperature and off at some other fixed temperature.

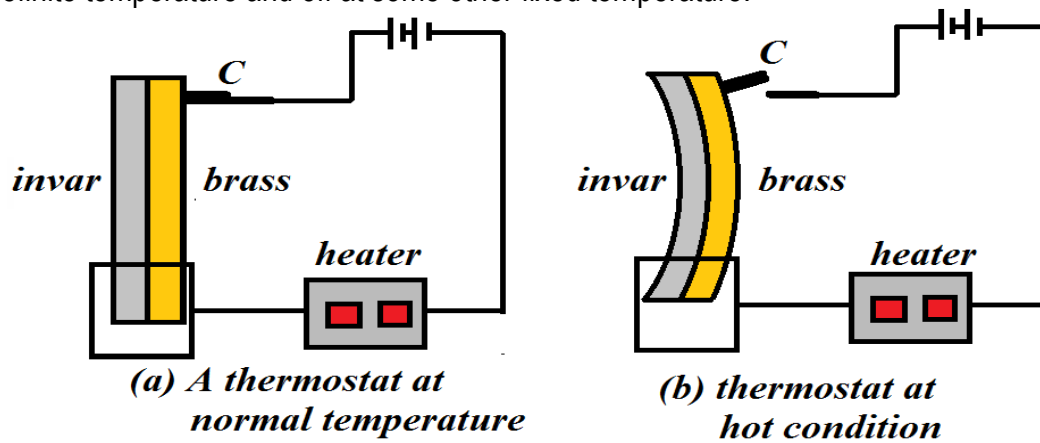


Fig. above shows the working of a thermostat.

- The two metals in the bimetallic strip are brass and invar. The contact point C of the thermostat is kept adjacent to the brass part of the bimetallic strip.

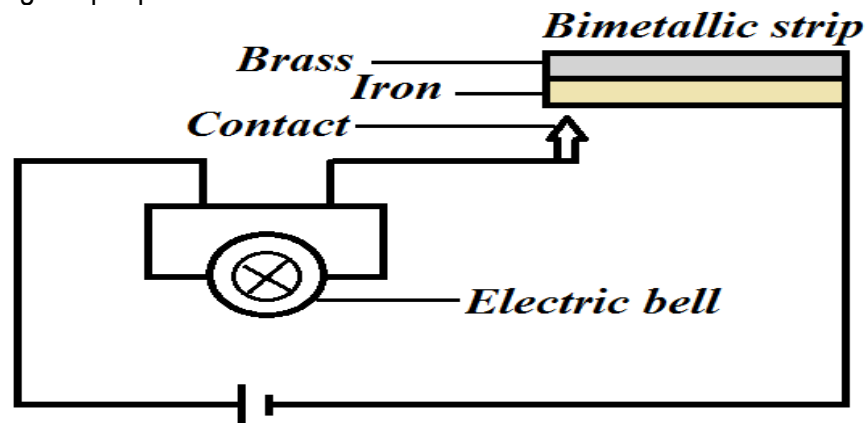
- At ordinary temperature the metal strip remains straight. This keeps the thermostat switched on and current flows through the heater. Heat from the heater warms up the air. The strip gets heated up and bends. Coefficient of linear expansion of brass is more than that of invar and so the bimetallic strip bends away from the contact point **C** and disconnects the circuit, stopping current flow through the heater.
- Hence, temperature of the heater falls. This also cools the bimetallic strip. The strip straightens up again restoring the electrical connection, and the heater is on again. Therefore, the heater cannot remain 'ON' above a fixed temperature as the thermostat controls the current flow. Thus, the temperature is also controlled.

Summary of working mechanism;

- When a thermostat is used in an electrical circuit, its bimetallic strip gets heated and bends if the temperature in the circuit exceeds a certain limit. Hence the circuit breaks. Since there is no current in the circuit now, the temperature of the strip decreases. The strip gets restored to its original position, and the circuit is completed, allowing current to flow again.

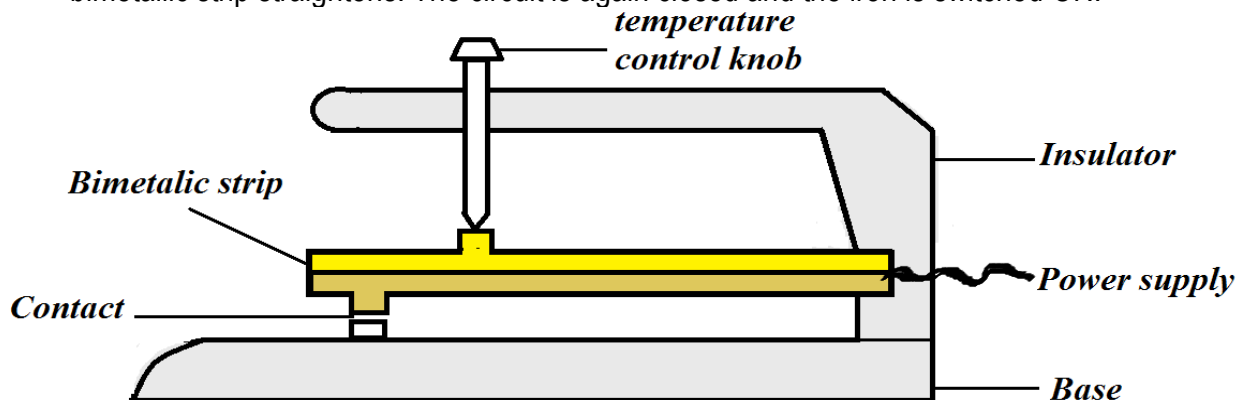
In Fire alarms circuit

- A bimetallic strip of brass and iron is connected to a battery and an electric bell as shown in the diagram. One terminal of the bell is connected to a screw which is at a very small distance from the iron side of the strip. In case, there is an accidental fire, the bimetallic strip bends towards iron and touches the screw. Thus, the circuit is completed and the bell rings, thereby warning the people of the accidental fire.



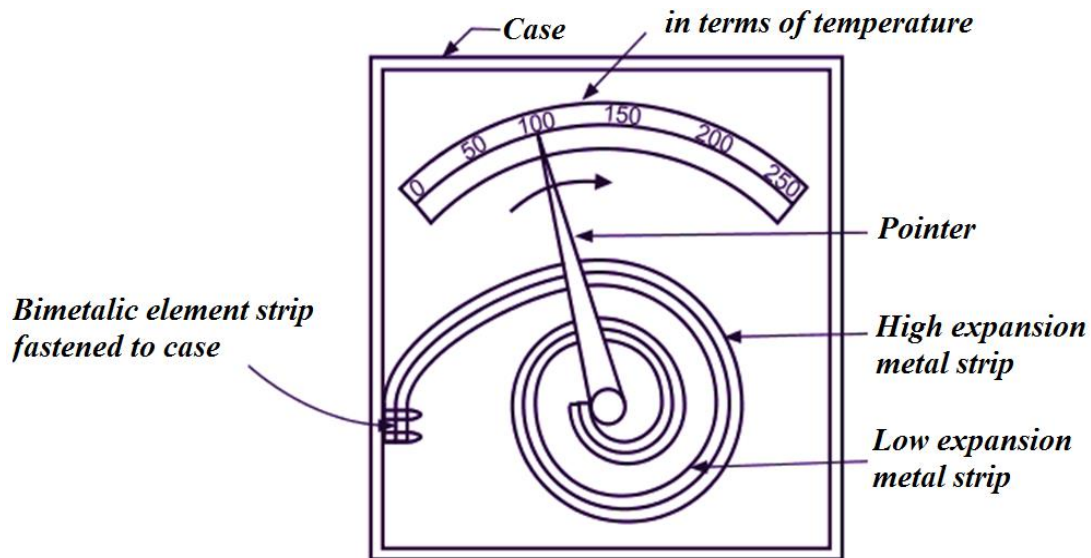
In Electric iron

- In an electric iron (see Figure below) when electric current flows through its heating element, it becomes hot. The bimetallic strip connected with the heating element through a spring also begins to heat up. On getting hot, bimetallic strip bends and is disconnected from the heating element. This makes the circuit open and switches OFF the electric iron. On cooling, the bimetallic strip straightens. The circuit is again closed and the iron is switched ON.



(b) **In Bimetallic thermometer**

- The figure below shows a spiral bimetallic thermometer instrument. The spiral bimetallic strip is attached to the pointer mounted in housing with a scale.



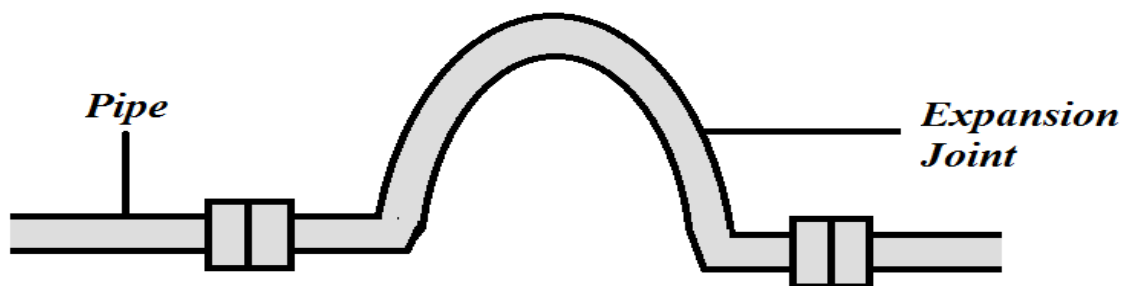
- When the spiral bimetal is heated, it bends in the direction of metal having low thermal coefficient of expansion, and rotates in clockwise direction. Thus, the pointer attached to it also moves on calibrated scale, indicating the temperature reading.

(c) **In Bimetallic Valve**

- Is also used to open and close temperature – sensitive valve
- The valve opens when the temperature rises and close when the temperature falls

5. **Applied in hot-water pipes**

- In certain industries, hot liquids or hot water is transported through metal pipes from one place to another. These pipes are subjected to expansion and contraction. To avoid cracks in the pipes when they expand and contract, they are arranged in the form of loops.

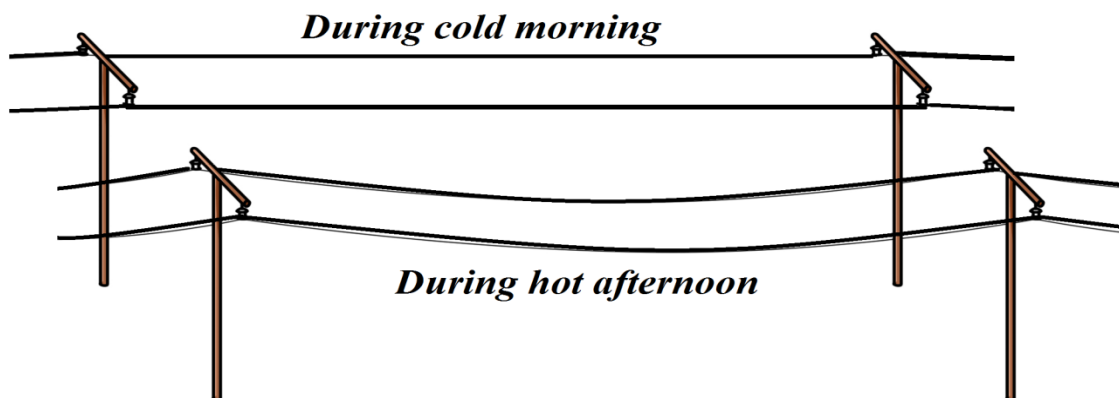


6. **Applied in designing of pendulum clocks**

- The pendulums of clocks expand in summer and contract in winter, therefore, they lose time in summer and gain time in winter. In order that the clocks should give correct time, the pendulums are made from invar which is an alloy having a very small coefficient of linear expansion in some clocks, compensating pendulums are used.
- A compensating pendulum is made of a number of iron and brass rods joined in such a way that the length of the pendulums remains constant even if there is a change in temperature. Such clocks show accurate time. If this is not possible the clocks require the adjustment in summer and winter.

7. Overhead telephone and electrical cables are loosely held

- In order to allow for contraction and expansion during the cold and hot day respectively
- During cold weather or at night, the metal can contract without breaking



Some effects of expansion and contraction

1. Vessels made of thick or ordinary glass break easily

- A thick walled glass pot often cracks when hot water is poured into it. The inner surface of the thick glass, in contact with the hot water, warms up and expands. Glass is a bad conductor of heat, and so the outer surface remains colder, and hence expansion is less. **This unequal expansion sets up a thermal stress and the glass cracks.**
- In case of a thin walled glass pot, heat transfer to the outer surface is easier and there is almost equal expansion of both the surfaces. The chance of cracking is reduced. Pyrex glass has a low coefficient of expansion. Hence, beaker, test tube, etc. for laboratory use are usually made of pyrex glass.

2. Riveting of two metal plates

- Rivets are used in shipbuilding, boiler making, etc to join metals sheets
- When ships and boilers are constructed, the steel plates are joined together firmly by riveting. This is also an application of thermal expansion and contraction. A rivet is heated to red hot and passed through the plates. Then it is hammered to fix it firmly. On cooling, the rivet contracts and holds the plates together firmly

3. Glass stoppers sometime stick in the necks of bottle.

- By warming the neck of such a bottle gently, the stopper often comes out easily. This is because the neck of the bottle expands when hot water is poured over it. As a result, a small gap is created between the stopper and glass, enabling the easy removal of the stopper.

4. Weathering of rocks

- This is the action of sun, air and water on rocks, causing them to break
- Hot sun shine makes the outside of a rock expand, and pieces break off due to the force of expansion

5. To Pass a Nail Through Hole in Metal Plate:

- To pass nail through a metal plate having a hole of a diameter slightly less than that of the nail, the plate is heated. so that the diameter of hole increases and the nail can easily pass through it.

6. Use of thick Bottles for Soft drinks:

- To avoid bursting of soft drink bottles containing gas, due to thermal expansion, their walls are made very thick.

7. Fitting the iron rim on the wooden wheel:

- The iron ring is always made **slightly lesser in diameter than that of the wooden wheel**. Therefore, the iron ring cannot easily slip onto the rim of the wooden wheel.



Iron rim

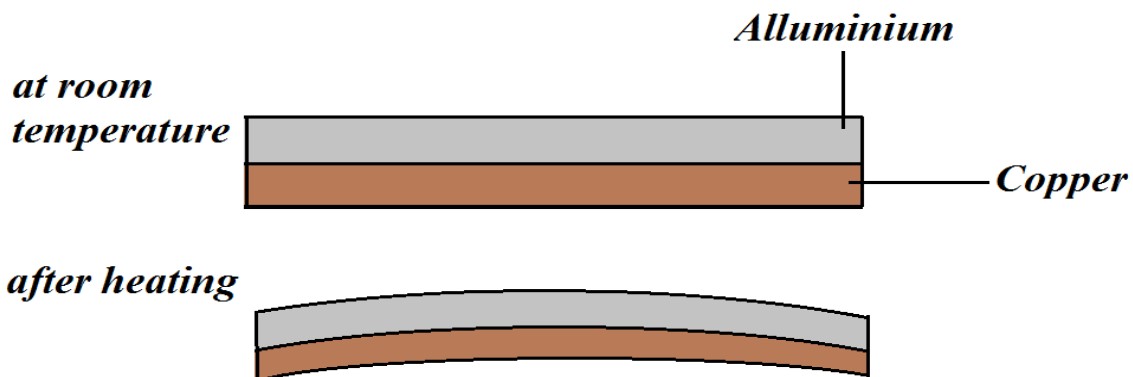


Wooden wheel

- Now the iron ring is heated to a higher temperature, it results in the **expansion in the size of the ring**, and the hot ring is then easily slipped onto the wooden wheel's rim. Cold water is then poured on the iron ring **to contract in size and hold the wooden wheel tightly**.

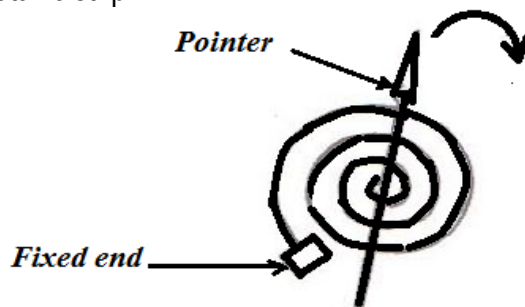
Class Activity – 6:3

- Explain why
 - The metal lid on a glass jam jar can be unscrewed easily if the jar is inverted for a few seconds with the *lid* in very hot water,
 - Furniture may creak at night after a warm day,
 - Concrete roads are laid in sections with pitch between
- Why is an iron rim heated before fixing a cart wheel?**
- Give** a reason why a concrete beam reinforced with steel does not crack when subjected to changes in temperature
- A blacksmith fixes iron ring on the rim of the wooden wheel of a bullock cart. The diameters of the rim and the iron ring are 5.443 m and 5.434 m respectively at 37°C. To what temperature the iron ring should be heated so that it can fit the rim of the wheel. **(Given that α for iron = $1.20 \times 10^{-5} \text{K}^{-1}$)**
ANS: 175°C
- Brass expands more than invar when heated equally. Sketch the bimetallic strip after being cooled several degrees below room temperature.
- Railway lines 25m long are being laid when the temperature is 15°. Gaps of 1.8cm are being left between them. If $\alpha = 12 \times 10^{-6} \text{ } ^\circ\text{C}^{-1}$, what is the temperature at which they will touch each other?
[ANS; 75°C]
- A bimetallic strip is made from aluminium and copper. When heated it bends in the direction shown in Figure below



- Which metal expands more for the same rise in temperature, aluminium or copper?
- Draw a diagram to show how the bimetallic strip would appear if it were cooled to below room temperature.

8. Figure below shows a bimetallic strip.



Explain how a rise in temperature causes the pointer to move in the direction shown.

Thermal Expansion of Liquids

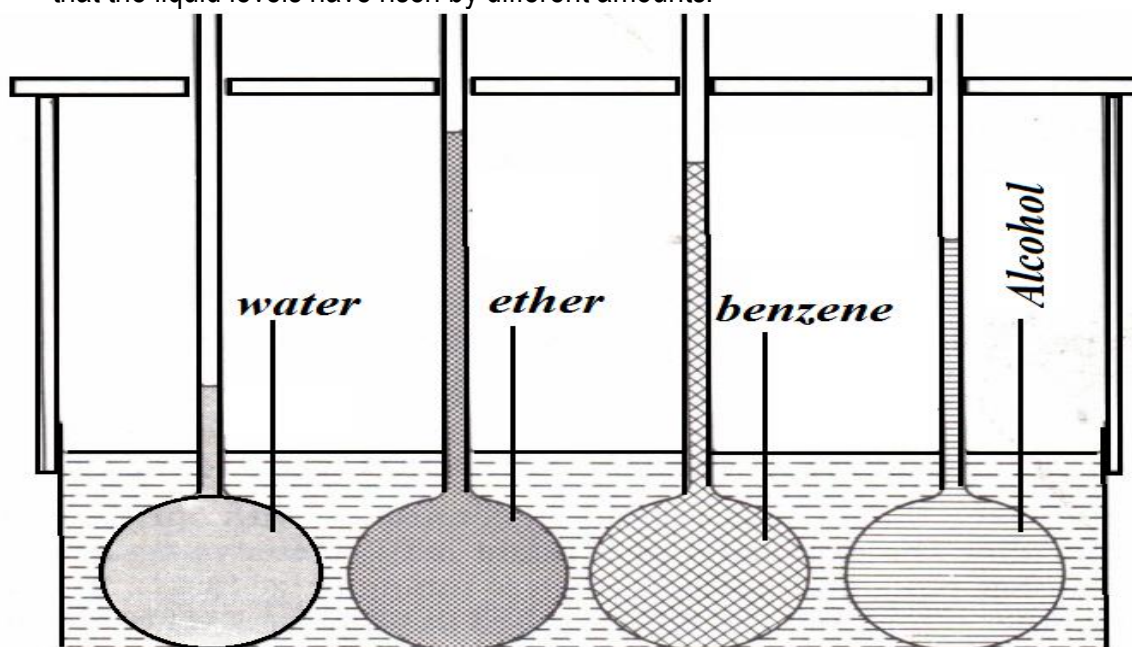
- It is easier to observe expansion in liquids than in solids. This is **because liquids expand much more than solids for equal change of temperature.**

Explain why when heating a liquid, its level initially decreases and then it increases to become more larger than the original level?

- The liquid level drops due to the expansion of its container which initially absorbed all the heat. After a while, the heat reaches the liquid it compensates for the expansion of the container and rises much more than the original level.

To compare the expansions of various liquids

- Different liquids have different thermal expansions. To demonstrate this, several fairly large glass bulbs with glass stems are filled to a short distance above the bulb with various liquids (see figure below).
- **In order to make a fair comparison**, the bulbs and stems must all be of the same size. The bulbs are immersed in a metal trough containing cold water and left until they have reached a steady temperature. A little extra liquid should now be added, where necessary, to make all levels the same.
- The bath is now heated and well stirred to ensure a uniform temperature. When the bulbs and their contents have acquired the new temperature of the bath it will be seen that the liquid levels have risen by different amounts.



Comparison of expansion of liquids

- Thus, for a given rise in temperature, equal volumes of different liquids show different expansions in volume (ie., some liquids expand more than others for a given rise in temperature).

Apparent volume expansion of a Liquid

- Is the difference between the initial volume of a liquid and its final volume without consideration of the expansion of the container

Absolute (Real) expansion of the liquid

- Is the difference between the final volume of a liquid and its intermediate volume

The intermediate volume of a liquid

- Is the volume of a liquid attains due to the change that was caused by the expansion of the container

Volume expansivity of a Liquid (coefficient of volume of expansion)

- Volume expansivity Is the fractional change in volume per unit temperature change.
- Its SI unit is K^{-1} or $^{\circ}C^{-1}$.
- Volume expansivity = $\frac{\text{increase in volume of liquid}}{\text{original volume} \times \text{rise in temperature}}$

$$\therefore \beta = \frac{V_2 - V_1}{V_1 \times \Delta\theta}$$

Whereby:

Original/initial volume of liquid = V_1 , Final volume = V_2

Increase in volume of liquid, $\Delta V = V_2 - V_1$

Initial temperature = θ_1 , Final temperature = θ_2

Rise in temperature = $\Delta\theta = \theta_2 - \theta_1$

Volume expansivity = β

Worked Examples;

1. The increase in volume of 10 cm^3 of mercury when the temperature rises by $100^{\circ}C$ is $0.182 cm^3$. What is the volume expansivity of mercury?

$$\text{ANSW: } \gamma = \frac{\Delta V}{V_1 \Delta T} = \frac{0.182}{10 \times 100} = 1.82 \times 10^{-4} C^{-1}$$

2. A 500 cm^3 Pyrex beaker is 95% full of methanol at $15^{\circ}C$. At what temperature will it be 100% with the methanol? (Volume expansion of methanol = $122 \times 10^{-5} K^{-1}$)

$$\text{ANS; } V_1 = 95\%, \Delta V = V_2 - V_1 = 100\% - 95\% = 5\%$$

$$\text{(OR; } V_1 = 95\% \times 500 = 475 cm^3, \Delta V = V_2 - V_1 = 100\% - 95\% = 5\% \times 500 = 25 cm^3)$$

$$\gamma = \frac{\Delta V}{V_1 \times \Delta T} \rightarrow \Delta T = \frac{\Delta V}{V_1 \times \gamma} = \frac{5\%}{95\% \times 122 \times 10^{-5}} = 43.14^{\circ}C$$

$$\therefore T_2 = T_1 + \Delta T = 15 + 43.14 = 58.14^{\circ}C$$

3. A hollow glass sphere has a density of $1.30 g/cc$ at $20^{\circ}C$. Glycerin has a density of $1.26 g/cc$ at $20^{\circ}C$. At what temperature would the sphere begin to float in Glycerine? Given coefficient of volume expansion of glycerine is $53 \times 10^{-5} C^{-1}$.

ANSW:

NB:

- Due to volume expansion of Glycerin which has been given from the question, it seems that glycerin was to be heated
- Therefore, in order for the glass sphere starts floating in Glycerin, the Glycerin must be heated so as its **density must change from 1.26g/cc to 1.3g/cc** (The glass sphere will float when it has the same density as glycerine *ie.* $\rho_1 = \rho_2 = 1.3\text{g/cm}^3$)
- When Glycerine is heated, it expands from volume V_1 to volume V_2 :
- The mass of Glycerine is the same before and after heating ($m_1=m_2$)

Now; $\rho_1 = 1.26\text{g/cm}^3$, $\rho_2 = 1.3\text{g/cm}^3$, $T_2 = 20^\circ\text{C}$, $\gamma_{GLY} = 53 \times 10^{-5} \text{C}^{-1}$, $T_2 = ?$

$$\text{From; } \gamma = \frac{\Delta V}{V_1 \times \Delta T} = \frac{V_2 - V_1}{V_1 \times (T_2 - T_1)} \rightarrow V_2 - V_1 = \gamma \times V_1 \times (T_2 - T_1)$$

$$V_2 = V_1 [1 + \gamma \times (T_2 - T_1)] \text{----- (i)}$$

$$\text{But; } m_1 = m_2 \rightarrow \rho_1 V_1 = \rho_2 V_2 \rightarrow V_2 = \frac{\rho_1 V_1}{\rho_2} \text{----- (ii)}$$

Substitute equation (ii) into (i)

$$\frac{\rho_1 V_1}{\rho_2} = V_1 [1 + \gamma \times (T_2 - T_1)] \rightarrow \frac{\rho_1}{\rho_2} = 1 + \gamma \times (T_2 - T_1)$$

$$\frac{1.26}{1.3} = 1 + 53 \times 10^{-5} (T - 20) \rightarrow \frac{-0.0308}{53 \times 10^{-5}} + 20 = T_2$$

$$T_2 = -58 + 20 = -38^\circ\text{C}$$

\therefore The glycerine must be cooled to -38°C for the hollow glass sphere start floating.

Alternatively;

$$\text{From; } \gamma = \frac{\Delta V}{V_1 \times \Delta T} = \frac{V_2 - V_1}{V_1 \times (T_2 - T_1)}, \text{ but; } v = \frac{m}{\rho}, \text{ then, } \gamma = \frac{\frac{m}{\rho_2} - \frac{m}{\rho_1}}{\frac{m}{\rho_1} \times (T_2 - T_1)}$$

$$\gamma = \frac{\frac{1}{\rho_2} - \frac{1}{\rho_1}}{\frac{1}{\rho_1} \times (T_2 - T_1)} = \frac{\frac{(\rho_1 - \rho_2)}{\rho_1 \rho_2}}{\frac{1}{\rho_1} \times (T_2 - T_1)} \rightarrow (T_2 - T_1) = \frac{\frac{(\rho_1 - \rho_2)}{\rho_1 \rho_2}}{\frac{1}{\rho_1} \times \gamma} = \frac{\rho_1 (\rho_1 - \rho_2)}{\gamma \rho_1 \rho_2}$$

$$T_2 = \frac{(\rho_1 - \rho_2)}{\gamma \rho_2} + T_1 = \frac{1.26 - 1.3}{53 \times 10^{-5} \times 1.3} + 20 = \frac{-0.04}{68.9 \times 10^{-5}} + 20$$

$$\therefore T_2 = -58 + 20 = -38^\circ\text{C}$$

NB;

- If the glass sphere was to be heated, its density could change from 1.3g/cc to 1.26g/cc
- But here it not the case, since its volume expansion has not been given.

Linear expansivities between (0 – 100)°C

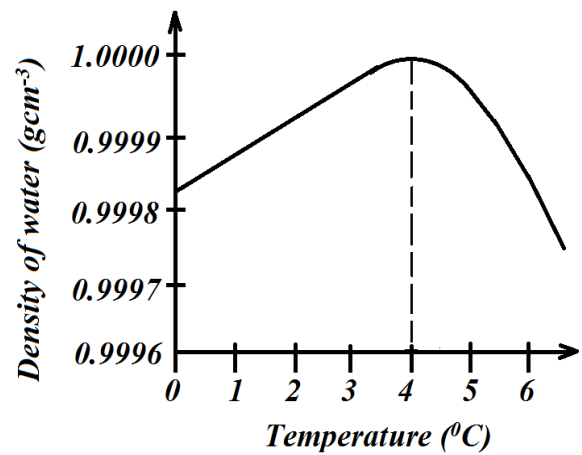
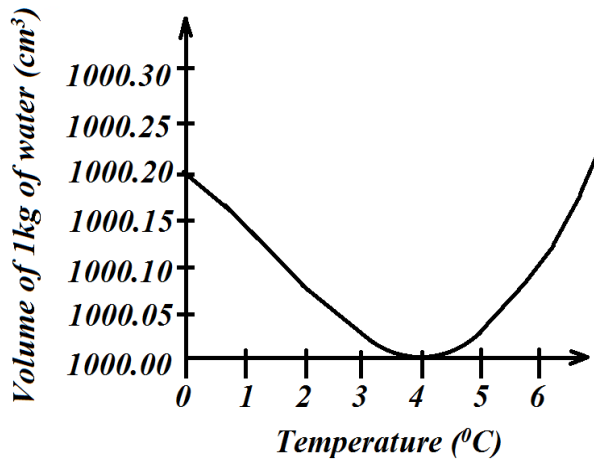
Liquid	Volume expansivity (K ⁻¹) x 10 ⁻⁵	Liquid	Volume expansivity (K ⁻¹) x 10 ⁻⁵
Benzene	124	Mercury	18
Gasoline	95	Methanol	122
Glycerin	53	Water at 20°C	21
Kerosene	99	Water at 90°	70

Anomalous expansion of Water

- When water is heated from 0°C to 4°C it **contracts** continuously instead of **expanding**. Conversely **expands** when cooled down from 4°C to 0°C . This unusual expansion of water from 4°C to 0°C is called **anomalous expansion**.
- When heated from 4°C to 100°C its expansion is normal like the expansion of other liquids. Since water contracts when heated from 0°C to 4°C and expands from 4°C to 100°C , its **volume is smallest and the density is maximum at 4°C** . Hence water is heaviest at 4°C .
- **Therefore;** the **anomalous expansion of water** is defined as the property of water that causes it to expand rather than contract when the temperature changes from 4°C to 0°C , causing it to **become less dense**.

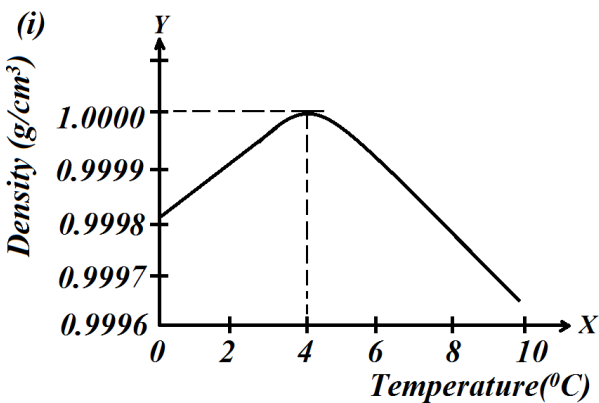
OR

- **Anomalous expansion of water** Is the unusual behavior of water where its volume decreases when the temperature rises from 0°C to 4°C and increases when the temperatures falls from 4°C to 0°C
- The two graphs below represent the change in volume and density of 1 g of water, with the increase in temperature

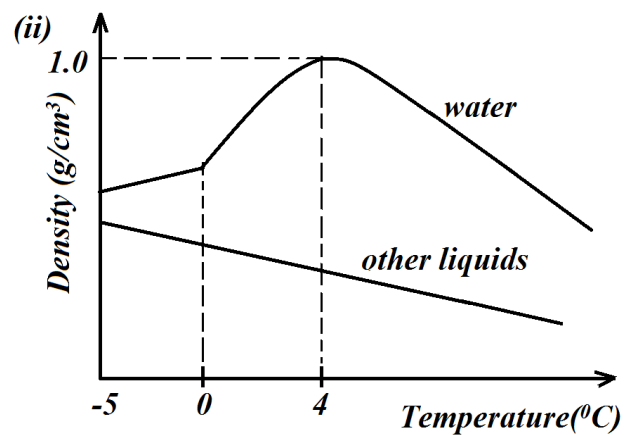


What happens below 4°C to 0°C

At 4°C , just above the freezing point, water reaches its maximum density. As the water cool further toward its freezing point, the liquid water expands to become less dense.



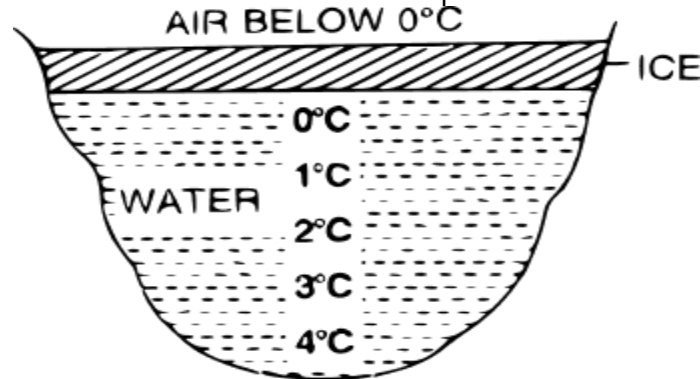
Variation of density of water with temperature



Graph of density against temperature of water compared with other liquids

Effect of Anomalous Expansion of water on Marine life

- Due to anomalous expansion of water, fishes and various living creatures can survive under frozen lakes, rivers or seas. In cold countries, with the fall in atmospheric temperature, upper surfaces of lakes, seas and various ponds gradually cool. Water of the upper surface, then being denser and heavier, moves down. Water below it, being comparatively warmer and lighter, moves up.
- **This convection process** in water continues until the density of the water in the lower part becomes maximum i.e., **the temperature of the lower water reaches 4°C**. As the temperature of the upper surface decreases further below 4°C, density begins to decrease. So water cannot move down further. It then begins to cool further and at last turns into ice. As ice is lighter than water, a thick layer of ice, thus formed, floats over the surface of water. Both ice and water are bad conductors of heat. So, a negligible amount of heat can be conducted from the lower levels of water to the atmosphere outside.



- So, the entire volume of water (top to bottom) in a pond cannot freeze. The temperature just below the floating ice-slab remains at 0°C [Fig. above]. The temperature of water gradually increases with depth and the lowest layer remains at 4°C, where the living creatures can easily survive.

Effects of Anomalous expansion of water

- A daily life example of the anomalous expansion of water is observed in deep freezers. Water containers (glass bottles) tend to bust in deep freezers at 0°C due to volume increase, as volume increases when cooling from 4°C to 0°C.
- **It supports the life of aquatic organism.** In cold countries during the winter season, the surface of the lakes will be at lower temperature than the bottom. Since the solid water (**ice**) has lower density than its liquid form, **below 4°C**, the frozen water will be on the top surface above the liquid water (**ice floats**). This is due to the anomalous expansion of water. As the water in lakes and ponds freeze only at the top the species living in the lakes will be safe at the bottom.
- **In winter season the water supply pipes open to the atmosphere after burst when the temperature of the surrounding falls below 4°C. This is due to the fact that water below 4°C expands (anomalous expansion) and exerts pressure on the walls of the pipes and thus causes damage to it**
- In rainy season a lot of water sweeps through numerous cracks and fissures in rocks. In winter season when temperature falls below 4°C going down to 0°C or below, expansion of water takes place, thus developing a high pressure. **This results in the breaking of the rocks.**
- Icebergs, being less dense than water, float in oceans thus posing a danger to ships.

Effects of Expansion of Liquids

- **The expansion of liquids used in liquid thermometers.** Liquid-in-glass thermometers are based on the principle of **thermal expansion of substances**. A liquid in a glass tube (called

a **capillary**) expands when heated and contracts when cooled. A calibrated scale can then be used to read off the respective temperature that led to the corresponding thermal expansion.

- **Weathering of rocks** .This happen when water freezes in the cracks of a rock the volume of water increases .This causes the rock to break into small pieces
- **Industrial bottling of liquids provides empty space to allow for expansion during freezing of the liquids.** Liquids exhibit thermal expansion in volume, due to this expansion the bottle might break, as the bottles are sealed. In order to avoid breakage of bottles and to allow the space for liquid to expand, industrial bottles are always provided with some empty space.
- When boiling water is poured into a thick tumbler its inner surface expands. However, due to low thermal conductivity of glass, the expansion of outer surface of the tumbler is quite small. Due to uneven expansion of the outer and inner surfaces, the tumbler breaks.

Class Activity – 6:4

1. What is the anomalous expansion of water?
2. An ice sheet 5m thick covers a lake that is 20m deep. What is the temperature of the water at the bottom of the lake? Explain your answer
3. How do fishes and aquatic animals survive when the pond gets covered with thick ice?
4. Deep pond of water has its top layer frozen during winter. State the expected temperature of water layer (i) just in contact with ice, (ii) at the bottom of pond. **[ANS; 0°C , 4°C]**
5. (a) If a 500-mL glass beaker is filled to the brim with ethyl alcohol at a temperature of 5.0°C, how much will overflow when its temperature reaches 22 °C, given that $\gamma = 110 \times 10^{-5} K^{-1}$ **[ANS; 9.35mL]**
 (b) How much less water would overflow under the same conditions?, given that $\gamma = 21 \times 10^{-5} K^{-1}$ **[ANS; V = V - V' = 9.35 - 1.785 = 7.565mL]**
6. Explain why filled bottles of water when placed in a freezer may burst when water is frozen?
7. Draw a labeled diagram for a set up of an experiment to show that alcohol expands more than an equal volume of water for the same rise in temperature
8. As an expert in thermal expansion, you are consulted by a company to construct a thermometer. The company has provided you with mercury, alcohol and gasoline. With justification, explain which liquid you will use in constructing the thermometer.
9. **Explain why the surface of a lake freezes while the deepest water stays at 4 °C?**
ANS;
 When the weather gets cold, the temperature of water at the surface drops down and it becomes more dense than the water underneath. The dense water starts to sink and warmer water is pushed up. The water will keep circulating until the temperature of all water in lake become 4 °C. When the water temperature at the surface drops below 4 °C its volume starts to increase and its density starts to decrease. At this stage, the water is less dense than the water underneath and it stays on the surface. In time, ice forms on the surface and the water temperature at the bottom of the lake will remain at 4 °C. Thus, fish can survive a severe winter by staying in the deep warm water.
10. **The density of mercury is 13.6 g cm⁻³ at 0°C and its coefficient of cubical expansion is 1.82×10⁻⁴ °C⁻¹. Calculate the density of mercury at 50°C**

ANSW: Given; $\rho_1 = 13.6 \text{ g/cm}^3$, $\gamma = 1.82 \times 10^{-4} \text{ } ^\circ\text{C}^{-1}$, $\Delta T = 50$, $\rho_2 = ?$

$$\gamma = \frac{\Delta V}{V_1 \times \Delta T} = \frac{V_2 - V_1}{V_1 \times (T_2 - T_1)}, \text{ but; } v = \frac{m}{\rho}, \text{ then, } \gamma = \frac{\frac{m}{\rho_2} - \frac{m}{\rho_1}}{\frac{m}{\rho_1} \times (T_2 - T_1)}$$

NB: When a body is heated its mass does not change, ie $m_1 = m_2 = m$

$$\gamma = \frac{\frac{1}{\rho_2} - \frac{1}{\rho_1}}{\frac{1}{\rho_1} \times (T_2 - T_1)} = \frac{\frac{(\rho_1 - \rho_2)}{\rho_1 \rho_2}}{\frac{1}{\rho_1} \times (T_2 - T_1)} = \frac{\rho_1(\rho_1 - \rho_2)}{\rho_1 \rho_2 (T_2 - T_1)} = \frac{(\rho_1 - \rho_2)}{\rho_2 (T_2 - T_1)}$$

$$\gamma = \frac{(\rho_1 - \rho_2)}{\rho_2 (T_2 - T_1)} \rightarrow \gamma \rho_2 (T_2 - T_1) = (\rho_1 - \rho_2)$$

$$\gamma \rho_2 (T_2 - T_1) + \rho_2 = \rho_1 \rightarrow \rho_2 = \frac{\rho_1}{\gamma (T_2 - T_1) + 1} = \frac{13.6}{1.82 \times 10^{-4} \times 50 + 1}$$

\therefore the density (ρ_2) of mercury at 50°C is 13.48 g/cm³

11. Describe an experiment to show that same volume of different liquids heated to same rise in temperature expand by different amounts.

ANS;

- Just like solids, liquids expand at different rates. In order to investigate this, a number of identical flasks are filled with different liquids ensuring that their initial levels are the same in the glass tubes. For a fair comparison, the tubes should be identical i.e. of same diameter. The flasks are then simultaneously immersed in a bath of hot water. The bath of water should be stirred continuously to ensure that temperature is uniform.
- It will be observed that the level of the liquids in the tubes differ after some time. If water, alcohol and methylated spirit were used, it would be observed that methylated spirit expanded the most, followed by alcohol and water the least.

Thermal Expansion in Gases

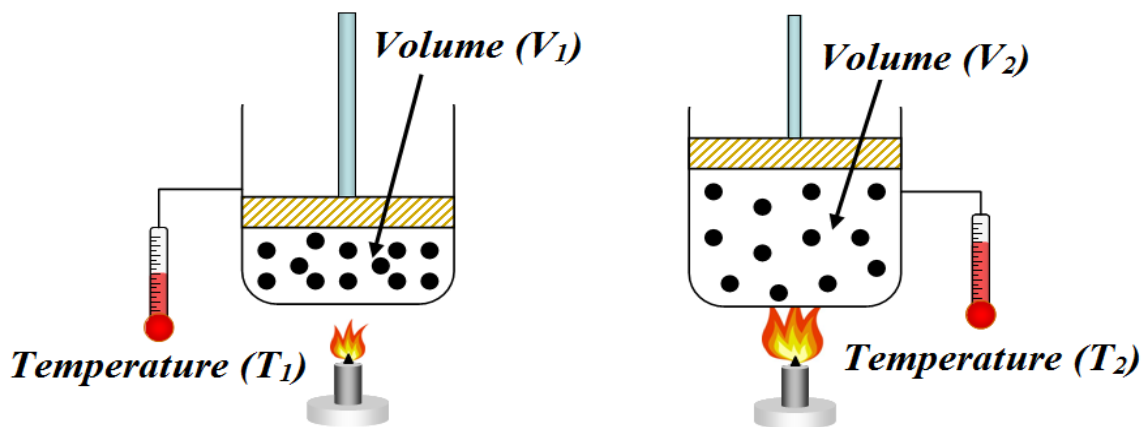
- **Gases expand much more than solids and liquids when heated.**
- **This is because** the particles in gases are not held closely together, as they are in solids and liquids, but are instead free to move in all directions.
- Three properties are important when studying the expansion of gases. These include **Pressure, Volume and Temperature.**
- The temperature must be converted into Kelvin scale

Charles' Law

- This law involves the relationship between the volume and the temperature of a fixed mass of a gas at constant pressure. The law state that

“The volume of a fixed mass of a gas is directly proportional to the absolute temperature provided the pressure remains constant”

- This means that if volume increases, the temperature will also increase.
- For example; Hot air rises, which is why hot-air balloons ascend through the atmosphere and why warm air collects near the ceiling and cooler air collects at ground level. Because of this behavior, heating registers are placed on or near the floor, and vents for air-conditioning are placed on or near the ceiling. The fundamental reason for this behavior is that gases expand when they are heated. See the figure below



- Mathematically

$$V \propto T \quad \rightarrow V = kT \quad \dots\dots\dots \text{make } K \text{ the subject} \quad \rightarrow K = \frac{V}{T}$$

Therefore: $\frac{V_1}{T_1} = \frac{V_2}{T_2} \rightarrow \frac{V_1}{V_2} = \frac{T_1}{T_2}$

Whereby:

V_1 = initial volume

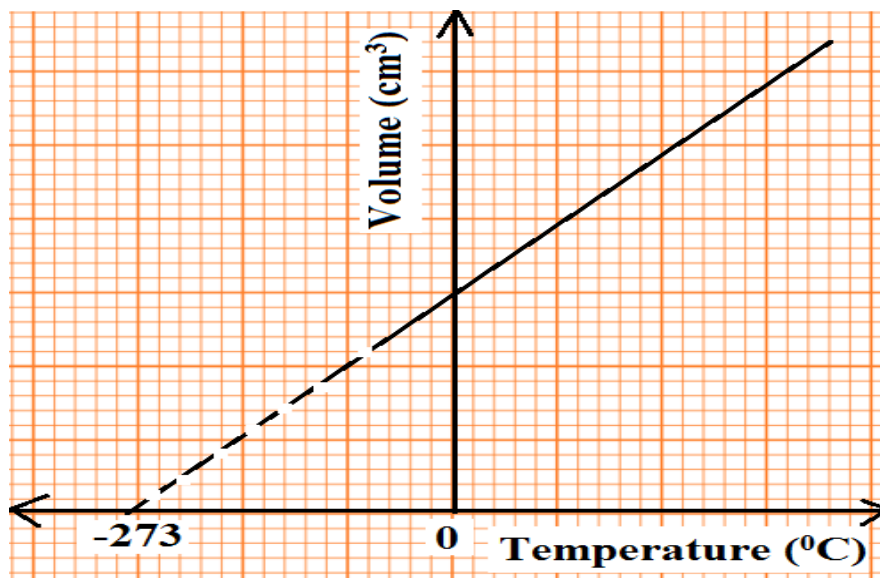
V_2 = final volume

T_1 = initial temperature

T_2 = final temperature

Graphical Representation of Charles law

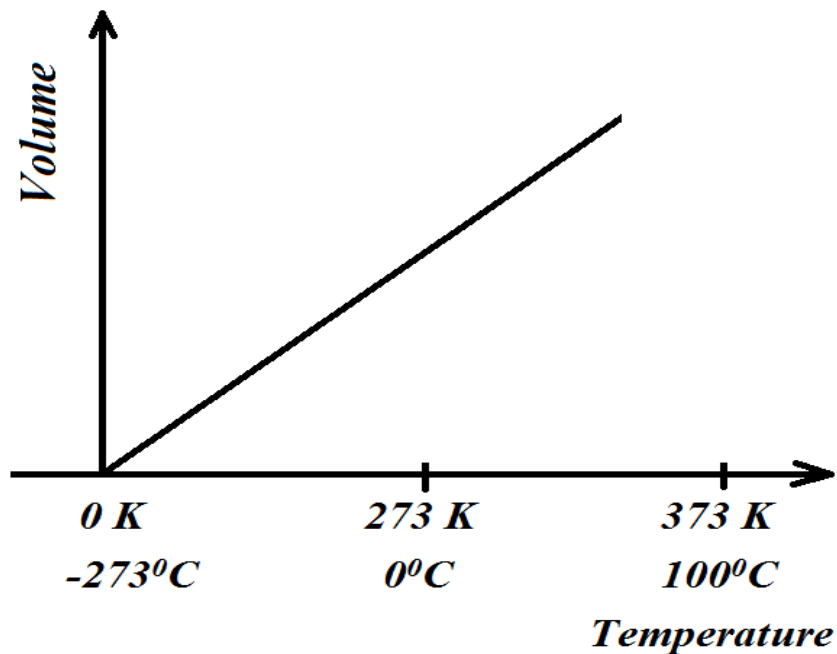
- Graph between Volume and absolute temperature of a gas at constant pressure is a **"straight line"**. This graph shows that at constant pressure, the volume of the given sample of the gas is directly proportional to the Kelvin temperature.
- If the graph between V and T is extra plotted, it intersects **T-axis** at $-273\text{ }^\circ\text{C}$. At $-273\text{ }^\circ\text{C}$ volume of any gas theoretically becomes zero as indicated by the graph.



- From the graph above it seems that as temperature increases also volume increases and vice versa

- The graph does not pass through the Celsius temperature origin (0 °C). If they are produced backward they cut the temperature axis at about –273 °C. This means at 0 °C the volume of the gas is non-zero, but at –273 °C the volume of the gas touches the zero mark.
- This temperature (–273 °C) is called **absolute zero** because it is found to be the lowest temperature possible. It is the zero of the absolute or Kelvin scale of temperature. Degrees on this scale are called kelvin and are denoted by K.
- **But practically volume of a gas can never become zero.** Actually no gas can achieve the lowest possible temperature and before -273.16 °C all gases are condensed to liquid. This temperature is referred to as absolute scale or absolute zero. At -273.16 °C all molecular motions are ceased.
- **Absolute zero is the temperature at which all particles of matter possess zero energy**

The figure below shows the relationship between the Kelvin scale and the Celsius scale of temperature



Conversion;

$$T(K) = 273 + \theta (^{\circ}C) \dots\dots\dots 1$$

$$\theta (^{\circ}C) = T(K) - 273 \dots\dots\dots 2$$

Worked Examples:

1. A gas of volume 300 cm³ was heated from 23^o C to 83^o C. Determine the volume at one atmospheric pressure

Solution:

Given: $T_1 = 23^{\circ} C = 23+273 = 296 K$, $T_2 = 87^{\circ} C = 87 + 273 = 360 K$

$V_1 = 300 \text{ cm}^3$, $V_2 = ?$

From: $\frac{V_1}{V_2} = \frac{T_1}{T_2}$

$$\frac{V_1}{V_2} = \frac{T_1}{T_2} \quad \rightarrow \quad V_2 = \frac{V_1 \times T_2}{T_1} = \frac{300 \times 360}{296} = 364.86 \text{ cm}^3$$

2. To what temperature must a gas at 127^oC be cooled, so that its volume is reduced to 1/5 of its initial volume? Assume pressure remains constant. [ANS: 80 K or - 193^o C]

$$\text{ANS; } V_1 = V, V_2 = \frac{V}{5}, T_1 = 127^\circ\text{C} = 127 + 273 = 400\text{K}, T_2 = ?$$

$$\text{From; } \frac{V_1}{V_2} = \frac{T_1}{T_2} \rightarrow \frac{V}{V/5} = \frac{400}{T} \rightarrow 5T = 400 \rightarrow T = \frac{400}{5} = 80\text{K}$$

Significance of Charles' Law

- It explains how gases behavior at constant pressure and the relation between the absolute temperature and the volume of the gas. According to Charles law, at a constant pressure, the volume and absolute temperature of a gas are directly proportional to each other
- At constant pressure, the density of a gas is inversely proportional to its volume, ie, $\rho \propto \frac{1}{T}$
- Using this concept hot air is used to fill the balloons used for meteorological purposes

Real life examples of Charles law

- Charles' Law describes the expansion of gases when they are heated. Keeping it simple, we can say that as the temperature of any particular gas increases, the molecules in that gas exhibit increased movement. As soon as the movement of the molecule increases, there is an increased number of collisions. What happens is that the molecules begin to hit the walls of the container more frequently, and, that too, with an increased amount of force.
- If the wall of the container is flexible, say, a balloon, the pressure will remain constant; thereby, allowing the volume to increase. However, if the container is inflexible, the more frequent collisions will result in increased pressure. In this article, we will talk about the real-life examples of Charles' Law..
 - (i) **It is a common observation that an inflated basketball shrinks in size when left under a cold environment.** This is true because a decrease in temperature results into a corresponding decrease in volume according to Charles's Law. Therefore, the volume of air inside the basketball shrinks on cold day.
 - (ii) **Tyre. In cold weather,** you might have regularly kept a check on the pressure of the tyres of your car. Driving increases the temperature of the tyres, and, therefore, the air inside the tyre warms and expands. When you measure the pressure of the tyres at the time when you have just driven the car, it will be high. However, in cold weather, the pressure of the tyres will be low. So, it is recommended that you should measure the pressure of the tyres regularly in colder climates, especially before **long trips** or during **cold snaps**.
 - (iii) **Explosion of Aerosols:** Aerosols refers to products like insecticides, perfumes, deodorants, spray paints and so on. If aerosol bottle is exposed to a very high temperature, it may explode. This is true because when heated, the pressurized gas expands and increases in volume. Since the gas cannot escape from its locked nozzle, it eventually explodes. **In fact, this is the reason behind the warning signs on its container,** indicating that it should be stored in a cool environment, and kept away from the sunlight and high temperature
 - (iv) **Turkey Timer.** The working of the Pop-Up Turkey Timer (Thermometer) is also based on Charles' law. Let's see how! If you remember what the Charles' law states, you might be familiar with the fact that gases expand when heated. The same principle applies to the Pop-Up Turkey Timer. The thermometer (or timer) is placed inside the turkey. As the temperature increases and the turkey cooks, the gas inside the thermometer also expands. As soon as the timer pops, it indicates that the turkey has been cooked.
 - (v) **Helium Balloon.** If you have had the chance to go out on a chilly day, you might have noticed that the balloon crumbles. However, if you take the balloon to a warm room, it regains its shape. Why does this happen? This happens because the temperature on a

cold day is low, and, so, the volume decreases. Now, in accordance with the Charles's Law, as soon as you enter a warm room, the temperature increases; with an increase in temperature, the volume also increases. Therefore, the balloon goes back to its original shape.

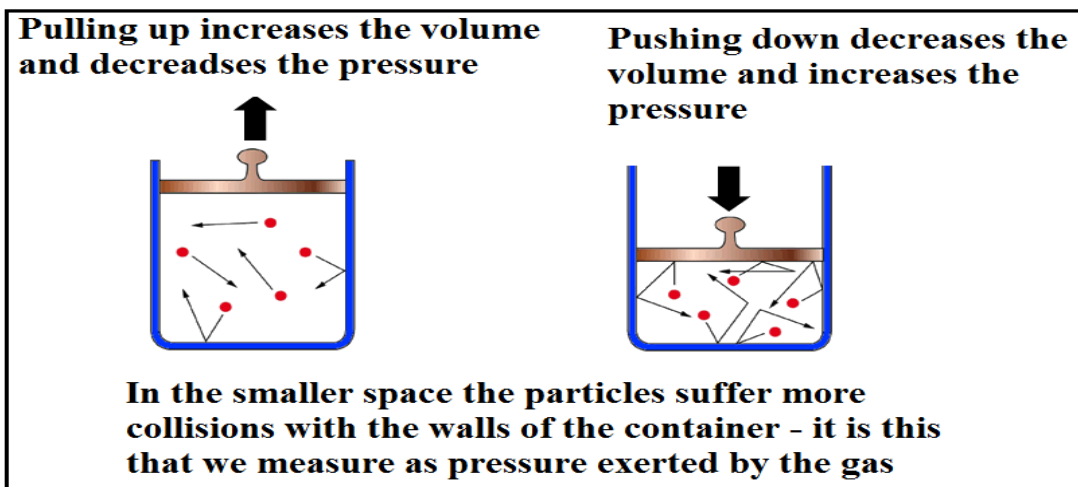
- (vi) **Bakery.** Charles's Law finds its way into our kitchens as well. In case you have ever tried your hand at baking, you might be familiar with the substance most commonly used in cooking, i.e., the yeast. Yeast is often used in baking to make the bakery products fluffy. Yeast is responsible for releasing carbon dioxide bubbles. These carbon dioxide bubbles expand further with high temperature. The expansion of the carbon dioxide bubbles with an increase in temperature works as a leavening agent and cause the bakery products to become fluffy.
- (vii) **Hot Air Balloon.** You might have wondered about the working of the hot air balloon. **Charles's Law** describes that temperature and volume are directly proportional to each other. When a gas is heated, it expands. As the expansion of the gas takes place, it becomes less dense and the balloon is lifted in the air. The warm air is less dense than the cold air, which means that it is lighter than the cold air.

Class Activity – 6:5

1. Use Charles's law to explain why cooler air sinks.
2. Change the following temperatures to Kelvin scale (a) 33°C (b) 57°C
ANS: (a) T (K) = 306 K (b) T (K) = 330 K
3. Change the following temperatures to Celsius scale (a) 4K (b) 292K
ANS: (a) $\theta^\circ \text{C} = -269^\circ \text{C}$ (b) $\theta^\circ \text{C} = 19^\circ \text{C}$
4. A 0.20m³ container with a movable piston holds nitrogen gas at a temperature of 20°C. What will be the volume of the gas if the temperature increased to 50°C? **(ANS: $V_2 = 0.22 \text{ m}^3$)**
5. (a) What is the need for the Kelvin scale of temperature?
(b) What is the boiling point of water on the Kelvin scale? Convert it into centigrade scale.
ANS;
(a) The behaviour of gases shows that it is not possible to have temperature below 273.15C. This act has led to the formulation of another scale known as Kelvin scale. The real advantage of the Kelvin scale is that it makes the application and the use of gas laws simple. Even more significantly, all values on the Kelvin scale are positive
(b) **373K, 100°C**
6. A gas occupies a volume of 20 cm³ at 27°C and at normal atmospheric pressure. Calculate the new volume of the gas if it heated to 54°C at the same pressure. **(ANS: $V_2 = 21 \text{ cm}^3$)**
7. A gas occupies 3 litres at 0°C. What volume will it occupy at -20°C, pressure remaining constant? **[ANS; 2.78L]**
8. A gas occupies 500 cm³ at normal temperature. At what temperature will the volume of the gas be reduced by 20% of its original volume, pressure being constant? **[ANS; 218.4K or -52.6°C]**

Boyle's Law

- This law involves the relationship between the volume and the pressure of a fixed mass of a gas at constant temperature. The law state that
"The volume of fixed mass of a gas is inversely proportional to its pressure if the temperature is kept constant"
- The figure below shows the demonstration of Boyles' Law by diagram



- Mathematically

$$P \propto \frac{1}{V} \rightarrow P = \frac{K}{V} \rightarrow PV = K \rightarrow PV = \text{Constant}$$

$$\therefore P_1V_1 = P_2V_2$$

Whereby:

P_1 = initial pressure

P_2 = final pressure

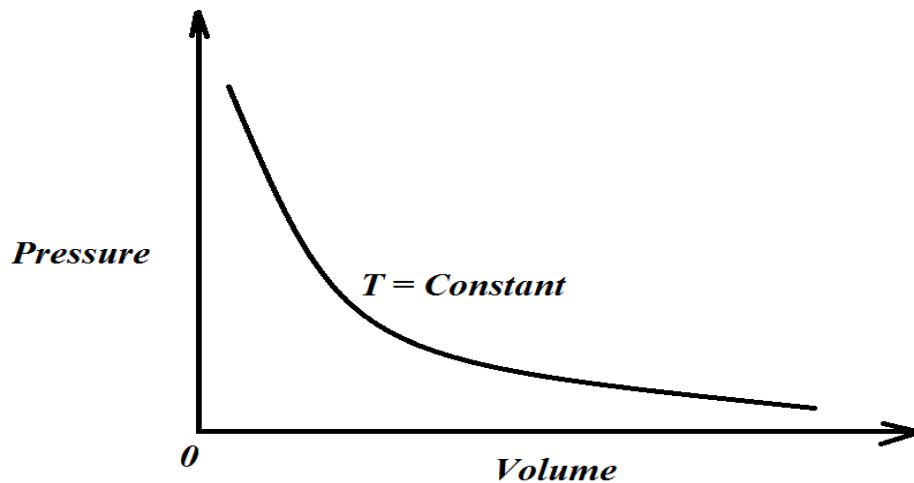
V_1 = initial volume

V_2 = final volume

- The equation above shows that as the pressure increases, the volume decreases and vice versa. For example, when the pressure doubles, the volume is decreased by half. Also, the units of pressure and volume must be consistent. P_1 and P_2 must be expressed in Pa or atm. V_1 and V_2 must be expressed in m^3 or L.

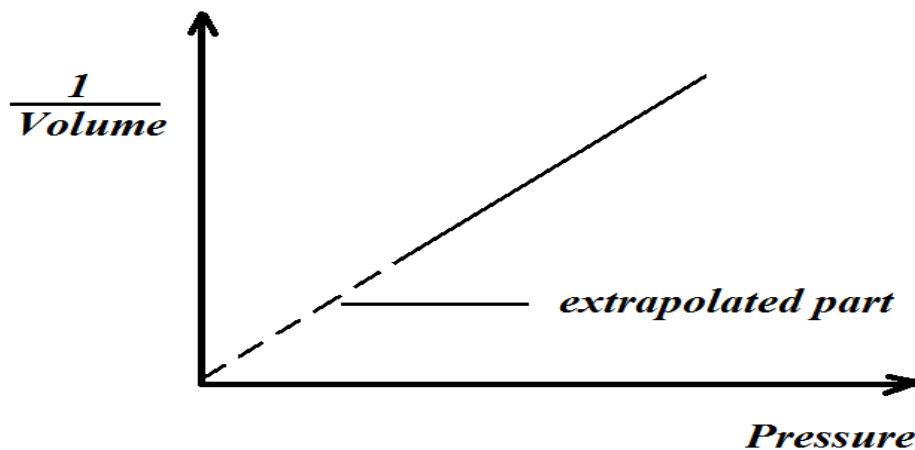
Graphical representation of Boyle's Law:

- A graphical representation of Boyle's law is typically seen as a curve. This curve is called the PV (Pressure – Volume) curve, and it is hyperbolic in nature. It shows the relationship between the pressure of a gas and its volume at constant temperature.



A graph of pressure against volume

- From the graph above it seems that as pressure increases also volume decreases and vice versa.
- The figure below shows the relationship between reciprocal of volume and pressure



- From the graph above it seems that as the pressure increased also inverse of volume increased and vice versa

Worked Examples

1. A gas occupies 250 cm³ when the pressure is 20 atmospheres. What will its volume be if pressure is reduced to 15 atmospheres while the temperature is kept constant?

Answer

$$P_1 = 20 \text{ atm}, P_2 = 15 \text{ atm}, V_1 = 250, \text{ cm}^3 \quad V_2 = V_2$$

$$\text{From: } P_1 V_1 = P_2 V_2$$

$$20 \times 250 = 15 \times V_2 \quad \rightarrow \quad V_2 = \frac{5000}{15} = 333.33 \text{ cm}^3$$

2. A bubble of air has a diameter of 2.0mm when it is 0.5 m below the water surface of a boiler, calculate the diameter of the bubble as it reaches the surface, assuming that the temperature remains constant. Take $g = 10\text{m/s}^2$, density of water = 1000kg/m^3 and atmospheric = 10^5 Nm^{-2}

ANSW: Given, $d_1 = 2.0\text{mm}$, $h = 0.5\text{m}$, $g = 10 \text{ m/s}^2$, $\rho = 1000\text{kg/m}^3$,

$$P_1 = P_2 + P \text{ (due to column of water)} = P_2 + \rho h g$$

$$P_1 = 10^5 + 0.5 \times 1000 \times 10 = 10^5 + 5000 = 1.05 \times 10^5 \text{ Nm}^{-2}, \quad P_2 = 10^5 \text{ Nm}^{-2}$$

- From Boyle's Law: $P_1 V_1 = P_2 V_2$, but assume that the bubble is spherical, then $V = \frac{4\pi R^3}{3}$, But;

$$R = d/2 = 2/2 = 1$$

- $P_1 V_1 = P_2 V_2 \rightarrow P_1 \times \frac{4\pi R_1^3}{3} = P_2 \times \frac{4\pi R_2^3}{3} \rightarrow P_1 \times R_1^3 = P_2 \times R_2^3$

- $R_2 = \sqrt[3]{\left(\frac{P_1 \times R_1^3}{P_2}\right)} = R_1 \sqrt[3]{\left(\frac{P_1}{P_2}\right)} = 1 \times \sqrt[3]{\left(\frac{1.05 \times 10^5}{10^5}\right)} = 1.0164 \text{ mm}$

- Therefore; diameter of bubble as it reaches the surface of water

$$d_2 = 2R_2 = 2 \times 1.0164 = 2.033 \text{ mm}$$

3. A gas is expanded, at a constant temperature, from a volume of 500 mL to a volume of 1.5 litre, where its final pressure is 150 mm of Hg. What was the original pressure? **[A; 450 mmHg]**
4. Find the volume of a sample of nitrogen at a pressure of 1.50 atm, if its volume is 3.15 L at 1.00 atm and the temperature is constant. **[ANS; 2.1L]**

Significance of Boyle's Law

- It explains how gases behavior at constant temperature and the relation between the pressure and the volume of the gas. **According to Boyle's law, at a constant temperature, the pressure and volume of a gas are inversely proportional to each other**

- At constant temperature, the density of a gas is directly proportional to its pressure, ie,.. $P \propto \rho$
- **Atmospheric pressure is low at high altitudes, so air is less dense. Hence,** a lesser quantity of oxygen is available for breathing. This is the reason why mountaineers have to carry oxygen cylinders with them.
- **It explains how people breathe and exhale air.** When the diaphragm expands and contracts, lung volume increases and decreases, changing the air pressure inside of them. The pressure difference between the interior of the lungs and the external air produces either inhalation or exhalation
- **Doubling pressure halves volume,** at constant temperature and mass. *Example: When you blow bubbles underwater, they expand as they rise to the surface.*

Class Activity – 6:6

1. Use Boyle's law to explain why it is dangerous to heat even a small quantity of water in a sealed container.
2. A gas in a cylinder occupies a volume of 465 ml when the pressure on it is equivalent to 725 mm of mercury. What will be the volume of the gas when the pressure on it rises to 825 mm of mercury while the temperature is held constant? **(ANS: $V_2 = 408.6$ ml)**
3. Bubble of gas, which has a volume of 0.4 cm^3 , released by a diver 30 m in under the surface of a lake, what will be the volume of the bubble when it reaches the surface? (Assume the barometric pressure is 10 m of water.) **(ANS: $V_2 = 1.2 \text{ cm}^3$)**
4. Sketch a graph of the volume of a gas versus the pressure on the gas. What would the graph of V versus P look like if volume was directly proportional to pressure?
5. What will be the minimum pressure required to compress 500 dm^3 of air at 1 bar to 200 dm^3 temperature remaining constant. **[ANS; 2.5 bar]**
6. A steel cylinder of internal volume 20 litres is filled with hydrogen at 29 atmospheric pressure. If hydrogen is used to fill a balloon at 1.25 atmospheric pressure at the same temperature, what volume will the gas occupy? **[ANS; 464 L]**
7. **2 litres** of a gas is enclosed in a vessel at a pressure of 760 mmHg. If temperature remains constant, calculate pressure when volume changes to 4 dm^3 . **[ANS; 380mmHg]**
8. 800 cm^3 of gas is collected at 650 mm pressure. At what pressure would the volume of the gas reduce by **40%** of its original volume, temperature remaining constant? **[ANS; 1083.33 mmHg]**
9. A cylinder of 20 litres capacity contains a gas at 100 atmospheric pressure. How many flasks of 200 cm^3 capacity can be filled from it at 1 atmosphere pressure, temperature remaining constant?
ANS; N = 10,000 flasks.
10. **Why Boyles' law is important?**
ANS; Boyle's law explains the behaviour of gases. It proves that the pressure and volume of a gas are inversely proportional. When pressure is applied to a gas, the volume shrinks and the pressure rises.
11. 88 cm^3 of nitrogen is at a pressure of 770 mm mercury. If the pressure is raised to 880 mmHg, find by how much the volume will diminish, temperature remaining constant. **[$V' = V_2 - V_1 = 11 \text{ cm}^3$]**

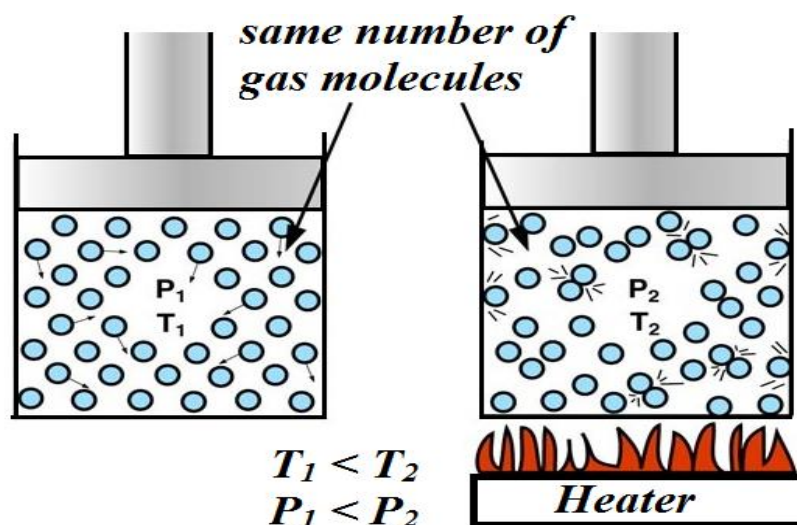
Application of Boyle's Law

- **Bubbles in water seem to grow as they ascend from the bottom of the water to the surface** .This is due to the decrease in Pressure
- **Death of deep sea creatures when brought to shallow waters** .This happens when the pressure inside their bodies is greater than the pressure of the surrounding water hence the balance is distracted causing a burst of the cells bladders and other internal biological structures due to increase in volume

- **Popping of ears at high altitude.** When the plane starts to rise it is going from an area of high pressure where your ears are accustomed to an area of low pressure causing the air inside increases in volume, this straining your eardrums
- **The operation of your lungs also can be explained using Boyle's Law.** When you **inhale** (breathe in), your **diaphragm** expands the volume of the lungs, causing a pressure drop that allows outside air to rush into the lungs (**inhalation**). When you **exhale** (breathe out), your diaphragm are relaxing, thus reducing the volume inside your lungs, increasing the pressure and forcing the air outwards.
- **Soda bottles or cans are consider a practical application of Boyle's law,** as all of us apply Boyle's Law but unintentionally. Note that when you open the bottle of soda quickly, the gas rushes from everywhere in the form of foam, causing a mess. So what is the cause of this mess? This mess occurs because the soda is pumped into the soda bottle by passing the water on carbon dioxide. **When you open the bottle, you are actually reducing the pressure on the gas, and the volume of the gas expands. If you remove the cap quickly, the gas pushes out of the bottle. Therefore, you should open the cap slowly and carefully until the gas comes out quietly.**
- **The working of a syringe can also be explained using Boyle's Law.** When the **plunger** of a syringe is pulled out, the volume inside the **barrel** increases, resulting in a decrease in the pressure inside the barrel. Fluids (such as water) flow from a high pressure area to a low pressure area. This means that once the pressure inside a syringe is lower than the pressure outside the syringe, a fluid near the **needle (e.g., water, medicine, etc.)** will flow into the syringe. The opposite is also true. When the plunger is pushed back in, the volume decreases and the pressure increases. Once the pressure is greater than that outside the syringe, the fluid inside the barrel will flow out.

Pressure Law (Gay Lussac's Law)

- This law involves the relationship between the temperature and the pressure of a fixed mass of a gas at constant volume. The law states that
“At constant volume, the pressure of a fixed mass of a gas is directly proportional to its absolute temperature”
- For example; If you heat a gas you give the molecules more energy so they move faster. This means more impacts on the walls of the container and an increase in the pressure. Conversely if you cool the molecules down they will slow and the pressure will be decreased. See the figure below



- Mathematically

$$P \propto T \quad \rightarrow \quad P = K T \quad \rightarrow \quad \frac{P}{T} = \text{Constant}$$

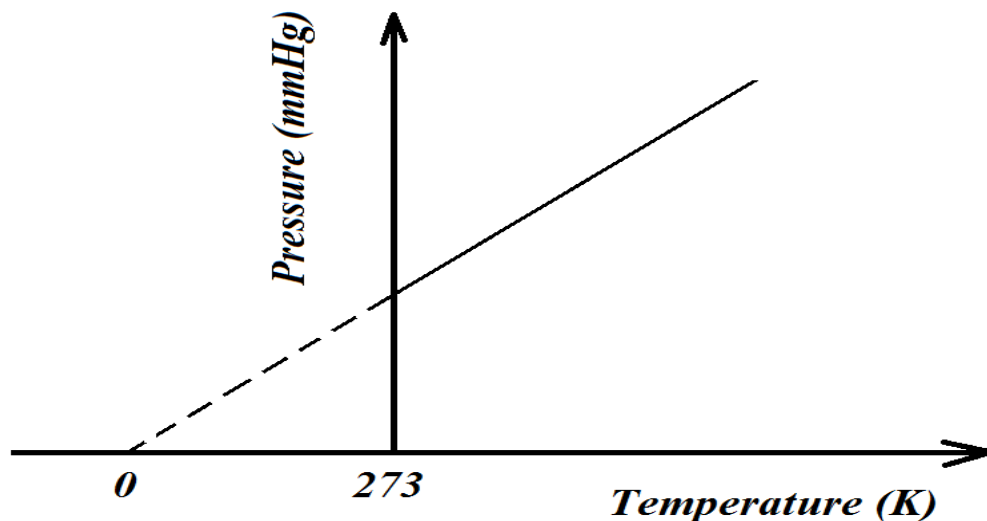
$$\therefore \frac{P_1}{T_1} = \frac{P_2}{T_2} \quad \rightarrow \quad \frac{P_1}{P_2} = \frac{T_1}{T_2}$$

Where:

P_1 = initial pressure, T_1 = initial temperature

P_2 = final pressure, T_2 = final temperature

Graphical representation of Gay Lussac's Law:



- From the graph above it seems that as pressure increases also temperature increases and vice versa

Worked Examples

- A vessel used for storing gas has a safety valve which blows off at 10^6 N/m^2 . It contains gas at a pressure of $8.0 \times 10^5 \text{ N/m}^2$ at 15°C . At what temperature would the valve start to blow off?
[ANS: 360K or 87°C]

ANS;

$$P_1 = 10^6 \text{ Nm}^{-2}, P_2 = 8 \times 10^5 \text{ Nm}^{-2}, T_2 = 15^\circ \text{C} = 273 + 15 = 288 \text{ K}, T_1 = ?$$

$$\text{From: } \frac{P_1}{P_2} = \frac{T_1}{T_2} \quad \rightarrow \quad \frac{10^6}{8 \times 10^5} = \frac{T}{288} \quad \rightarrow \quad T = \frac{10^6 \times 288}{8 \times 10^5} = 360 \text{ K}$$

- A car tyre is at an air pressure of $4.0 \times 10^5 \text{ Pa}$. at a temperature of 27°C . While it is running, the temperature rises to 75°C . What is the new pressure in the tyre? (Assume the tyre does not expand)

Answer

Since the tyre does not expand, this implies that the volume is constant

$$P_1 = 40000 \text{ Pa}, P_2 = P, T_1 = (27 + 273) = 300 \text{ K}, T_2 = (75 + 273) = 384 \text{ K}$$

$$\text{From: } \frac{P_1}{P_2} = \frac{T_1}{T_2} \quad \rightarrow \quad \frac{40000}{P} = \frac{300}{384} \quad \rightarrow \quad P = \frac{40000 \times 384}{300} = 51,200 \text{ Pa}$$

Significance of Pressure Law (Gay Lussac's Law)

- Pressure in well-inflated tyres of automobiles is almost constant. But on a hot summer day, **this increases considerably and the tyre may burst if pressure is not adjusted properly.** Similarly, during winters, on a cold morning, we find the pressure in the tyres of a vehicle decreased considerably.

Real life examples of Gay Lussac's Law (Pressure law)

- (i) **Pressure cooker:** Applying heat to a pressure cooker increases the pressure inside the device. Increasing pressure **raises the boiling point of water**, shortening cooking times. Because the container is sealed, flavors aren't lost to the air with steam. When a pressure cooker is kept on a heating source (stove). As per Gay-Lussac's law, the pressure of the fluid in the cooker increases with the rising of the temperature.
- (ii) **Aerosol can:** The reason you shouldn't store aerosol cans under hot conditions or dispose of them by burning is because heating the can increases the pressure of its contents, potentially causing the can to burst.
- (iii) **Gun bullet.** When the bullet from a gun is ignited, the chemical energy stored in the shell of the bullet is converted into heat by chemical reactions. This heat increases the temperature which as per Gay-Lussac's law increases the pressure. Because of the high pressure, the bullet is fired from the gun.
- (iv) **Automobile tyres.** The rupture of automobile tyres on subjection to high temperature is a classic example of Gay-Lussac's law. The high temperature pressurizes the air inside the tyres and beyond a point, they explode.
- (v) **Soda or soft drinks bottles are made of thick glass.** This is because, when gas is heated in a closed container its pressure increases. Hence they are made of thick glass to withstand pressure increase.

NB;

Gay-Lussac's law is very similar to Charles's law, with the only difference being the type of container. Whereas the container in a Charles's law **experiment is flexible**, while **it is rigid** in a Gay-Lussac's law experiment

Class Activity – 6:7

1. A rigid metal container holds carbon dioxide gas at a pressure of 2×10^5 Pa and a temperature of 30°C . What temperature the gas be lowered for the pressure to reduce to half (1×10^5 Pa)?
ANS: $T_2 = 151.5\text{K} = -121.5^\circ\text{C}$
2. A gas in a fixed-volume container has a pressure of 1.6×10^5 Pa at a temperature of 27°C . What will be the pressure of the gas if the container heated to a temperature of 277°C ?
ANS: $P_2 = 2.93 \times 10^5$
3. A student comes to school by a bicycle whose tire is filled with air at a pressure 240 kPa at 27°C . She travels 8 km to reach the school and the temperature of the bicycle tire increases to 39°C . What is the change in pressure in the tire when the student reaches school?
[ANS; 249.6kPa]
4. An LPG cylinder can withstand a pressure of 14.9 atmosphere. The pressure gauge of the cylinder indicates 12 atmosphere at 27°C . Because of a sudden fire in the building, the temperature rises. At what temperature will the cylinder explode? **[ANS; $T=99.5^\circ\text{C}$]**
5. A 30l sample of nitrogen inside a rigid metal container at 20°C is placed inside an oven whose temperature is 50°C . The pressure inside the container at 20°C was at 3.00 atm. What is the pressure of the nitrogen after its temperature is increased to 50°C ? **[ANS; 3.31atm]**

The General Gas Equation

- Any two of the three gas laws can be used to derive the general gas law or equation

$$V \propto T \quad (\text{Charles' law}) \dots\dots\dots 1$$

$$P \propto \frac{1}{V} \quad (\text{Boyle's law}) \dots\dots\dots 2$$

$$P \propto T \quad (\text{Pressure law}) \dots\dots\dots 3$$

Combine equation 2 and 3

$$P \propto \frac{T}{V} \rightarrow P = k \frac{T}{V} \dots\dots\dots \text{make } k \text{ the subject} \quad \text{Then; } \frac{PV}{T} = \text{Constant}$$

$$\boxed{\therefore \frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}}$$

Standard Temperature and Pressure (STP)

- STP is a set of conditions for experimental measurements to enable comparisons between sets of data.
- The standard temperature is 0 °C (273 K) while the standard pressure is 1 atmosphere (1.013 x 10⁵ Pa or 760 mm of mercury).

Worked examples

1. **125 cm³ of gas are collected at 15 °C and 755 mm of mercury pressure. Calculate the volume of this gas at s.t.p. (standard temperature and pressure).**

ANS: given; P₁ = 755 mmHg, V₁ = 125 cm³, T₁ = (273+15) = 288 K.

P₂ = 760 mmHg, T₂ = (273 + 0) = 273 K, V₂ = ?

Since $\frac{PV}{T} = \text{Constant}$, it follows that $\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$

$$V_2 = \frac{P_1 V_1 T_2}{P_2 T_1} = \frac{125 \times 755 \times 273}{760 \times 288} = 117.7 \text{ cm}^3$$

Therefore, the volume of gas at s.t.p. $\approx 118 \text{ cm}^3$

2. A bicycle pump contains 50 cm³ of air at 17 °C and at 1.0 atmosphere pressure. Find the pressure when the air is compressed to 10 cm³ and its temperature rises to 27 °C.

From; $\frac{PV}{T} = \text{Constant}$, it follows that $\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$

$$P_2 = \frac{P_1 V_1 T_2}{V_2 T_1} = \frac{1 \times 50 \times 300}{10 \times 290} = 5.17 \text{ atm}$$

Therefore, the pressure at 10 cm³ and 27°C $\approx 5.2 \text{ atm}$

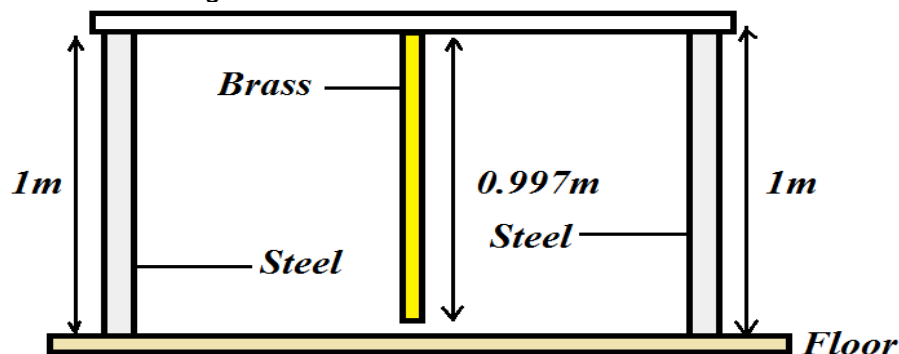
Class Activity – 6:8

1. A sample of oxygen gas has a volume of 0.11 m³ at a temperature of 12 °C and a pressure of 8.1 x 10⁵ Pa while a sample of nitrogen gas has a volume of 0.18 m³ at a temperature of 22 °C and a pressure of 1.03 x 10⁵ Pa. Which gas will have the larger volume at STP?
(ANS: V₂O = 0.084 m³, V₂N = 0.17 m³ At STP, nitrogen gas would have a volume that is more than twice the volume of oxygen gas.)
2. A fixed mass of gas has a volume of 1.25 litres at a pressure of 76.0 cm of mercury and a temperature of 27.0°C. The gas expands to a volume of 1.55 litres raising the pressure to 80.0 cm of mercury. What is the final temperature of the gas in °C? **(ANS: T₂ = 391.58K = 118.58°)**
3. A fixed mass of gas occupies a volume of 0.001 m³ at a pressure of 76 cmHg. What volume does the gas occupy at 17.0°C if its pressure is 72 cmHg? **(A: V₂ = 1.12 x 10⁻³ m³)**
4. It is found that on heating a gas its volume increases by 50% and its pressure decreases to 60% of its original value. If the original temperature was -15°C, find the temperature to which it was heated. **[ANS; T₂ = -40.8°C or 232.2K]**
5. 2500 cm³ of hydrogen is taken at STP. The pressure of this gas is further increased by two and a half times (temperature remaining constant). What volume will hydrogen occupy now?
[ANS; V₂ = 714.29 cm³]

6. 50 cm³ of hydrogen is collected over water at 17 °C and 750 mmHg pressure. Calculate the volume of a dry gas at STP. The water vapour pressure at 17 °C is 14 mmHg.
ANS; Pressure of dry hydrogen, $P_1 = 750 - 14 = 736\text{mmHg}$, $P_2 = 760\text{mmHg}$
 $V_1 = 50\text{cm}^3$, $T_1 = 17 + 273 = 290\text{K}$, $T_2 = 273\text{K} \rightarrow V_2 = 45.58\text{cm}^3$
7. 100 cm³ of gas A was collected at 10°C and 78.0 cmHg pressure, while 120 cm³ of gas B was collected at 50°C and 70.0 cmHg pressure. Which of the two gases is denser at STP? **ANS:** **At STP, gas B has large volume than gas A so gas A is denser than gas B .** That is $V_{A2} = 99.00\text{ cm}^3$ and $V_{B2} = 93.42\text{ cm}^3$
8. 250 cm³ of a gas are collected at 25°C and 750 mm of mercury. Calculate the volume of the gas at STP **(ANS: $V_2 = 226.01\text{ cm}^3$)**
9. A sample of carbon dioxide occupies 30 cm³ at 15°C and 740 mm pressure. Find its volume at STP. **[ANS; 27.7cm^3]**
10. At 0°C and 760 mmHg pressure, a gas occupies a volume of 100 cm³. Kelvin temperature of the gas is increased by one-fifth and the pressure is increased one and a half times. Calculate the final volume of the gas **[ANS; $V = 80\text{ cm}^3$]**

Self Assessment – 6

- Define coefficient of linear expansion. A copper pipe which is 1 meter long at 15° C increases in length by 0.15% when carrying steam at 100° c, find the coefficient of linear expansion of copper
- Explain why a compound metal bar made up of two strips, one of iron and another of brass, bends when heated**
- A beaker containing water is heated a temperature of 23° C to 90° C. State and explain what happens to the Mass, volume and density of water
- An aluminium lid on an ordinary glass jar fits so tightly that it cannot be unscrewed .Should the jar and lid be immersed in hot or cold cold water to loosen the lid?. Explain your answer
- A steel tower has a height of 324 m at a temperature of 18° C .How tall is the tower on a day when the temperature is 35° C?
- A brass rod of length of 0.997 m at 20° C is hung from a steel framework with a height of 1.00 m at 20° C as shown in the figure below.



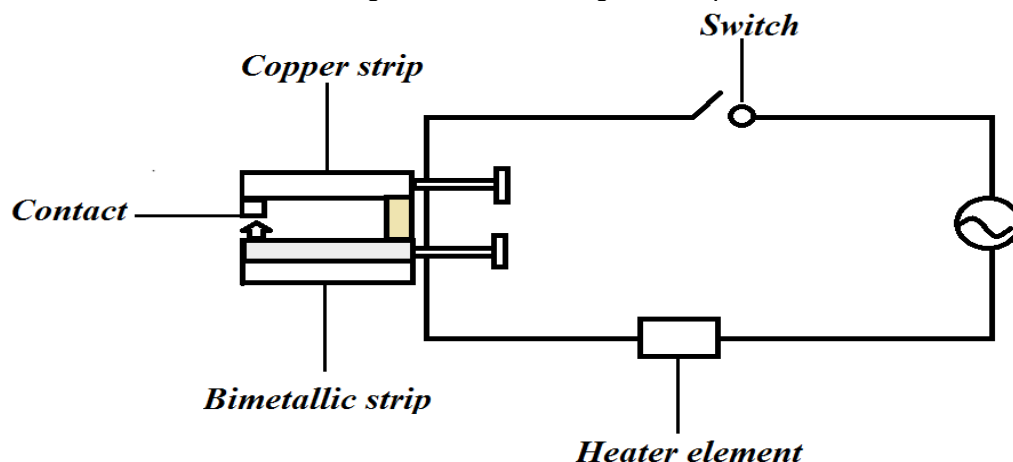
- At what temperature would the brass rod just touch the floor?
- A brass has a hole whose radius is too small for an iron rivet to fit in .Explain two ways the rivet can be made to fit in the hole
 - Convert the following temperatures on the Celsius scale to temperatures on the Kelvin or absolute scale (a) 100° C (b) 25° C (c) -100° C**
 - Convert the following temperatures on the Kelvin scale to temperatures on Celsius scale: (a) 273K (b) 400K (c) 100K
 - State Charles law and describe how it is verified in laboratory.1000 cm³ of air at 0°C are heated to 70°C. What volume will the air occupy if the pressure remains at atmospheric throughout?**

11. The pressure of 440cm^3 of the gas is 80cm of mercury. What will be the new pressure of the gas if its volume is reduced to 400cm^3 at constant temperature?
12. The pressure in a metal glass cylinder at 15°C is 2 atmospheres. At what temperature will the pressure be doubled?
13. When is a given mass of a gas said to be at s.t.p? The volume of a gas collected at a temperature of 36°C and pressure of 78cm of mercury is 230cm^3 . Find its volume at s.t.p.
14. The figure below shows a brass invar bimetallic strip at room temperature

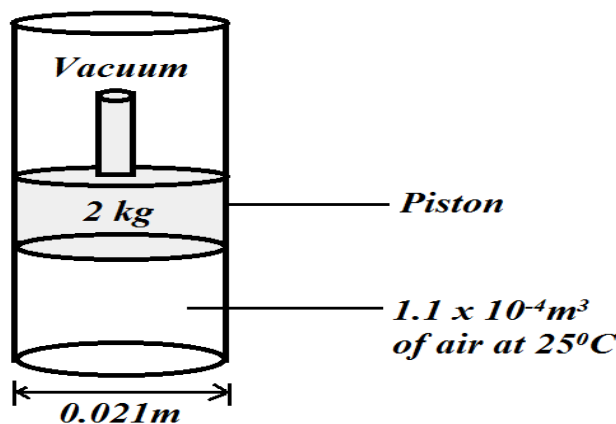


Given that brass expands more than invar when both are heated equally, sketch the appearance of the strip after being cooled to several degrees below room temperature

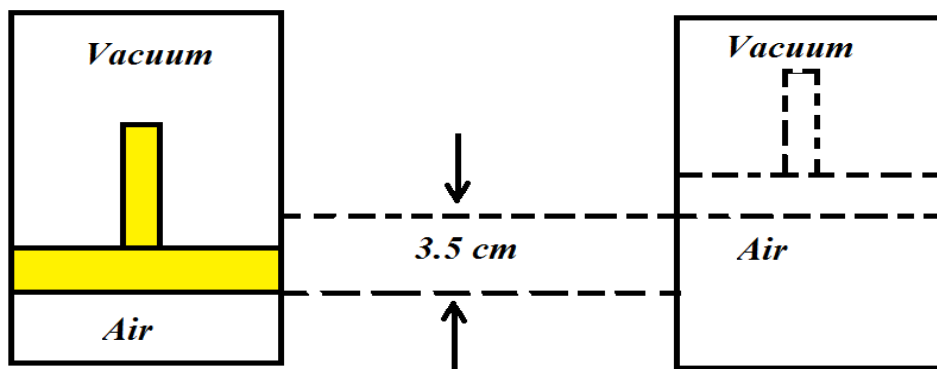
15. A glass test tube was heated over a Bunsen burner flame. Cold water was then quickly poured into the test tube. Explain why the test tube would break when cold water is poured in.
16. A rally car tyre is at an air pressure $3 \times 10^5\text{ Pa}$ and a temperature of 27°C at start of the rally. The temperature rises to 57°C when the car is racing. Assuming the tyre does not expand, what is the new pressure in the tyre?
17. The pressure of 3 m^3 of a gas at 27°C is 3 atmospheres. What will be the pressure of the gas if it is compressed into a half the volume and heated to 102°C ?
18. (a). Define the term linear expansivity of a solid .
(b).The original length of a metal bar is 101.5cm at 15°C . Determine the linear expansivity of the metal if the bar increases in length by 1.41mm when the temperature is raised to 100°C
19. Why electrical cables are left sagging during installation?
20. A metal rod 80cm long increased in length by 0.09 cm when the temperature was raised by 93.6°C .Determine linear expansivity of metal.
21. The air in a bicycle tyre occupies a volume of 1000 cm^3 when it is at a pressure of 2.5 atmospheres the air is released to the atmosphere
 - (i) Assuming that the temperature of the gas does not change, what volume does it occupy at the atmosphere
 - (ii) A pump with a volume of 150 cm^3 per stroke is used to inflate the tyre .What is the pressure of the tyre after two strokes?
22. An iron rod is 100 cm long at 0°C . What must be the length of aluminium rod at 0°C if the difference between the length of the two rods are to remain the same at all temperatures? (Linear expansivities of iron and aluminium are $1.2 \times 10^{-5}\text{ K}^{-1}$ and $2.4 \times 10^{-5}\text{ K}^{-1}$, respectively)
23. A gas occupies a volume of 2m^3 when its pressure is 1140mmHg at a temperature of 27°C . What volume will it occupy at s.t.p?
24. The figure below shows a circuit diagram for controlling the temperature of a room



- (a) State and explain the purpose of the bimetallic strip.
 (b).Describe how the circuit controls the temperature when the switch is closed.
25. A container holds a gas at 0°C .To what temperature must be heated to its pressure to double? (Assume that the volume of the container does not change)
26. Explain why a glass container with thick walls is more likely to crack than one with thin walls when a very hot liquid is poured in each of the glasses
27. A balloon is filled with air to a volume of 200ml At a temperature of 20°C .The balloon is then dipped in water at 80°C .Assuming two leakage occurs and ignoring the pressure change due to the water. Calculate the new volume of air.
28. A compound strip of brass and iron, 10 cm long at 20°C ,is held horizontally with iron uppermost. When heated from below with a Bunsen burner the temperature of the brass is 820°C and that of the iron is 770°C .Calculate the difference in lengths of the iron and brass
 (ANS: $\Delta l = 0.062\text{ cm}$)
29. State any three applications of bimetallic strip
30. Using the kinetic theory of gases explain how a rise in the temperature of gases causes a rise in the pressure of a gas when a volume is held constant.
31. The pressure indicated by the gauge on a constant -volume gas thermometer in a thermal equilibrium with a room is 97 kPa .When the thermometer was immersed in a bath of ice water, the pressure was 90kPa .What is the temperature in the room in $^{\circ}\text{C}$?.
32. Helium gas at temperature of -30°C is held in a rigid metal container at a pressure of 1.5×10^5 pa. The container is heated to a temperature of 25°C .W hat is the new pressure of the gas?
33. A metal rod is 10 m long at 20°C .At what temperature would its length increases by 5cm if its linear expansivity is $2 \times 10^{-6}/\text{K}$ [ANS; $T_2 = T_1 + \frac{\Delta L}{L_1 \alpha} = 20 + \frac{0.05}{10 \times 2 \times 10^{-6}} = 2520^{\circ}\text{C}$
34. A cylinder closed at both ends as an inner diameter of 0.021 m. The cylinder is fitted with a movable piston of mass of 2 kg. The space between the piston and the bottom of the cylinder contains $1.11 \times 10^{-4}\text{ m}^3$ of air at 25°C while the space above the piston has been evacuated as shown in the figure below.



- (a) Determine the pressure of the air in the cylinder given that the pressure comes from the weight of the cylinder
- (b) The cylinder is placed over a source of heat causing the air to expand and push the piston upward a distance of 3.5cm as shown in the figure below



Assuming that the pressure of the air remained constant, what was the change in volume of the air? (Volume of a cylinder = $\pi r^2 h$)

35. Distinguish between heat and temperature
36. (a) State Charles' Law ,Boyle's Law and the Pressure Law
 (b) Write down the ideal gas equation
 (c) The volume of a certain gas at 10^0 C is 100 cm^3 .Calculate the volume of the gas if it is warmed at a temperature of 30^0 C at constant pressure
37. Explain each of the following observations:
 (a) A lid on a metal can be unscrewed easily if the can is immersed in hot water for a few minutes
 (b) Corrugated iron – sheet roofs make cracking noises on a night preceded by a hot day
 (c) It is difficult to unscrew wheel nuts in the morning, while it is relatively easy to unscrew them on a hot day.
38. Define ; (a) Thermal expansion (b) Linear expansivity
 (c) Absolute zero temperature (c) Anomalous expansion of water
39. Differentiate between
 (a) Apparent expansion of liquid and absolute expansion of liquid
 (b) Heat and temperature
40. What do you understand by intermediate expansion of liquid?
41. Show how the combined gas law is obtained
42. Explain the following observations:
 (a) An inflated balloon hung in the open at a wedding bust when the temperature of the environment rises
 (b) A whale cannot survive in a shallow water
 (c) Soda bottles are thick
 (d) Water bubbles seem to increase in size as they rise from the bottom of a tank
 (e) Electric wire are seem to sag when its hot but look very straight when its cold
 (f) Fish living in polar regions such as Antarctica do not die even when the temperature fall below 0^0C
43. Mention two demerits of anomalous expansion
44. (a) State Boyle's law.
 (b) Sketch the graph of pressure (P) against the reciprocal of volume ($1/v$) for air at constant temperature
 (c) A bubble of air of volume 50.0mm^3 is released by a diver at a depth where the pressure is 304.0 cm Hg .Assuming that the temperature remains constant ,what is its volume just before reaches the surface where the pressure is 76.0 cm
45. A piece of copper is dropped into water, if the temperature of the water is rising what is happening to the copper?

46. Why are the over head power cables more likely to break and fall during the cold season of the year than during the warm season of the year even though they carry the same weight all year round?
47. A grandfather's clock is controlled by a swinging brass pendulum of length 1.3 m at a temperature of 20°C. ($\alpha = 19 \times 10^{-6} \text{ } ^\circ\text{C}^{-1}$)
- (a) What is the length of the pendulum rod when the temperature drops to 0°C?
- (b) If the period of the pendulum is given by $T = 2\pi \sqrt{\frac{l}{g}}$, where L is its length, does the change in the length or the brass rod cause the clock to run fast or slowly?
48. A gas is contained in an 8 litre vessel at a temperature of 20°C and pressure of 9 atm. At what temperature will the gas fill 3 litre vessel at a pressure of 13 atmospheres?
49. The temperature of a body is 47°C. What would this temperature be in the absolute scale?
50. Given that at s.t.p a gas occupies 5600cm³, determine the pressure at which it will occupy the volume of 28.5 litres at a temperature of 220°C.
51. States Charles' Law. An ideal occupies a volume of 500 cm³ at a temperature of 30⁰ C. At what temperature will it occupy a volume of 456 cm³
52. Explain why inflated balloon swells up and even bursts when in the open on a hot day?
53. A form three student carried out an experiment on one of the gas laws. She obtained the following results

Temp T (°C)	10	35	60	80	90	110
Volume V (cm ³)	5.0	5.8	6.4	7.0	7.2	7.8

- (a) Plot a graph of volume V against temperature
- (b) From the graph, determine the volume of the gas at 0⁰ C
- (c) Determine the slope of the graph
54. State Boyle's Law. A gas occupies a volume of 600 cm³ at a pressure of 760 mmHg. Determine its volume at a pressure of 1085 mmHg
55. The volume of a bubble at the base of a container of water is 3 cm³. The depth of water is 30 cm. The bubble rises up the column until the surface.
- (a) Explain what happens to the bubbles as it rises up the water column
- (b) Determine the volume of the bubble at a point 12 cm below the water surface
56. Explain the following
- (a) Deep sea animals cannot survive at regions with shallow waters
- (b) Determine the volume of the bubble at a point 12 cm below the water surface

Chapter: 07

Transfer of Thermal Energy

Methods of transfer of thermal energy

- Heat transfer may be defined as the transmission of the energy from one region to another region as a result of the temperature gradient.
- Heat can be transferred from one place to another in three ways, these include
 1. Conduction
 2. Convection
 3. Radiation

Heat transfer by conduction

- Have you ever left a **metal spoon** in a pot of soup being heated on a stove? After a short time, **the handle of the spoon** will become hot. This is due to transfer of heat energy from molecule to molecule or from atom to atom.
- Conduction is a very effective method of heat transfer in metals. However, air conducts heat poorly.
- **Thus; Conduction** is the transfer of heat energy from one substance to another or within a substance **OR;**
- **Conduction** is the transfer of heat through matter from a region of higher temperature to a region of lower temperature

How conduction takes place?

- **In solids**, molecules are closely arranged so that they cannot move freely. When one end of the solid is heated, molecules at that end absorb heat energy and vibrate fast at their own positions. These molecules in turn collide with the neighboring molecules and make them vibrate faster and so energy is transferred. This process continues till all the molecules receive the heat energy.

*molecules in solid objects don't
"move" (they just vibrate or "jiggle")*



- The process of transfer of heat in solids from a region of higher temperature to a region of lower temperature without the actual movement of molecules is called **Conduction**.

Good and Bad Conductors

Good Conductors

- Are the materials that allow heat to flow through them easily.
- For example, silver, aluminium, iron etc.
- **Silver** – you can find spoons made of silver. If use a spoon to eat some hot soup, heat will be transferred to the spoon from the soup and it will become very hot.
- **Aluminium** – some saucepans are made of aluminium, allowing the food in the saucepan to get heated quickly.
- **Iron** – when we iron a shirt on an ironing board, heat from the shirt is conducted to the shirt to remove any creases.
- Conductors have different rate of heat conduction. Example, copper is the best conductor of heat while steel is the poorest conductor.

Bad Conductors (Thermal Insulators)

- *Are materials that cannot allow heat to flow through them easily*
- For example, air, wood, plastic, wool, glass etc.
- **Air** – air is a very bad conductor of heat. This explains why we find it in between the two panes of glass in double-glazed windows, as less heat will be lost.
- **Wood** – we often see the handles of saucepans made of wood. This means less heat will be transferred from the metal pan to the handle, so we don't burn our hands when we hold the handle.
- **Plastic** – found on the handle of an electric kettle. So when you boil water to make a cup of tea, the plastic acts as an insulator so you won't burn yourself when pouring the water into your mug.
- **Wool** – insulators like wool can trap air to reduce heat loss, for example in fleece winter jackets

Factors affecting the rate of Conduction

1. Length of the material

- The length of the material the energy must flow through can affect the rate at which it flows. The shorter the length, the faster it will flow. Therefore, the conducted heat, **Q** is inversely proportional to the length of the bar, l : $Q \propto \frac{1}{l}$

2. Cross-sectional area that is perpendicular to the heat flow

- The larger the cross-sectional area, the faster the rate of heat conduction. A bar twice as wide conducts twice the amount of heat.
- In general, the amount of heat conducted, **Q**, is proportional to the cross-sectional area, **A** i.e., $Q \propto A$

3. Difference in temperature between the two ends of the material

- The greater the difference in temperature between the two ends of the bar, the greater the rate of thermal energy transfer, so more heat is transferred. The heat, **Q**, is proportional to the difference in temperature, i.e., $Q \propto \Delta T$

4. Thermal conductivity of the material.

- This is a measure of the rate at which a material conducts heat. The higher the thermal conductivity of the material, the faster the rate of conduction

Minimizing heat losses by Conduction

- In a system where heat needs to be conserved, heat losses by conduction can be minimized by thermal insulation

- This involves the use of poor conductors of heat used in boilers, hot – water pipes and in the textile industry
- In the house is achieved by using double – glazed windows , carpets curtains and draught excluders

Application of Good and Bad Conductors in everyday

Application of Good conductors

- Metals are good conductors of heat, therefore cooking utensils, kettles, irons, boilers, are always made up of iron, copper, Aluminum, or their alloys.
- Aluminium is used in making motor engines, pistons and cylinders due to its low density and high thermal conductivity
- Soldering iron is made of iron with a tip made of copper **because copper is a much better conductor of heat than iron.**
- When ironing a piece of clothing, **the iron is hot and the heat is transferred to the clothing**
- A heat exchanger uses a hot fluid to conduct heat **to a cooler fluid without the two actually coming into direct contact**

Application of Bad conductors

- The handles of saucepans, teapots, kettles, etc, are always made from substances like wood, plastic, etc, **to prevent heat from entering the hand so that it can be held comfortably**
- Air is a poor conductor of heat, **this is why heat from our bodies does not escape easily to the atmosphere**
- Fur, wool, cotton wool, cork, feathers, sawdust, trapped air in birds' feathers etc. act as good insulators.
- Fur and feathers keep most animals and birds warm during winter.
- Birds flutter their feathers in cold weather to trap air between their feathers
- Woolen clothes or blankets are used to keep people warm on cold days (winter).
- In cold countries, **glass windows in houses are double paned**
- Refrigerators, ice boxes, ovens, solar heater panels and connecting pipes are made of double walls with filling of cork, air, cotton wool or polyurethane in between their walls.
- Sawdust is used to cover up ice blocks **because it has good insulating properties.**
- Building materials like **brick, asbestos, mud, grass, etc., are bad conductors of heat.** They do not permit heat and cold to pass through the walls of bricks. They keep the houses warm in winter and cool in summer. **Roof sheds are made of asbestos for the same reason**
- Vehicles carrying inflammable materials such as petrol **are covered with materials of bad conductors of heat.** Otherwise, the petrol can get heated up and catch fire.
- The bottoms of cooking pots need periodic cleaning to remove layers of soot, which impede the flow of heat

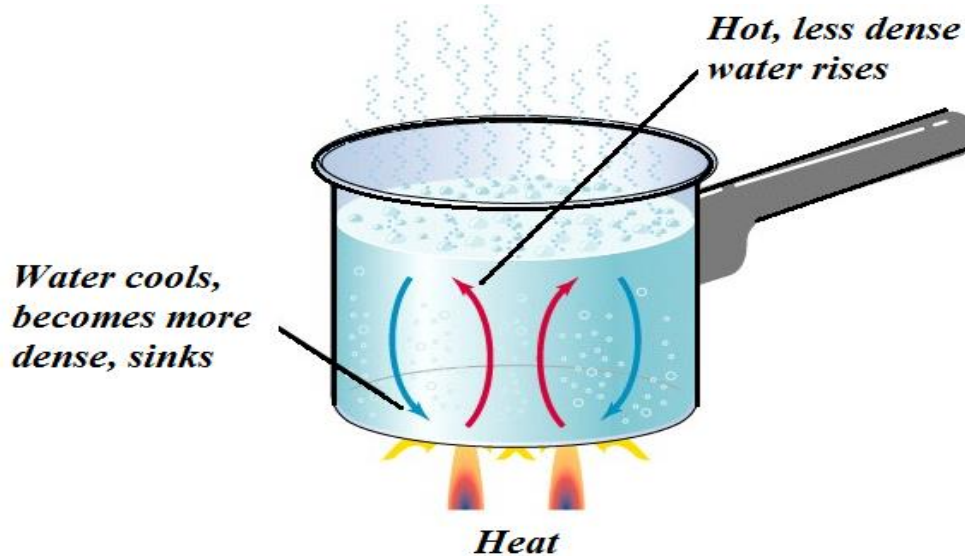
Convection of Heat

- **Convection** Is the movement of heat through fluids (liquids or gases) caused by movement of liquid from the hotter to the colder parts

How does convection Occurs?

- The **Figure** below shows how convection occurs, using hot water in a pot as an example. When particles in one area of a fluid (in this case, the water at the bottom of the pot) gain thermal energy, they move more quickly, have more collisions, and spread farther apart.

- This decreases the density of the particles, so they rise up through the fluid. As they rise, they transfer their thermal energy to other particles of the fluid and cool off in the process. With less energy, the particles move more slowly, have fewer collisions, and move closer together. This increases their density, so they sink back down through the fluid. When they reach the bottom of the fluid, the cycle repeats. The result is a loop of moving particles called a **convective current**.



How can we minimize Convection?

- ❖ It can be minimized by keeping the place vacuum, Example, vacuum flask minimizes heat loss by convection.
- ❖ It can also be reduced by filling air cavities with insulating materials

Application of Convection

(i) Chimneys

Smoke and gases from fires in houses and factories rise up chimneys and the flow is convection current. Gases pass up a tall chimney faster than up a short one, this is because high chimneys have greater pressure difference

(ii) Ventilation

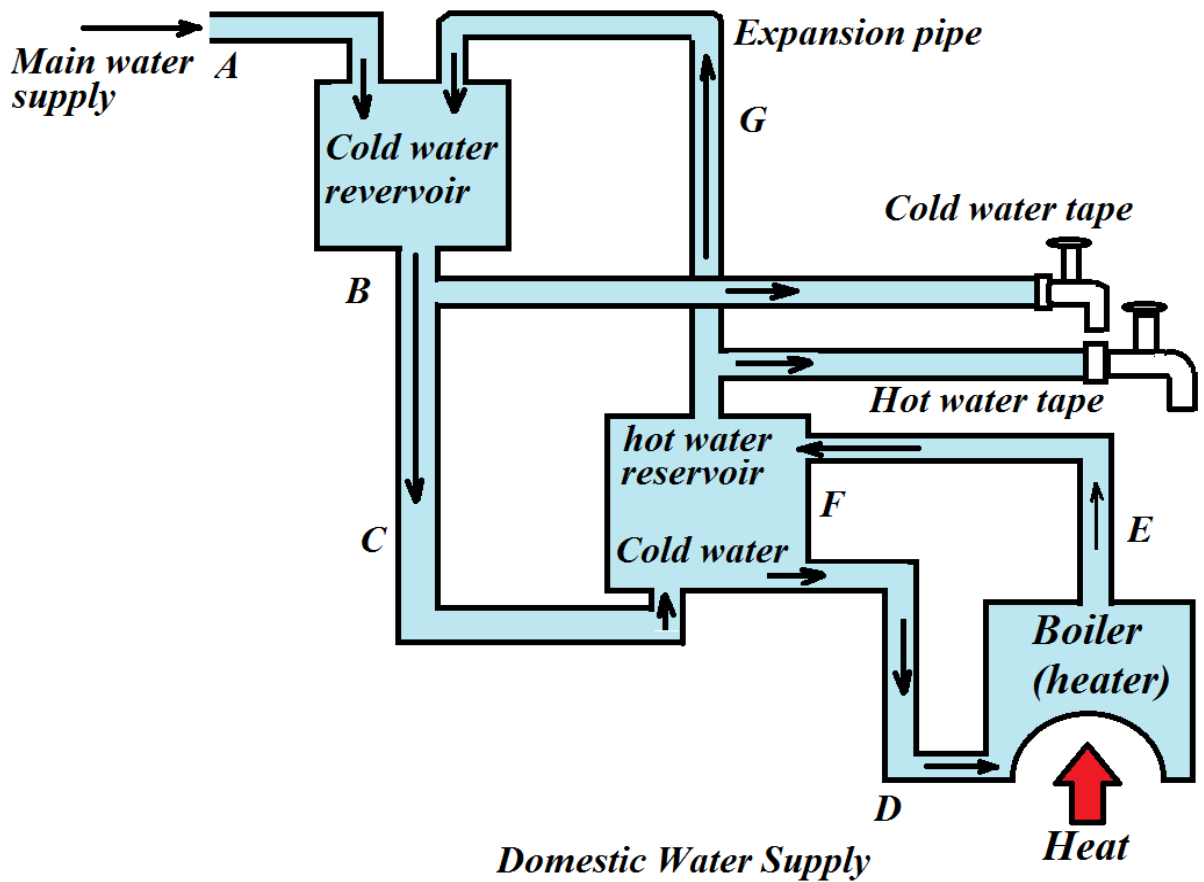
Damp, warm, breathed – out air is less dense than ordinary air; it rises and can escape through openings near the roof. Houses and cinemas have openings in or near their ceilings.

(iii) Air condition

- **On a hot summer day**, air-conditioners are used constantly. The process of cooling air in air-conditioners employs the principle of convection. The air which is cold is released by the air-conditioners. Now, this cold air is denser than the warm air, and, hence, it sinks. The warm air, being less dense, rises and is drawn in by the air-conditioner. As a result, a convection current is set up and the room is cooled.
- **When it is cold**, the heated air is turned on heating the surrounding air. The heated air rises up and cold air moves down to take the place of the rising warm air. This forms convectional current which continue to circulate until the air in the room is at the desired temperature

(iv) Domestic hot water supply system

This system works on the principle of Convection Current. Hot water moves from the boiler to where it is used under convection current. Inside the boiler water is heated making it less dense, water rises and flow into the hot water reservoir where it is stored until needed for use main water supply. See the figure below:



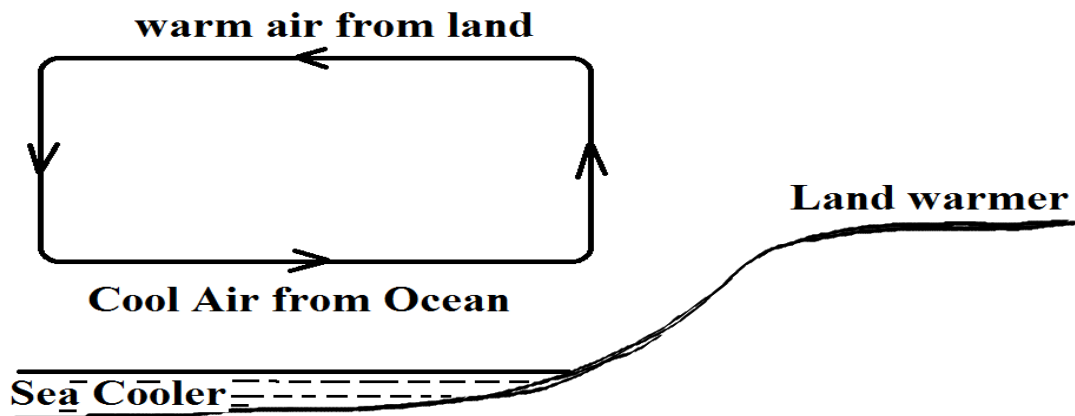
(v) **Winds**

Air flows from area of high pressure to area of low pressure. The warm air molecules over hot surface rise and create low pressure. So, cooler air with high pressure flows towards low pressure area. This causes wind flow.

(vi) **Land and sea breeze**

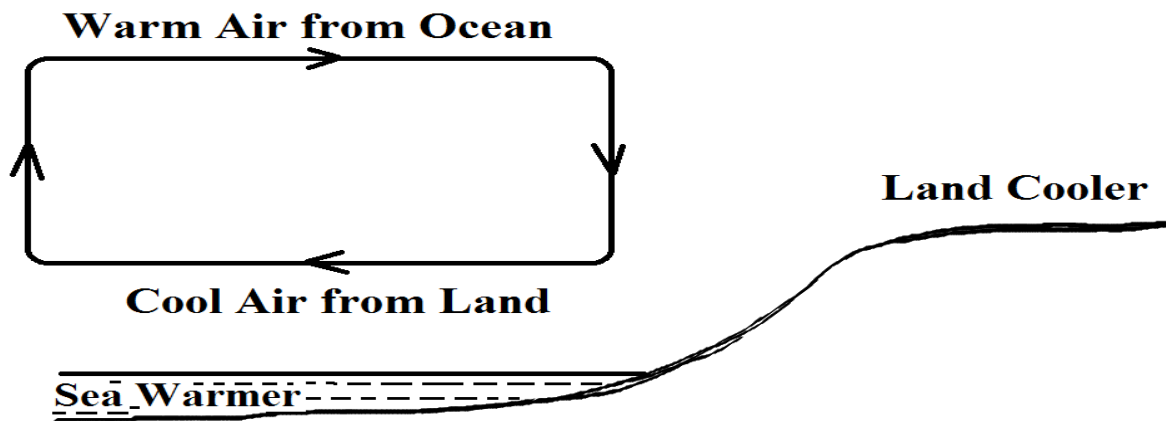
Land and sea breezes are a result of expansion of air caused by unequal heating and cooling of adjacent land and sea surfaces

- During the day time the land is warmer than the sea thus the air over the land surface becomes less dense and so it rises. The space left is occupied by cooler air from above the sea surface. **Thus sea breeze occur**



Sea Breeze- Day time

- During the night time the sea is hotter than the land thus the air over the sea becomes less denser and therefore replaced by cooler, denser air from the land . **Thus land breezes occur**

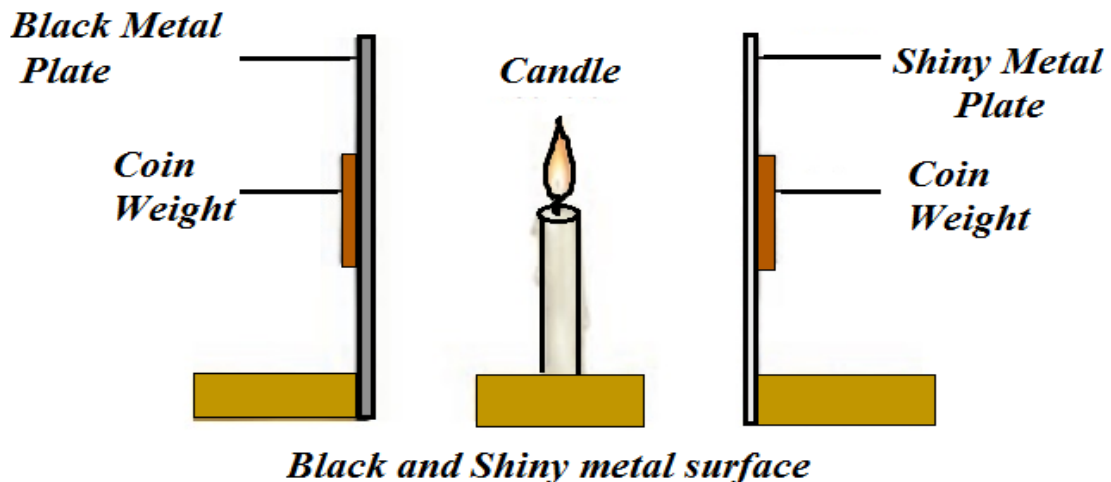


Land Breeze - Night time

- **N.B:** The land gets warmer quickly than the sea, because sea water has a higher heat capacity than the land (sea water needs more time to raise its temperature)
- (vii) **Motor – car cooling system**
Car engines are cooled by convection currents in the water pipes. **Water is a very good substance to carry the unwanted heat away from the engine to the radiator.**
- (viii) **Hot Air balloons**
Air molecules at the bottom of the balloon get heated by a heat source and rise. As the warm air rises, cold air is pushed downward and it is also heated. When the hot air is trapped inside the balloon, it rises.

Radiation of heat

- If you have stood in front of a fireplace or near a campfire, you have felt the heat transfer **known as radiation**. The side of your body nearest the fire warms while your other side remains unaffected by the heat. Although you are surrounded by air, the air has nothing to do with this transfer of heat.
- **Therefore; Radiation** is the transfer of heat energy through space by electromagnetic radiation.
- Transfer of heat energy from the sun reaches us in the form of radiation. Radiation is emitted by all bodies **above 0 K**.
- Some objects **absorb radiation** and some other **objects reflect them**.
- This can be shown using the demonstration set up shown in Figure below. In this figure the inside surface of one plate is shiny and of the other is dull black. Coins are stuck on the outside of each plate with candle wax. If the heater is midway between the plates they each receive the same amount of radiation.



- Therefore, after few minutes the wax on the black plate melts and the **coin falls off** while the shiny plate stays cool and the wax on it is **un-melted** and the coin does not falls off.

NB;

- Radiant heat travels with the speed of light and can be **reflected , absorbed and transmitted**
- **Heat** loss by radiation is minimized by covering a body (surrounding) **with shiny surfaces (polished surface).**

Radiant Detector

- There are two instruments which can be used to detect radiations. These are Thermopile and Liquid in thermometer
- **Thermopile T** is an instrument used to converts radiant heat energy into electrical energy

Absorbers, Emitters and Reflectors

Absorber

- Absorber is a material that delivers/gain all radiant energy.
- A surface that absorbs all radiant energy is called **black body.**
- Good radiators of heat are also good absorbers
- Example, black cooking vessel, **black clothes dry faster than others colored clothes etc**

Emitter

- Emitter is a material that delivers out all radiant energy.
- A surface that emits all radiant energy is called **black body**

Reflector

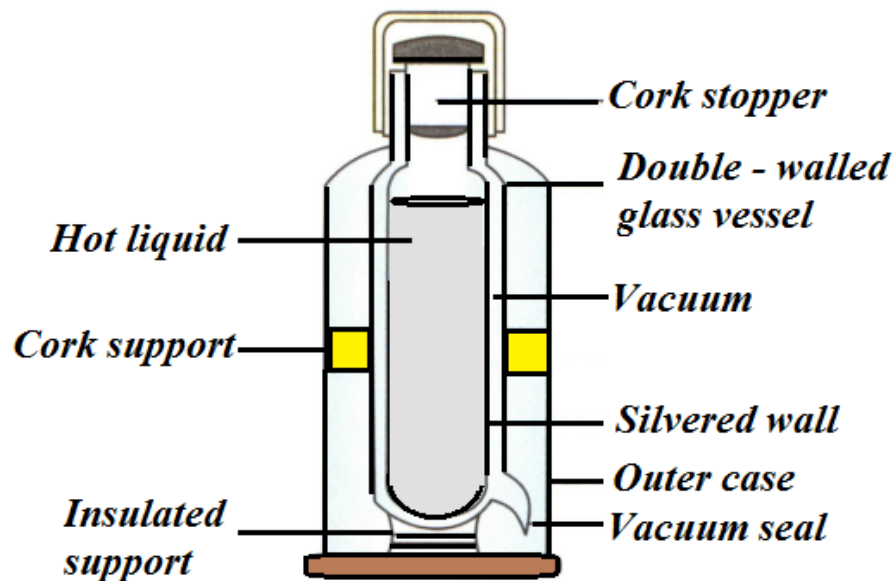
- Reflector is a material (surface) that bounces back all radiant energy.
- Example, white or bright – colored surfaces are poor emitters and absorbers of heat, solar cookers etc

Uses of good and bad radiators or absorbers

- **White clothes.** In hot countries white clothes are slightly cooler than other clothes
- **White buildings.** These are cooler than buildings with dark colours on the outside. Storage tanks containing oil or petrol are usually painted with aluminium paint. The polished, silvery surface is a bad absorber.
- **Vacuum flask.** This is used to keep hot liquids hot and cold substances cold

Thermos Flask (Vacuum Flask)

- Thermos flask is a device used to hold hot or cold liquid for long period of time.
- It consists of a double-walled glass (polished by coated with a thin layer of aluminium), container with vacuum between the walls. It has a stopper made of insulating materials or cork (See the fig. below)



How thermos flask prevents heat Loss?

- **Vacuum flasks** are designed to keep liquids hot by minimizing heat losses in three ways that is, conduction, convection and radiation.
 - Heat losses by conduction is minimized by
 - The stopper which is a bad conductor (made of wood, rubber or cork)
 - Glass flask (Poor conductor of heat)
 - The supporting pad (separator). It is also a poor conductor of heat
 - Heat losses by convection can be minimized by
 - Vacuum between the walls of the flask
 - The stopper which is a bad conductor (made of wood, rubber or cork)
 - **Heat loss by radiation** is harder to stop as radiant heat can pass through a vacuum. **To minimize heat loss through radiation, the walls of the glass are silvered so as to reflect the radiant heat back into the hot liquid.**

Metal Foil

- **Metal foil** is a polished material used to cover items to minimize heat loss by radiation. It is used to prevent heat loss by reflecting radiant energy
- For example, aluminium foils are used in wrapping hot food.

Applications of radiation

- Electric kettles, iron boxes and cooking pans have polished surfaces to reduce heat loss through radiation
- Houses in hot regions are painted white to minimize heat absorption through radiation
- Green houses act as heat trap by preventing longer wavelength radiations from passing through glass
- Cloudy nights are warmer than clear nights because clouds reflect radiation back to the earth
- We wear white or light-coloured clothes in summer **because they are poor absorbers and good reflectors of heat.** This way they keep us cool. On the contrary, we prefer to wear dark-coloured clothes in winters because **they absorb most of the heat of sun and keep our body warm.**

- Radiators of heat in cars, machines and air conditioners are painted black **so as to have cooling effect by radiating most heat.**
- Base of cooking utensils is made black. **Such a black surface absorbs more heat from the surroundings.** This is because conduction, convection and radiation of heat are minimum.
- In cold areas, a greenhouse is used for the better growth of plants. Radiation from the sun passes through the glass or plastic and warms up the soil and plants. **Plants and soil absorb and emit radiation and increase the temperature in the greenhouse. Plants grow well in the increased temperature of the greenhouse.**
- Teapots, the shiny surfaces are bad emitters of radiation, **shiny teapots can keep tea warmer than a black teapot for a longer time.**
- The cooling fins at the back of our refrigerator need to radiate their heat quickly to the surroundings. **Its surface is made rough and painted black.**
- When we sit beside a fire, **the heat of fire reaches us by radiation.**
- **Room (electric) heaters have bright polished surfaces which act as good reflectors of heat.** Such surfaces absorb very little heat and reflect towards us most of the heat radiations. These surfaces remain cool even after continuous use of heaters. The highly polished surfaces of spacecraft reflect most of the heat radiated from the sun.
- **Cloudy nights are warmer than clear nights sky**
Radiation from the sun reaches the earth and heats the earth's surface. The clouds act as a good reflector of heat energy. At night, the heat radiated from the earth is reflected by the clouds. This in turn makes the night warmer. On clear nights, there are no clouds in the sky to trap heat. So clear night become cool.

Self Assessment – 7

1. Explain the following:

- A wooden knob and a metal latch on a door are both at room temperatures. Why the latch is colder to touch.**
- Water is heated generally from below.**
- Land becomes warmer than water during the day.
- Why in cold countries windows have two glass panes with a thin layer of air between them?
- In winter the human body covered with a blanket keeps warm.**
- It is better to use thin blankets to keep the body warm rather than using a single blanket of thickness equal to their combined thickness.**
- In winter the birds fluff up their feathers.
- Old quilts are less warm than the new ones.
- On a hot sunny day, it is advisable to wear light coloured clothes.
- We feel warmer in a fur coat. Why?**
- If air is bad conductor of heat, why do we not feel warm without clothes?**
- Why snow is a better heat insulator than ice?**
- When glass and copper rods are heated up to some temperature and then if the hot areas are touched, why copper piece seems warmer than the glass rod?
- Ventilators are provided in rooms just below the roof. Why?
- Why coolant coil fitted on the ceiling of the refrigerator?**
- Why is it hotter at the same distance over the top of a fire than in front of it?**
- A body with a large reflectivity is a poor emitter. Why?**
- Why are steam pipes wrapped with insulating materials?**
- Animals curl into a ball when they feel very cold. Why?**

- (t) An ink dot on a cup of porcelain appears dark. When the same cup is heated to a high temperature, the dot becomes brighter than rest of the cup?
- (u) Why in summer days does the fan give hot air first in a closed room when it is switched on?**
- (v) Why are houses often built with high ceilings?**
- (w) A fan rotating in a room may cause you to feel much cooler but a thermometer in the room will show the same temperature as before the fan was switched on.

2. Explain the following:

- (a) Why is radiator of motor car painted black?
- (b) Water should not be used to put off electrical fires.
- (c) Heat from the sun heats up earth, but the air in the atmosphere does not get heated up.
- (d) The top of a roof made of asbestos is painted white**
- (e) A tile floor feels colder than a wooden floor when you walk on it, even when both are at same temperature. Why?
- (f) Stainless steel pans are usually provided with copper bottoms. Why?
- (g) When a pan is removed from the fire, it slowly cools down. Why does it cool down?**
- (h) A brass tumbler feels much cooler than wooden tray on a chilly day**
- (i) Why should air coolers be kept higher up in the room, and not down like the heaters?
- (j) why in hot water sugar is dissolved**
- (k) Why room heaters should be placed at the floor of the room?
- (l) It is hotter at the same distance over the top of a fire than it is on the side of it. Why?**
- (m) The bottoms of the cooking utensils are painted black. Give the reason.
- (n) Why is the freezing chest in a refrigerator fitted near its top**
- (o) Why is an oven made of double walls with the space in between filled with cork?
- (p) Why do we use cooking utensils made up of copper?
- (q) Why is a tea kettle provided with an ebonite handle?
- (r) In summer, ice is kept wrapped in a gunny bag. Explain the reason.**
- (s) the water pipes are covered with cotton during very cold water
- (t) Why are chimneys provided over furnace in factories?**
- (u) A house with thick walls is likely to be cooler during hot seasons.**
- (v) Eskimos make double walled ice houses. Why?**
- (w) Why no heat sensation is felt when a burning match stick is held in the fingers?
- (x) At what temperature would a block of wood and a block of metal feel equally cold or equally hot, when touched?
- (y) A cloudy night is hotter than a clear sky night. Why?**
- (z) Why do fire workers wear bright clothes?
3. Explain what is meant by the conduction of heat .Use the kinetic theory to explain how heat is transferred along a metal rod
4. Explain briefly the reason why steam pipes are covered with felt or asbestos
5. A piece of iron and a piece of wood are both cooled to a temperature of 0°C .When touched with a finger ,the iron feels colder than the wood .Why ?
- 6. Why are cloudy nights generally warmer than clear ones?**
7. Why are thermometers that are used in weather stations shielded from the sunshine? What does a thermometer measure if it is shielded from the sunshine and also if it is not?

ANS;

- Sunshine emits electromagnetic radiation that increases the heat energy surrounding the thermometer. As a result, the mercury in the thermometer will show a higher reading than the normal temperature of the environment when it is exposed to sunshine. So, if you are

shielding it from the sunshine, the thermometer will show the exact temperature that is required here. Therefore, if a thermometer is shielded from sunshine, it will measure the exact temperature of the environment.

- However, if it is not shielded from sunshine, the thermometer will show a higher reading than the actual temperature of the environment.

8. On average, would Earth be warmer or cooler without the atmosphere? Explain your answer.

Ans;

- **Atmosphere plays a crucial role in maintaining Earth's temperature.** It contains greenhouse gases that are transparent to solar radiation. These gases allow the sun's radiation to reach the Earth's surface. When the Earth's surface is heated, it emits infrared radiation. The greenhouse gases in the atmosphere trap this radiation and re-emit it back to the Earth. This helps to maintain Earth's temperature, and is responsible for a phenomenon known as global warming.

- **In the absence of an atmosphere,** the radiation from the sun hitting the Earth would heat up the ground, and then escape into space. Since there would be no greenhouse gases to trap this radiation, heat would quickly dissipate into space. As a result, the temperature of the Earth would decrease and it would become cooler.

9. Discuss the uses of good and bad conductors of heat energy in everyday life

10. What do you understand by convection? Describe how you would demonstrate the formation of convection currents in a liquid

11. Explain the following:

- (a) Cork is packed between the double walls of refrigerating chambers
- (b) Most cooking pots are made of Aluminium
- (c) Ice blocks are wrapped in paper when they are stored
- (d) Water tanks in the tropics are painted white
- (e) White clothes are worn in the Arctic in preference to dark ones

12. A thermometer having a blackened bulb records a higher than an ordinary thermometer when they are both held at an equal distance from a fire Explain this

13. Explain briefly how a fire can assist in the ventilation of a room

14. (a) Explain how heat transfer by radiation takes place.

(b) Why does heat transfer by radiation not require a medium?

(c) A good cooking vessel should be black on the outside and not shiny white. Explain.

(d) Give one way through which heat losses by radiation can be prevented.

15. List three areas where heat transfer through each of the following methods is applied:

- (a) conduction
- (b) convection
- (c) radiation

16. (a) explain how heat transfer by conduction takes place

(b) Why are gases poor conductors of heat

(c) Why are cooking vessels made of aluminium and not iron?

(d) Why are stadium seats made of plastic and not steel?

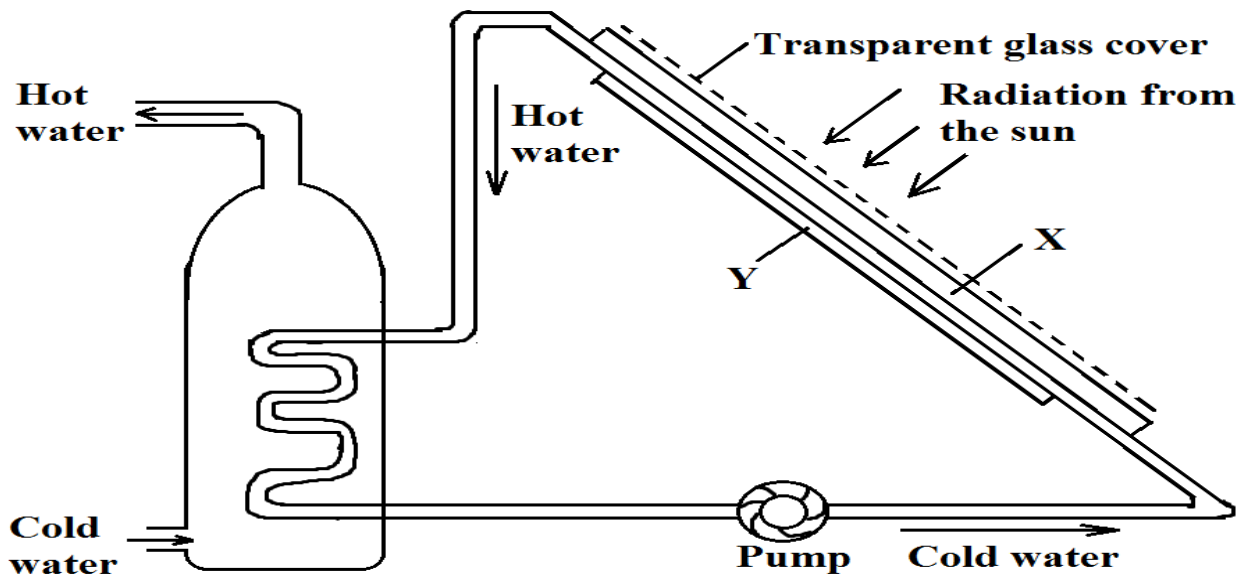
(e) Give two ways through which heat losses by convection can be prevented?

17. Why is it not possible for heat transfer by convection to take place in solids?

18. Explain the importance of making ventilation on the top of the walls in a room

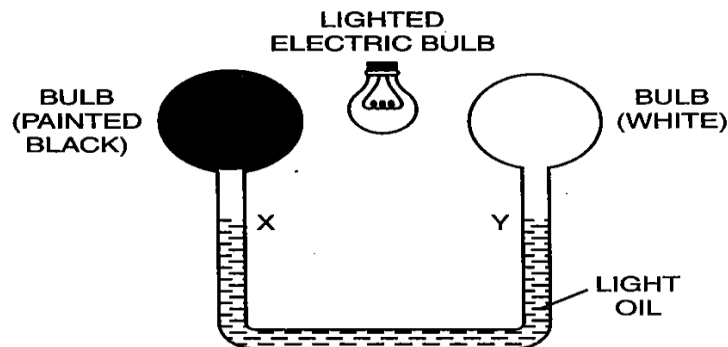
19. Explain briefly how heat travels in metals

20. **The figure below shows a solar panel used for heating water**



- What is the name of this type of electromagnetic radiation from the sun? What is the significance of this radiation?
- What color should the surface covered with a transparent glass sheet?
- Why is the surface covered with a transparent glass sheet?
- If no pump was available, how could the water be made to circulate naturally?
- Why is the solar panel tilted at an angle to the horizontal?
- What is the material Y behind the panel? Why is it used?

21. The diagram below shows two air filled bulbs connected by U – tube partly filled with a low density (or light) oil.



- Describe how the levels of oil in the limbs X and Y change after the electric bulb is lighted.
- Account for your observation.

22. How is heat loss by conduction ,convection and radiation reduced in a vacuum flask

23. Explain, how land and sea breeze occurs

Chapter: 08

Measurement of Thermal Energy

Heat Content (internal thermal energy)

- Heat content is the energy possessed by a body due to its temperature.

Factors that determine the heat Content

- You might have felt that the land is cool in the morning and hot during day time. But, water in a lake will be almost at a particular temperature both in the morning as well as in the afternoon.
 - Both are subjected to same amount of heat energy from the Sun, but they react differently. It is because both of them have different properties. In general the amount of heat energy absorbed or lost by a body is determined by three factors.
- (i) **Mass of the body** – The amount of heat energy required is directly proportional to the mass of the substance.
 - (ii) **Nature of material of the body** – The amount of heat energy required depends on the nature on the substance and it is expressed in terms of its specific heat capacity C.
 - (iii) **Rise in temperature of the body** – The amount of heat energy required is directly proportional to the rise in temperature.

Heat Capacity of a Substance

- ✓ Heat Capacity – Is the amount of heat required to raise the temperature of a given mass of a substance by 1Kelvin (K). It is denoted by letter **C**,
- ✓ The SI unit for the heat capacity of a substance is **Joule per Kelvin (J/K or JK⁻¹)**. When the temperature is increased by **1°C** is the same as an increase of **1 K**, which is the same unit as **J°C**.
- ✓ **Mathematically:**

$$\text{Heat capacity, } C = \frac{\text{quantity of heat absorbed}(Q)}{\text{change in temperature}(\Delta\theta)}$$

Worked Examples,

1. In an experiment to determine the heat capacity of steel, 100KJ of heat energy was supplied to a block of steel initially at 22°C. If the final temperature of the block was 219°C, determine the heat capacity of steel.

Soln:

Given: Q = 100 KJ = 100000 J, T₁ = 22°C, T₂ = 222°C, C = ?

From: Heat capacity, C = $\frac{\text{quantity of heat absorbed}(Q)}{\text{change in temperature}(\Delta\theta)}$

$$\therefore \text{Heat capacity, } C = \frac{100000}{222-22} = 500 \text{ J/K}$$

Class Activity – 8:1

1. Define the term heat capacity. 3000 J of heat is lost when the temperature of an iron rod reduces from 50^o C to 30.5^o C. Determine its heat capacity. (ANS: C = 153.85 J/K)
2. A solid with a heat capacity of 320 J/K requires 2000 J of heat to raise its temperature to 80^o C. Find its original temperature (ANS: T₁ = 73.75^o C)
3. An iron ball requires 5000 J heat energy to raise its temperature by 20^oC. Calculate the heat capacity of the iron ball. [ANS; 250J/K]

Specific heat Capacity

- Consider the following two observations below
 - (i) Quantity of heat required to raise the temperature of 1 litre of water will be more than the heat required to raise the temperature of 500 ml of water. If Q is the quantity of heat absorbed and *m* is the mass of the body, then; $Q \propto m$
 - (ii) Quantity of heat energy (Q) required to raise the temperature of 250 ml of water to 100 °C is more than the heat energy required to raise the temperature to 50 °C. Here, $Q \propto \Delta T$, where ΔT is the change in temperature of the body.
- Hence, from the above two observations, heat lost or gained by a substance when its temperature changes by ΔT is; $Q \propto m\Delta T$
- When removing the proportionality sign, then; $Q = mC\Delta T$
- From the above equations, the absolute temperature and energy of a system are proportional to each other. The proportionality constant is the specific heat capacity (C) of the substance.
- In order to understand the specific heat capacity of the substance, think of heating 500 ml of water and 500 ml of oil. Which will be heated first? Why? It is because heat gained by a body depends upon the nature of the substance (specific heat capacity), i.e.,.. **A substance with low specific heat capacity heats up and cools down rapidly while a substance with high specific heat capacity heats up and cools down slowly**
- The capacity of a substance to gain heat energy is denoted by the term specific heat capacity.
- Mathematically it is derived from the equation above as,

$$\text{Specific heat capacity, } C = \frac{\text{heat energy supplied}(Q)}{\text{mass} \times \text{temperature change}(\Delta T)}$$

- **Therefore;** Specific heat capacity of a substance is defined as the amount of heat required to raise the temperature of 1 kg of the substance by 1^oC or 1K.
- The SI unit of specific heat capacity is $\text{Jkg}^{-1} \text{K}^{-1}$. The most commonly used units of specific heat capacity are $\text{J/kg}^{\circ}\text{C}$ and $\text{J/g}^{\circ}\text{C}$.
- Among all the substances, water has the highest specific heat capacity and its value is **4200 J/kg °C**. So, water absorbs a large amount of heat for unit rise in temperature. Thus, water is used as a coolant in car radiators and factories to keep engines and other machinery parts cool. It is because of the same reason the temperature of water in the lake does not change much during day time.
- Specific heat capacities of some common substances are given in Table below.

Materials	Specific heat capacity (J/kgK)	Materials	Specific heat capacity (J/kgK)
Water	4200	Glass	700
Sea water	3900	Steel	500

Paraffin	2200	Copper	390
Methylated spirit	2500	Brass	320
Ice	2100	Iron	480
Mercury	1395	Lead	130
Aluminium	900	Zinc	380

Worked Examples:

1. An energy of 84000 J is required to raise the temperature of 2 kg of water from 60° C to 70° C. Calculate the specific heat capacity of water.

Solution;

From: **Specific heat capacity, C** = $\frac{\text{heat energy supplied}(Q)}{\text{mass} \times \text{temperature change}(\Delta T)}$

$$C = \frac{84000}{2 \times (70 - 60)} = 4200 \text{ J kg}^{-1} \text{ K}^{-1}$$

2. The specific heat capacity of a metal is **160J kg⁻¹K⁻¹**. Calculate the amount of heat energy required to raise the temperature of 500 gram of the metal from 125° C to 325° C.

Solution:

$$\text{From; } Q = m\Delta T C = 0.5 \times (325 - 125) \times 160 = 16,000 \text{ J}$$

3. Calculate the specific heat capacity of a body of mass 3 kg if it requires 6000 J of heat to raise its temperature from 30° C to 34° C

Soln:

Given: m = 3 kg, H = 6000 J,

T₁ = 30° C, T₂ = 34° C

From: **Specific heat capacity, C** = $\frac{\text{heat energy supplied}(Q)}{\text{mass} \times \text{temperature change}(\Delta T)}$

$$\therefore \text{Specific heat capacity, } C = \frac{Q}{m\Delta T} = \frac{6000}{3 \times (34 - 30)} = 500 \text{ J/kgK}$$

Class Activity – 8:2

1. Calculate the heat energy required to raise the temperature of 2kg of water from 10°C to 50°C. Specific heat capacity of water is 4200 JKg⁻¹ K⁻¹. **[ANS; 336,000J]**
2. How much heat is required to raise the temperature of a 25kg sample of mercury from 20°C to 30°C? **(ANS: H = 348750J)**
3. The temperature of a 6kg block of copper rises from 15°C to 30°C on being heated. Determine the amount of heat energy supplied to the block. (Specific heat capacity of block is 390Jkg°C) **(ANS: H = 35 100J)**
4. How much heat energy is given out by an iron block of 20g mass when it cools from 920°C to 20°C. **(ANS: H = 8 640J)**
5. How much heat energy is required to raise the temperature of a 3 kg sheet of glass from 24°C to 36°C?
[Specific heat capacity of glass = 840 J kg⁻¹ °C⁻¹] **[ANS; 30240J]**
6. Water in an ice maker of a refrigerator has a mass of 0.4 kg and a temperature of 22°C. What is the temperature of the water after 33 600 J of heat has been removed from it?
[Specific heat capacity of water = 4200 J kg⁻¹ °C⁻¹] **[ANS; 2°C]**
7. A piece of copper of mass 40 g at 200° C is immersed into a copper calorimeter of mass 60 g containing 50 g of water at 25° C .Neglecting heat losses ,what will the final temperature of the mixture be ? **(ANS: 36.0° C)**

8. A brass cylinder of mass X was heated to 100°C and then transferred into a thin aluminium can of negligible heat capacity containing 150 g of paraffin at 11°C . If the final steady temperature of the paraffin attained was 20°C , Determine the value of X (**A: $X = 0.116\text{ Kg}$**)
9. Some heat energy is given to 120 g of water and its temperature rises by 10K . When the same amount of heat energy is given to 60 g of oil, its temperature rises by 40K . The specific heat capacity of water is $4200\text{J Kg}^{-1}\text{ K}^{-1}$. Calculate:
 - (i) The amount of heat energy in joule given to water. **[ANS; 5040J]**
 - (ii) The specific heat capacity of oil. **[ANS; 2100J/kgK]**

Determination of Specific Heat Capacity of Substance

- In determination of specific heat capacity of substance two methods are used
 - a) Method of mixtures
 - b) Electrical method

(a) Method of mixture

- If the heat loss controlled when mixing the water, the heat energy gained by the cold water is equal to the heat energy lost by hot water due to the principle of conservation of energy.

Calorimetry

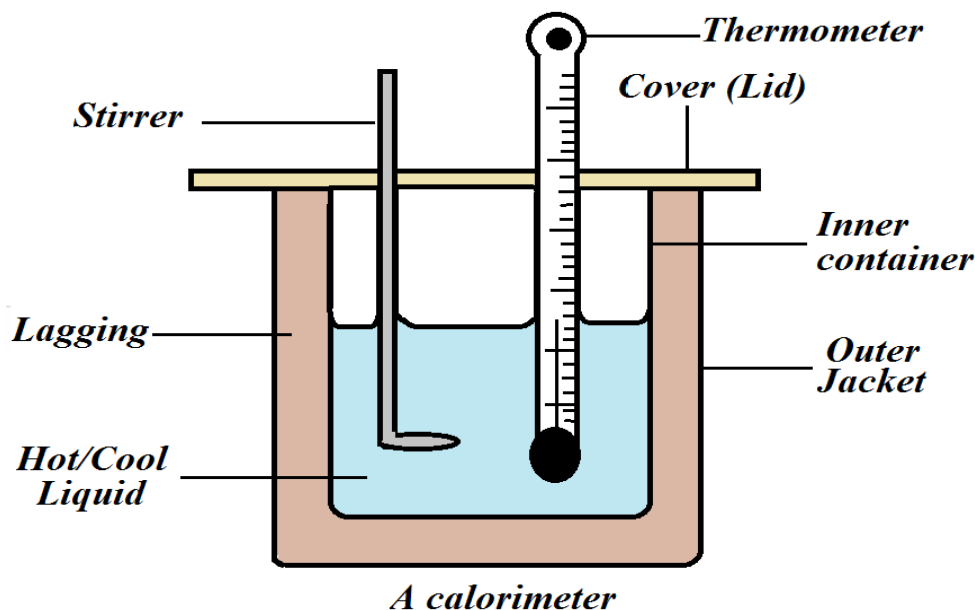
- Calorimetry means the measurement of the amount of heat released or absorbed by thermodynamic system during the heating process.
- When a body at higher temperature is brought in contact with another body at lower temperature, the heat lost by the hot body is equal to the heat gained by the cold body. No heat is allowed to escape to the surroundings. It can be mathematically expressed as

$$Q_{\text{Gain}} = -Q_{\text{Lost}} \quad \rightarrow \quad Q_{\text{Gain}} + Q_{\text{Lost}} = 0$$

- Heat gained or lost is measured with a calorimeter. Usually the calorimeter is an insulated container of water as shown in Figure below.

Calorimeter

- Is a device used to control the losses of heat energy when determining specific heat capacities of substances



How Specific heat Capacity is determined

- If a liquid of known mass (m_L) and initial temperature (θ_i) is put in the inner container (Let's say a calorimeter of mass m_C at θ_i) and a **hot substance** of known mass (m_S) at its initial temperature, (θ_S) is added to the liquid.
- **N.B:** the initial temperature of a liquid (θ_i) = Initial temperature of the calorimeter (θ_i)
- If θ is the final temperature of the mixture **and assume some heat is being absorbed by the calorimeter**, then to determine the specific heat capacity of **A SUBSTANCE**, C_S can be calculated as follows:

Heat lost by substance = heat gained by liquid + heat gained by calorimeter

$$m_S C_S (\theta_S - \theta) = m_L C_L (\theta - \theta_i) + m_C C_C (\theta - \theta_i)$$

$$C_S = \frac{m_L C_L (\theta - \theta_i) + m_C C_C (\theta - \theta_i)}{m_S (\theta_S - \theta)}$$

- Therefore the specific heat capacity of the substance is given by:-

$$C_S = \frac{m_L C_L (\theta - \theta_i) + m_C C_C (\theta - \theta_i)}{m_S (\theta_S - \theta)}$$

N.B

- Under the assumption that, if heat is not absorbed by any of the apparatus used to carry out this experiment. Then

Heat lost by substance = heat gained by liquid

$$m_S C_S (\theta_S - \theta) = m_L C_L (\theta - \theta_i)$$

$$C_S = \frac{m_L C_L (\theta - \theta_i)}{m_S (\theta_S - \theta)} \dots \dots \dots (\text{case II})$$

Assumption made when using this method:

- *No heat is lost to the surrounding*
- *Heat is not absorbed by any of the apparatus used to carry out this experiment. (case II)*

Precautions to be taken when carrying out such experiments

- Use a highly polished calorimeter so as to minimize heat loss by radiation
- The calorimeter should be heavily lagged so as to minimize heat loss by conduction
- The calorimeter should be covered with a lid of poor conductor so as to prevent heat loss by evaporation and convection.

Worked Examples

1. A block of metal of mass 5 kg is heated to 110⁰ C and then dropped into 1.5 kg of water. The final temperature is found to be 50⁰ C. What was the initial temperature of the water? (Specific heat capacity of metal = 460 Jkg⁻¹ K⁻¹)

Soln: Let initial temperature of the water be θ

$$\begin{aligned} \text{Heat lost by block} &= mC\Delta\theta \\ &= 5 \times 460 (110 - 50) = 5 \times 460 \times 60 = 138\,000\text{J} \end{aligned}$$

$$\begin{aligned} \text{Heat gained by the water} &= mC\Delta\theta \\ &= 1.5 \times 4200 \times (50 - \theta) = 6300 \times (50 - \theta) = 315000 - 6300 \theta \end{aligned}$$

From: **Heat gained by water = Heat lost by the metal block**

$$315\,000 - 6300 \theta = 138\,000$$

$$6\,300\theta = 315\,000 - 138\,000$$

$$\therefore \theta = \frac{177000}{6300} = 28.1^{\circ}\text{C}$$

2. Determine the final temperature when a 25.0 g piece of iron at 85.0⁰C is placed into 75.0 grams of water at 20.0 ⁰C. Given specific heat capacity of iron is 480J/kg/K and that of water is 4200J/kg/K **[ANS; 22.25⁰C]**
3. If 5 L of water at 50 ⁰C is mixed with 4L of water at 30 ⁰C, what will be the final temperature of water? Take the specific heat capacity of water as 4184 J kg⁻¹K⁻¹. **[ANS; 41⁰C]**
NB; Suppose if we mix equal amount of water ($m_1 = m_2$) with 50⁰C and 30⁰C, then the final temperature is average of two temperatures.

(b) **By electrical method**

Electrical energy supplied by heater = heat gained

$$\therefore ItV = mC\Delta\theta$$

Worked Examples:

1. A block of metal of mass 1.5 kg which is suitably insulated is heated from 30⁰ C to 50⁰ C in 8 minutes and 20 seconds by an electric heater coil rated 54 W. calculate:
 - (a) The quantity of heat supplied by the heater
 - (b) The specific heat capacity of the block

Solution:

$$(a) \quad \text{Heat energy supplied} = \text{power} \times \text{time} \rightarrow Q = Pt = IV \times t$$

$$Q = 54 \times [(8 \times 60) + 20] = 54 \times (480 + 20) = 54 \times 500 = 27\,000\text{ J}$$

$$\therefore \text{heat energy supplied} = 27\,000\text{J}$$

$$(b) \quad \text{From: } Q = mc\Delta\theta$$

$$27\,000\text{J} = 1.5 \times C \times (50 - 30) \rightarrow 27\,000 = 1.5 \times C \times 20$$

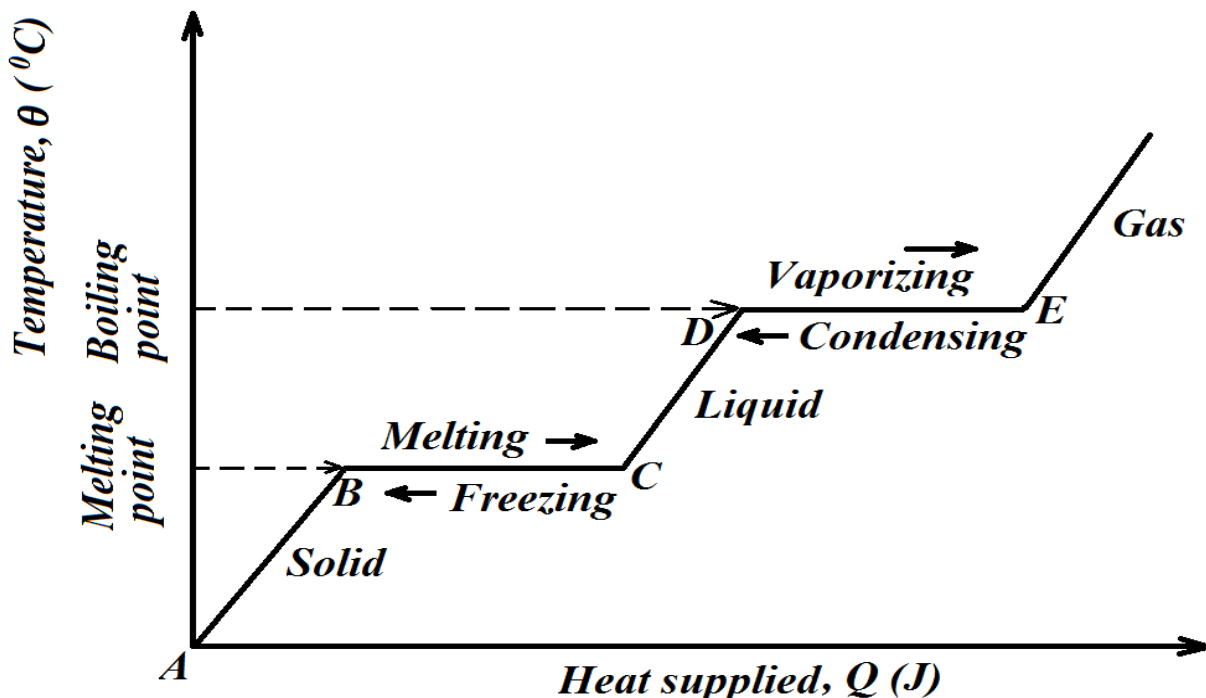
$$\therefore C = \frac{27000}{30} = 900\text{ J/kgK}$$

Class Activity – 8:3

- 2.5 kg of a metal at 200^o C is immersed into 1.2 kg of cold water of temperature 20^o C. The mixture, after thoroughly stirring attained a final temperature of 34.5^o C. Given that the specific heat capacity of water is 4200 J/kgK, determine the specific heat capacity of the metal. (ANS: $C_{\text{metal}} = 176.63 \text{ J/kgK}$)
- A block of metal of mass 0.20kg at a temperature of 100^oC is placed in 0.40kg of water at 20^oC. If the final steady temperature of the water is 24^oC, determine the specific heat capacity of the metal. (Neglect heat absorber by the container) (ANS: $c = 442.1 \text{ J/kg K}$)
- A block of aluminum of mass 0.5kg at a temperature of 100^oC is dipped in 1.0kg of water at 20^oC. Assuming that no thermal energy is lost to the environment, what will the final temperature of the water be at thermal equilibrium? (A: $\theta_F = 27.7^{\circ}\text{C}$)
- A mechanic dropped a steel nut of mass 0.02 kg and temperature 90^oC into 0.25 kg of water at 24^oC in a polystyrene cup. What is the temperature when the steel nut and water have come to thermal equilibrium?
[Specific heat capacity of water = 4200 J kg⁻¹ °C⁻¹; Specific heat capacity of steel = 450 J kg⁻¹ °C⁻¹] Assume that the exchange of heat is between the steel nut and water only.
[ANS; 24.56^oC]
- An electric kettle with a power rating, P can heat up 4.0 kg of water from 30^oC to 100^oC in 10 minutes.
 - Calculate the power, P of the kettle.
 - What assumption must you make to arrive at the answer?
[Specific heat capacity of water = 4200 J kg⁻¹ °C⁻¹]
ANS;
(c) $Pt = mC\Delta\theta \rightarrow P = \frac{mC\Delta\theta}{t} = \frac{4 \times 4200 \times 70}{10 \times 60} = 1960W$
 - All the heat supplied by the heater of the kettle is absorbed by the water. No heat is lost to the surroundings or absorbed by the kettle
- At a certain section of the Victoria Falls in Africa, water drops vertically through a height of 480 m.**
 - Explain why the water at the base of the waterfall has a temperature slightly higher than the water at the top.
 - Estimate the maximum possible difference in the temperature between the water at the base and at the top of the waterfall. (Take $g = 10 \text{ m s}^{-2}$)
ANS;
(a) As the water falls, it loses potential energy. Some of its potential energy is converted heat energy to increase the temperature of the water
(b) Assume all the potential energy is lost by the falling water is converted to heat energy;
 $\therefore mgh = mC\Delta\theta \rightarrow \Delta\theta = \frac{gh}{c} = \frac{480 \times 10}{4200} = 1.14^{\circ}\text{C}$
- In a ballistics test, a bullet travelling at a velocity of 360 m s⁻¹ is stopped by a stationary sand bag. If 20% of the energy lost by the bullet is converted to heat energy that is absorbed by the bullet. What is the increase in temperature of the bullet? [Specific heat capacity of the bullet = 150 J kg⁻¹ °C⁻¹]
ANS; $\frac{20}{100} \times \frac{1}{2}mv^2 = mC\Delta\theta \rightarrow \Delta\theta = \frac{20\% \times v^2}{2c} = \frac{0.2 \times 360^2}{2 \times 150} = 86.4^{\circ}\text{C}$
- A 10kW drilling machine is used to drill a bore in a small aluminium block of mass 8.0kg. How much is the rise in temperature of the block in 2.5 minutes, assuming 50% of power is used up in heating the machine itself or lost to the surroundings? Specific heat of aluminium = 910Jkg⁻¹ K⁻¹. [ANS; 103^oC]

Change of State

- As you know, matter can exist in different states. These states include solids, liquids and gases
- When matter changes its state, its internal energy changes, so the kinetic energy of its constituent particles changes. As it is changing from one state to another, the change in energy is reflected in the bonds between the particles, and therefore the temperature of the object doesn't change.
- Once the state change is complete, however, changes in energy are again observed in the form of changes in temperature. (see the figure below)



Explanations from the figure above

- ✓ From point A the temperature is rising steadily as more heat is added until point B where the solid starts to melt with steady temperature from point B to C
- ✓ Once the melting is over, addition of heat leads to a steady rise in temperature in region CD. D is the boiling point of the liquid. Then the liquid is transforming into gas (after vaporization occurs in DE)
- ✓ As the point E is reached, all the liquid has been converted into gas
- ✓ Reversing the process is also possible by removing the heat from gas reduces its temperature from point E to D, D to C, C to B and finally from B to A (Cooling takes place)

Melting and freezing Point of a substance

- Melting and freezing refer to the changes in state which occur when the solid and liquid states interchange. Melting occurs when a solid is heated and turns to a liquid and freezing occurs when a liquid is cooled and turns to a solid.

Melting Point

- **Melting point** is the temperature at which a substance changes from a solid to a liquid.
- At melting point the substance absorbs heat but the temperature does not change until the substance has completely melted.

- **Pressure affects melting.** Increase in pressure lowers the melting point of a substance while decreasing in pressure raises the melting point

Freezing Point

- **Freezing point** is the temperature at which a liquid changes into a solid without a change in temperature.
- During solidification a substance loses heat to its surrounding but its temperature does not fall
- Freezing occurs at the same temperature as melting, hence, the melting point and freezing point of a substance are the same temperature. Example, water freezes and melts at 0°C

Factors affecting freezing point

- **The presence of impurities** in a substance leads to a lower freezing point, this process is called freezing point depression. Freezing point depression is the reason why adding salt to frozen substances help to melt the ice. Freezing point depression occurs to the character of a material's solid state.
- In terms of physical changes, you can also alter the freezing point by changing the pressure. Normally, if you reduce the pressure, you can reduce the temperature at which a liquid freezes. However, in the case of water, the more you increase the pressure, the more its freezing point decreases.

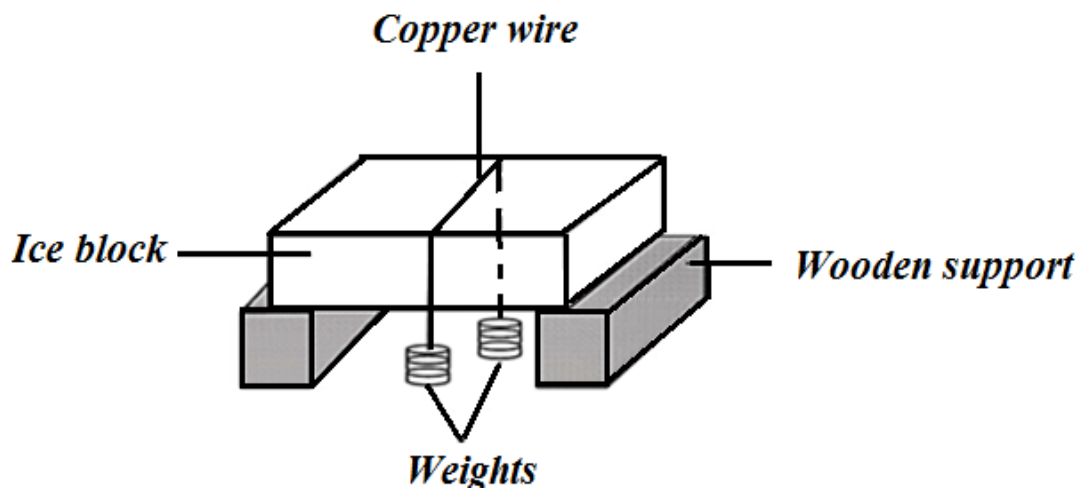
Regelation

- **Regelation** is basically the phenomenon where the ice melts to the water below 0°C when pressure is applied and it refreezes back to ice when the pressure is removed (reduced).

Consider an illustrative diagram below;

In this experiment, an ice block is set on the table, a copper wire connected to weights is placed over the ice block.

- Because of the hanging weight, **the wire exerts pressure** on the ice block, the high pressure lowers the melting point of ice at the point of contact
- The ice block absorbs the heat from the wire so the region in contact with the wire melts and lowers itself to water below 0°C and as the wire passes through, the water freezed back again resulting in the successful passing through of the wire without cutting the ice block in half.
- At the end of the experiment, the ice block will still be in one piece, even though the wire would have completely passed through it.



Effects of Regelation:

- Glacier acts as a source of a river due to **Regelation**. The mass of the glacier exerts pressure on the lower surface lowering the melting point of the ice at its base. This results in the melting of ice and propels the glacier to slide over the liquid. Under appropriate conditions, liquid water flows from the base of the glacier to lower altitudes when the temperature of the air is above the freezing point of water.
- The ice slab is shredded into pieces, and the shredded pieces are pressurized around the tip of a stick to prepare the ice ball. If two small pieces of ice are taken and pressed against each other, they stick to each other.
- Skating is possible on snow due to the formation of water only for the Regelation. Water is formed due to the increase of pressure and it serves as a lubricant.

Boiling and Evaporation

Boiling (Ebullition)

- Is the process by which a liquid turns into a vapor when it is heated to its boiling point

Boiling Point

- Is the temperature at which all of a liquid changes into a gas

OR

- Is the temperature at which its saturated vapour pressure becomes equal to the external atmospheric pressure.

Mechanism of Boiling

- The molecules at the surface of the liquid gain more kinetic energy move faster and are able to overcome intermolecular forces holding them together and hence escape.

What happens when a liquid boils?

- If a liquid is heated its temperature begins to rise, and therefore the saturated vapour pressure will increase. Ultimately, the saturated vapour pressure becomes equal to the external atmospheric pressure
- At this stage the further addition of heat will cause bubbles of vapour to form inside the body of the liquid and rise to the surface.

Factors affecting boiling point

- The factors that affect the boiling point of a liquid are its pressure and Impurities.

(i) Boiling Points Change with Changes in Pressure

- Since liquids boil when their vapor pressure becomes equal to surrounding pressure, then if the surrounding pressure is lower, liquids will boil at lower temperatures.
- At sea level water boils at 100°C (The atmospheric pressure is at its maximum). As one goes to higher altitude, the atmospheric pressure keeps decreasing, thus decreasing the boiling point of water. For example, in cities whose altitude is around 5,000 feet, water boils at 95°C instead of at 100°C and at 10,000 feet, water boils around 90°C .
- Water boils faster at the top of mountain than at the bottom, **this is because the higher the altitude, the lower the pressure**. Thus at the top water will boil at lower temperature than it does at sea level (bottom). This means that it requires less energy and therefore a shorter heating time to **reach its boiling point**.

- Water in a pressure cooker boils at a very high temperature nearly 120⁰ C due to the high pressure created in the cooker. That is why food cooked in a pressure cooker or rice cooker takes less time to get ready than food cooked in common cooking pots

(ii) Boiling point change with presence of impurities

- The boiling point of a liquid can be increased by adding impurities in the liquid. This occurs because the presence of impurities decreases the number of molecules available to become vaporized during boiling. Impurities include salt, sugar, and other dissolving molecules. Generally, impurities increase the boiling point of a liquid.
- **This explains why sea water boils at a higher temperature than pure / distilled water**

Applications of boiling at increased and reduced Pressure

- Boiling under increased pressure is important for fast cooking
- Boiling under reduced pressure is used in preparation of drug tablets

Evaporation

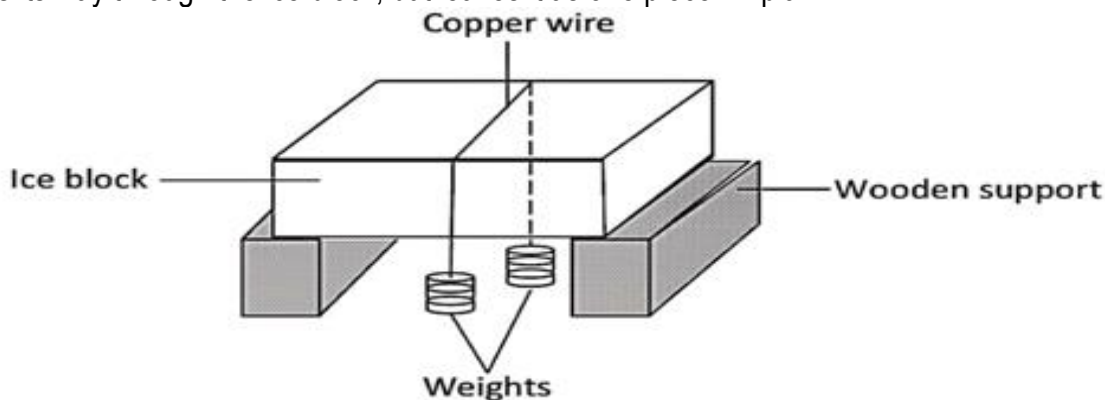
- Evaporation – Is the process through which a liquid changes to vapour (gas) at a temperature below its boiling point
- Evaporation occurs when heat energy forces the bonds which hold the water molecules to break, causing the water to shift from its liquid state to a gaseous state
- If the water is instead kept in a closed container, the water vapor molecules do not have a chance to escape into the surroundings and so the water level does not change.
- **Evaporation** is more rapidly when there is **windy, sunny** and **less humidity**

Difference between Boiling and Evaporation

Boiling	Evaporation
Occurs at a definite temperature which is boiling point	Occurs at any temperature
Accompanied by formation of bubbles	No bubbles
Occurs throughout the liquid	Occurs at the surface of the liquid
Has no cooling effect	Has cooling effect
Takes place rapidly	Takes place slowly

Class Activity – 8:4

1. The figure below shows a block of ice with two heavy weights hanging such that the copper wire/string connecting them passes over the block of ice. It is observed that the wire gradually cuts its way through the ice block, but leaves it as one piece. Explain



2. Explain why does food cook faster in a pressure cooker rather than in common cooking pot?
3. Why it is advisable to use pressure cooker at high altitude?

ANS; At **higher altitude** atmospheric **pressure** is lower than near the sea. This causes the water to boil at a lower temperature (below 100). Due to this it takes longer time for food to be cooked. A **pressure cooker** prevents boiling of water at lower temperature and leads to quicker **cooking** of the food. That's why it is advisable to use pressure cooker at high altitude

4. Explain the variation of boiling point with altitude and why are we using a pressure cooker at high altitude for cooking?

Latent heat (Hidden heat)

- The energy required to change a specific material's state is known as **the material's latent heat of transformation (L)**. When an object changes from the solid to liquid phase, you use the latent heat of fusion (L_F).
- When an object changes from the liquid to the gaseous phase, you use the latent heat of vaporization (L_V).
- You can calculate the energy required for a material to change phases using the following formula, where Q is the heat added, m is the mass of an object, and L is the specific latent heat of transformation of a material.

$$Q = mL$$

NB;

- Latent heat is the heat absorbed or released when matter changes its state of matter without change in temperature.
- The latent heat associated with melting a solid or freezing a liquid is called **The Latent Heat of Fusion WHILE**. The latent heat associated with vaporizing a liquid or a solid or Condensing a vapor is called **The Latent Heat of Vaporization**

Latent heat of fusion

- Latent heat of fusion – Is the quantity of heat energy required to change a solid to liquid at melting point without any change in temperature

Specific Latent Heat of Fusion of a substance

- **Specific latent heat of fusion** – Is the quantity of heat energy required to change completely a unit mass (1kg) of the solid to liquid at its melting point.
- **Its SI Unit is J/kg**
- **Mathematically:** $L_F = \frac{H}{m} \quad \rightarrow H = mL_F$
- **Whereby, L_F = Specific latent heat of fusion**

Melting /Freezing Point of some Substances at STP

Substance	melting/freezing point (°C)	latent heat of fusion (J/kg)
Aluminum	659	396000
Copper	1086	134000
Iron	1535	293000
Water	0	335000
Mercury	-39	11000
Ethyl alcohol	-117	105000

Latent heat of Vaporization

- **Latent heat of Vaporization** – Is the quantity of heat energy required to change a liquid to vapour at boiling point without any change in temperature

Specific Latent heat of Vaporization

- **Specific latent heat of vaporization** – Is the quantity of heat energy required to change completely a unit mass (1kg) of the liquid to vapour at its boiling point.
- Its SI Unit is J/kg
- Mathematically: $H = mLv \rightarrow Lv = \frac{H}{m}$
- Where by $Lv =$ Specific latent heat of vaporization

Worked Examples

1. Calculate the quantity of heat required to convert 50 grams of ice at 0° C to water at 0° C. (Specific latent heat of fusion of ice = 3.4×10^5 J/kg)

Solution:

$$\text{From: } Q = mL_F \rightarrow m = 50\text{g} = 0.05\text{ kg}, L_F = 3.4 \times 10^5 \text{ J/kg}$$
$$\therefore H = mL_F = 0.05 \times 3.4 \times 10^5 = 17\,000\text{ J} = 17\text{kJ}$$

Why steam is hotter (dangerous) than boiling water (liquid)?

- Steam is hotter than boiling water because it has enough energy (Latent heat of vaporization) to escape the boiling water. The water remains at the boiling temperature of 100° C and cannot get hotter till all the water has changed to steam.
- **Since Steam has much more thermal energy than liquid that is why steam is used in engines to convert thermal energy to mechanical energy**

Class Activity – 8:5

1. How much heat is required to change a 500 g ice cube at 0° to water at 0°? ($L_F = 3.36 \times 10^5$ J/kg) [ANS: 186 kJ]
2. How much heat is required to change 100 g of ice from ice at 0°C to vapour at 100°C ($Lv = 2.26 \times 10^6$ J/kg, $L_F = 3.36 \times 10^5$ J/kg, $c = 4200$ J/kgK). (ANS:301600 J)
3. How much heat would be required to change 1.5kg of ice at -10°C to steam at 120°C.? The specific heat capacities of ice, water and steam are 2144J/kg° C, 4186 J/kg° C and 2010 J/kg° C respectively (ANS: $H_t = 4, 627, 860\text{J}$)
4. How much heat energy is required to melt 5 kg of ice? (Specific latent heat of ice = 336 Jg^{-1})
ANS; $H = mL = 5000 \times 336 = 1680000\text{J} = 1.68 \times 10^6\text{J}$
5. How much boiling water at 100°C is needed to melt 2kg of ice so that the mixture which is all water is at 0°C? [Specific heat capacity of water 4.2 JKg^{-1} and specific latent heat of ice 336 Jg^{-1}].
A; heat lost = heat gained $\rightarrow mC\Delta\theta = m'L \rightarrow m = \frac{m'L}{C\Delta\theta} = 1600\text{g} (1.6\text{kg})$
6. A copper block of mass 2.5kg is heated in a furnace to a temperature of 500°C and then placed on a large ice block. What is the maximum amount of ice that can melt? (Specific heat capacity of copper = $0.39 \text{ Jg}^{-1} \text{ K}^{-1}$; latent heat of fusion of water = 335 Jg^{-1}). [ANS; 1.5KG]
7. Two accidents have happened. The first one with water at 100°C and the second one with steam at 100°C
 - (a) Which is dangerous burn due to water at 100°C, and burn due to steam at 100°C? Why?
 - (b) Latent heat of vapourisation of water is $2.26 \times 10^6 \text{ J/kg}$. Explain the idea of latent heat of vapourisation.

(c) Find the heat required to convert 1g of ice at 0°C to steam at 100°C

[ANS; $H = mL_f + mC\Delta T + mL_v$

The Cooling effect of evaporation and mechanism of the Refrigeration

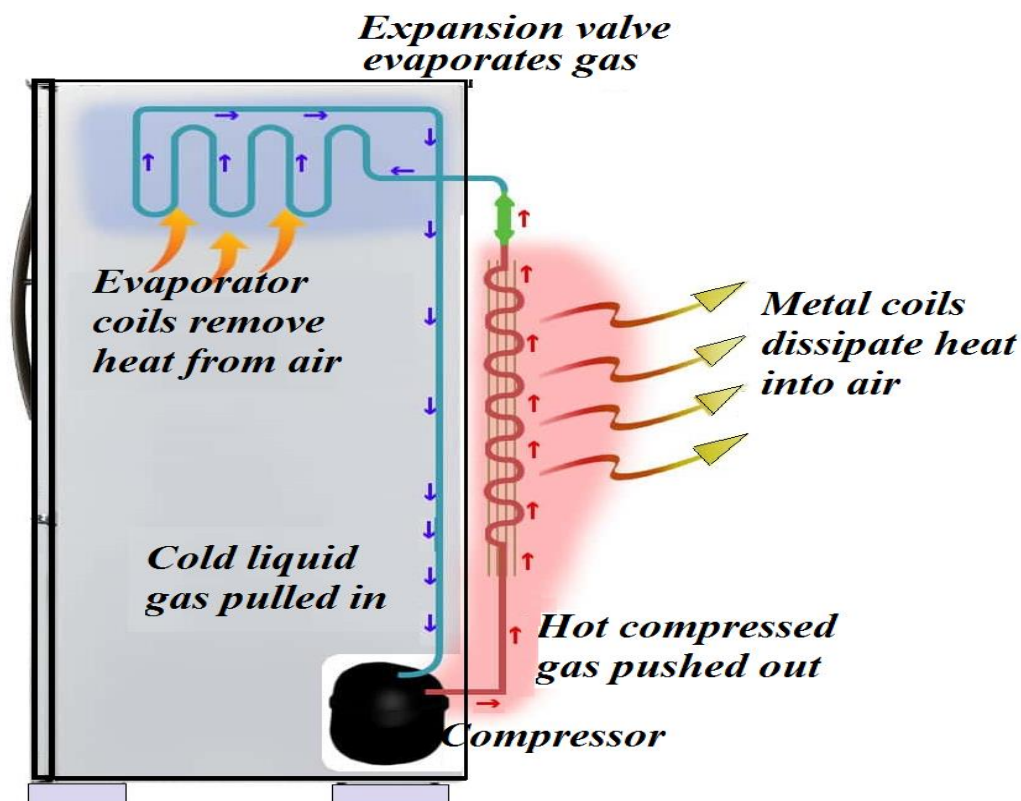
- **Evaporation has a cooling effect** because it takes thermal energy away from the surface. This is because the faster particles escape from the liquids surface leaving behind the slower ones, and that the faster ones have more energy, causing the temperature of the liquid to drop.
- **Sweating (Perspiration) uses the cooling effect** that comes from evaporation. The sweat, which is secreted from pores on the skin, can evaporate very easily. As the sweat evaporates, it takes away thermal energy from the skin to the surrounding air. This process causes the body to cool.
- For example, Dogs remove excess heat from their bodies by panting.
- Also, a volatile liquid like alcohol or ether poured on the skin feels much colder than water at the same temperature .**This is because the alcohol evaporates quickly taking latent heat of vaporization from the skin ,thus cooling the skin**
- **Another example of cooling effect by evaporation is that;** During the summer we wear cotton clothes. Cotton, being a good absorber of water allows more sweat to be in contact with the atmosphere, consequently helping in more evaporation. It is for this reason that we feel cooler when we wear cotton clothes. Similar example is when water is stored in earthen pots so as to make it cool. The pores of the earthen pot, just like the pores of cotton cloth provide a larger surface area for evaporation

Mechanism of Refrigerator

- The basic principle used in refrigeration is **cooling by absorption of latent heat (cooling effect by evaporation)**
- Refrigerator – Is a machine which can enable heat to flow from a cold region to a hot region
- **Refrigeration** is the process in which work is done to move heat from one location to another

In the refrigeration cycle ,there are five basic components:

- (i) Fluid refrigerant
- (ii) A compressor (controls the flow of refrigerant)
- (iii) The condenser coils (on the outside of the fridge)
- (iv) The evaporator coils (on the inside of the fridge)
- (v) Expansion valve (coolant)



The domestic refrigerator

- The figure above represents a typical refrigeration system. The electric compressor motor forces a gas at high pressure through a heat exchanger (condenser) on the rear outside wall of the refrigerator, where thermal energy (Q_H) is given off, and the gas cools to become liquid. The liquid passes from a high-pressure region, via a valve, to low-pressure tubes on the inside walls of the refrigerator; the liquid evaporates at this lower pressure and thus absorbs heat (Q_L) from the inside of the refrigerator cooling down the air.
- Lastly, the refrigerant evaporates to a gas, then flows back to the compressor, where the cycle begins again.

Class Activity – 8:6

1. Why does spirit poured on the skin feel much colder than water at the same temperature?
2. Ice cream appears colder to the mouth than water at 0°C . Give reason
3. Why does a bottle of soft drink cool faster when surrounded by ice cubes than by ice cold water, both at 0°C ?

Class Activity – 8

Use the following constants where necessary when solving the questions below

- ❖ Specific heat capacity of water = $4200\text{J}/(\text{kg } ^\circ\text{C})$
- ❖ Specific heat capacity of ice = $2100\text{J}/(\text{kg } ^\circ\text{C})$
- ❖ Specific heat capacity of steam = $2000\text{ J}/(\text{kg } ^\circ\text{C})$
- ❖ Specific latent heat of fusion of ice $3.3 \times 10^5\text{ J/kg}$
- ❖ Specific latent heat of steam = $2.26 \times 10^6\text{ J/kg}$
- ❖ Specific latent heat of vaporization of water = $2.3 \times 10^5\text{ J/kg}$
- ❖ Acceleration due to gravity ,g = 10 m/s^2

1. Which contains the greater amount of heat – a lake of water at 20 °C or a bowl of water at 90°C? Explain
2. An iron bar of mass 80 g is heated from a temperature of 15° C to a temperature of 65° C .How much heat is absorbed by the bar ,given that iron has a specific heat capacity of 460 J/(kg °C)
3. Water of mass 20g at a temperature of 42°C is poured into a well lagged copper vessel of mass 27 g at a temperature of 20°C.find the final temperature of the water.(specific heat capacity of copper = 400J/(kg °C)
4. Calculate the heat content of a piece of brass of mass 120g at a temperature of 20°C .Find the final temperature of water. (specific heat capacity of copper = 400J/(kg °C)
5. In an experiment to determine the specific heat capacity of a piece of metal, the following results were obtained:
 - Mass of piece of metal =200g
 - Initial temperature =25°C
 - Final temperature=80°C
 - Heat absorbed by the piece of metal = 1430 J
 Calculate the specific heat capacity of the piece of metal.
6. Distinguish between latent heat of fusion and the specific latent heat of fusion of a substance. Find the amount of heat required to change 1kg of ice at 0°C to water at the same temperature.
7. Define the term latent heat of vaporization and specific latent heat of vaporization. Find the quantity of heat required to change 5kg of water at 60°C into steam at 100°C
8. Determine the final temperature obtained when 500 g of water at 100° C was mixed with 500 g of water at 10° C and well stirred. **(ANS: T_f = 55° C)**
9. (a) Differentiate between heat and temperature
 - (b) The specific heat capacity of a certain substance is 800J/kg° C; what does this statement mean?
 - (c) Calculate the specific heat capacity of mercury, if 980 J of heat is required to raise the temperature of 7 g of mercury from 0° C to 1000° C
10. A piece of metal of specific heat capacity 840 J/(kg °C) and mass 30 g is heated to a temperature of 99°C and then dropped into a cavity in a block of ice at 0°C. Find the amount of ice that will melt
11. Discuss how high specific heat capacity of water helps in formation of land and sea breeze.

ANS;

 - Specific heat capacity of land is 5 times less as compared to water. Thus, air above land becomes hot and light and rises up resulting in drop in pressure of land mass during day time. Thus, cool air from sea starts blowing towards and forming sea breeze.
 - During night, land as well as sea radiates heat energy. However, temperature of land falls faster than sea water, because of high specific heat capacity of sea water. So, at night the temperature of sea water is more than land. Warm air above the sea rises and cold air from land starts blowing towards sea resulting in land breeze.
12. A refrigerator can convert 0.4 kg of water at 20°C to ice at -10°C in 4 hours .Find the average rate of heat extraction from the water in joules per second.
13. (a) what is meant by the following terms (i) melting point (ii) freezing point
 - (b) Describe how to find the melting point of a substance by means of cooling curve
14. Explain why:
 - (a) Heat energy has to be supplied to a solid in order to change it into liquid.
 - (b)Heat energy has to be supplied to a liquid in order to change it to vapor

15. A 0.2kg block of ice at 0°C is placed into a Styrofoam calorimeter cup with unknown mass of water at 20°C. When thermal equilibrium is reached, the final temperature is measured to be 5°C. What was the mass of the water initially in the cup?
16. A pressure cooker is a pot with a tight fitting lid that does not allow steam to escape until a preset pressure is reached. Explain how the pressure cooker can cook food faster than a sufuria with a loose – fitting
17. A container holds 1.5kg of ice initially at 40°C. Heat is supplied to the container at the rate of 12.6kJ per minute for 120 minutes.
- Plot a graph of temperature versus time for the 120 minutes during which heat is supplied.
 - What will the temperature of the contents of the container be at the end of the 120 minutes?
 - What will the mass of the steam in the container be at the end of the 120 minutes?
18. A 0.15kg aluminium cup holds 0.2kg water at 18°C. A 0.12 kg iron block at 85°C is placed into the water and the entire system surrounded by an insulating jacket. What will be the final temperature of the system when thermal equilibrium is reached?
19. The temperature of 500 g of a certain metal is raised to 100°C and it is then placed in 200 g of water at 15°C. If the final steady temperature rises to 21°C, Calculate the specific heat capacity of the metal. **(ANS: C = 128 Jkg⁻¹K⁻¹)**
20. How much thermal energy is required to raise the temperature of 3kg of aluminium from 15°C to 25°C?
21. Explain the following:
- When the brakes of a moving car are applied for an applicable time, they get hot
 - When the tyre of a car is pumped up, the pump gets warm
22. A car of mass 1000 kg travelling at 72 km/h is brought to rest by applying the brakes. Assuming that the kinetic energy of the car becomes transferred to internal energy in four steel brake drums of equal mass, find the rise in temperature of the drums if their total mass is 20 kg, the specific heat capacity of steel is 450 J/kgK, and the work done is equal on all four drums. **(ANS: $\Delta\theta = 22.2\text{ K}$)**
23. A bath contains 100 kg of water at 60°C. Hot and cold taps are then turned on to deliver 20 kg per minute each at temperatures of 70°C and 10°C respectively. How long will it be before the temperature in the bath has dropped to 45°C? Assume complex mixing of the water and ignore heat losses. **(ANS: t = 7.5 mins)**
24. Some hot water was added to three times its mass of water at 10°C and the resulting temperature was 20°C. What was the temperature of the hot water. **(ANS: T = 50°C)**
25. A piece of lead of mass 500 g and at air temperature falls from a height of 25 m. What is (a) Initial potential energy (b) Its kinetic energy on reaching the ground. Assume that all the energy becomes transferred to internal energy in the lead when it strikes the ground, calculate the rise in temperature of the lead if its specific heat capacity is 130 J/kgK. State the energy changes which occur from the moment the lead strikes the ground until it has cooled to air temperature again. **(P.E = 123 J, K.E = 123 J, $\Delta\theta = 1.89\text{ K}$)**
26. A waterfall is 100 m high and the difference in temperature between the water at the top and that at the bottom is 0.24 K. Obtain a value for the specific heat capacity of water in J/kgK explaining the steps in your calculations. Mention any assumptions you make. **(C = 4100 J/kgK)**
27. A 0.5 kg block of aluminium at a temperature of 100°C is placed in 1.0 kg of water at 20°C. Assuming that no thermal energy is lost to the surroundings, what will the final temperature of the aluminium and the water be when they attain the same temperature?

28. When a certain quantity of heat was supplied to a substance, its temperature rose from 5°C to 20°C . What will the final temperature of the substance be if twice the amount of heat is removed from the sample?
29. Why is water used as a coolant in car engines?
30. State what changes, if any, take place in the following:
- Melting point of ice when salt is added to the ice
 - The volume of water if it changes into ice
 - The boiling point of a liquid when the pressure on the liquid is reduced
31. Two substances A and B have the same mass and are at the same temperature. Substance A has a higher specific heat capacity than substance B. Which substance will have a higher final temperature if the same amount of heat is supplied to each substance?
32. An electric heater is rated at 250 W. Calculate the quantity of heat generated in 10 minutes
(ANS: H = 150 kJ)
33. A tin contains water at 290 K and is heated at constant rate. It is observed that the water reaches boiling point after 2 minutes and after further 12 minutes it is completely boiled away. Calculate the specific latent heat of steam.**(ANS: 2092 kJ/kg)**
34. An insulated cup holds 0.1 kg of water at 0°C . 0.1 kg of boiling water at a temperature of 100°C is poured into the cup. What will be the final temperature of the mixture at thermal equilibrium?
35. A 50 watt heater is immersed in a 2 kg block of aluminium which also holds a thermometer. The temperature of the block rises by 8 K in 5 minutes. Neglect heat losses, Calculate the specific heat capacity of aluminium. **(AN: 937.5 J)**
36. A metal sphere of unknown composition has a mass of 0.4 kg. The sphere is heated in a furnace to a temperature of 150°C and then dropped into an insulated cup holding 0.35 kg of water at 20°C . Upon reaching thermal equilibrium, the temperature of the system is measured to be 32.4°C .
- Calculate the specific heat capacity of the metal.
 - Use the values of specific heat capacity in table 8.1 to identify the metal.
37. Which contains the greater amount of heat – a lake of water at 20°C or a bowl of water at 90°C ? Explain.
38. An iron bar of mass 80 g is heated from a temperature of 15°C to a temperature of 65°C . How much heat is absorbed by the bar, given that iron has a specific heat capacity of $460\text{J}/(\text{kg}^{\circ}\text{C})$
39. Water of mass 20 g at a temperature of 42°C is poured into a well lagged copper vessel of mass 27 g at a temperature of 20°C . Find the final temperature of the water. (specific heat capacity of copper = $400\text{J}/(\text{kg}^{\circ}\text{C})$)
40. Calculate the heat content of a piece of brass of mass 200 g at a temperature of 20°C . Find the final temperature of water. (specific heat capacity of copper = $400\text{J}/(\text{kg}^{\circ}\text{C})$)
41. Differentiate between
- Melting point and boiling point
 - Freezing and vaporization
 - Evaporation and boiling
 - Melting and cooling
42. Explain the following
- The boiling point of water in Dar es Salaam is higher than at the top of Mt. Kilimanjaro
 - Why does water boil faster at the top of a mountain than at the bottom?
 - Water being heated while covered boils faster than uncovered water.
 - When one wipes spirit on the skin he feels cold
 - When snow is pressed by the hands, it melts to water. The water then immediately freezes.
 - The use of ammonia as a household refrigerant is discouraged
43. How much heat is required to change 40 g ice cube from ice at -10°C to steam at 110°C ?

44. Describe how a household refrigerator preserves food.
45. If 200 g of water is contained in a 500 g aluminium CAN at 10°C then an additional 100 g of water at 100°C is added into the CAN, what is the final equilibrium temperature of the mixture?
46. An unknown liquid of mass 400 g at a temperature of 80°C is poured into 400 g of water at 40°C . The final temperature of the mixture is 49°C . What is the specific heat capacity of the unknown liquid?
47. 20 g of steam at 100°C is added to 50 g of ice at 0°C . Find the amount of ice that is melted and the final temperature.
48. Explain how the following factors affect the melting and boiling points:
 - (a) Pressure
 - (b) Impurities
49. Explain how a refrigerator works.
50. An electric heater rated 1500 W is used to heat water in an insulated container of negligible heat capacity for 10 minutes. The temperature of water rises from 20°C to 40°C . Calculate the mass of water heated.
51. An electric kettle rated 2 kW is filled with 2.0 kg of water and heated from 20°C to 98°C . Calculate the time taken to heat the water assuming that all the electrical energy is used to heat the water in the plastic kettle and the kettle has negligible heat capacity.
52. The following data was obtained from an experiment. Mass of copper metal block = 200 g, initial temperature of the block = 22°C , ammeter reading = 0.5 A, voltmeter reading = 3.0 V, final temperature of the block = 30°C , time of heating = 7 minutes. Use the data to calculate the specific heat capacity of copper. What does this value mean? (**ANS: $C_c = 394\text{ J kg}^{-1}\text{K}^{-1}$**)
53. In an experiment, the following data was obtained. Use the data to calculate the time taken by the heater to raise the temperature of water container and the stirrer from 20°C to 23°C . What assumption have made in your calculations? Power of electric heater = 30 W, mass of the container and the stirrer = 200 g, specific heat capacity of the container and the stirrer = 400 J/kgK, mass of water in the container = 100g, specific heat capacity of water = 200 J/kgK
(**ANS: $t = 50\text{ s}$**)
54. A class of Physics students decided to determine the specific heat capacity of water in a waterfall. They used a sensitive thermometer to find the difference in temperature of water at the top and the bottom of the waterfalls and obtained the following results; height of the waterfalls = 52 m, temperature of the water at the top = 21.54°C and that at the bottom = 21.67°C . Stating any assumptions made, calculate a value for specific heat capacity of water.
55. A 200 g of liquid at 21°C is heated to 51°C by a current of 5 A at 6 V for 5 minutes. What is the specific heat capacity of the liquid? (**A: $C = 1500\text{ J/kg }^{\circ}\text{C}$**)
56. An electric kettle rated 1500 W is used to boil 500 g of water into steam at 100°C . Calculate the time required to boil off water.
57. Why steam is hotter than boiling water?
58. Explain as fully you can what happens when a liquid boils. Why would you expect the boiling point of a liquid to be lowered when the pressure above the free surface is vapour?

Chapter: 09

Vapour and Humidity

Evaporation of Liquids

- Evaporation is a gradual change of state from liquid to gas that occurs at the surface of a liquid.

Factors affecting the rate of Evaporation

- There are four primary factors that affect the rate of evaporation; temperature, surface area, wind, and humidity.

Effect of Temperature on Evaporation

- Temperature is directly proportional to the rate of evaporation
- The wet clothes dry faster on a hot sunny day because of the high temperature. When the temperature is high, the kinetic energy of the water particles increases, leading to the quicker conversion of liquid water into its gaseous state. This explains why hot water evaporates more quickly than cold water.

Effect of Surface Area on Evaporation

- Surface area is directly proportional to the evaporation increase in Surface area increases the rate of evaporation
- Suppose there is water in a wide shallow bowl, and the same amount of water is also kept in a narrow glass. In which one of the two situations will the water evaporate quickly? The water in the bowl will evaporate much faster than in the glass because the larger surface area gives more space for the water particles to evaporate.
- You might have also noticed that the wet clothes dry quickly when they are spread out properly on the clothesline compared to the clothes that are not spread out properly. This is again due to the larger surface area of the spread-out wet clothes, leading to faster water evaporation.

Effect of humidity on Evaporation

- Evaporation is also affected by humidity, or the amount of water vapor in the air. The lower the relative humidity, the dryer the air is and the faster it evaporates. The higher the humidity, the closer the air is to saturation, and less evaporation is possible
- **Thus; Humidity and rate of evaporation are in inverse relation to each other.** ie,. As the humidity decreases, the rate of evaporation increases.

Effect of Wind (The rate of flow of air) on Evaporation

- Wind is directly proportional to the evaporation. With rising wind speeds, the rate of evaporation of liquid increases. When the wind speed increases, the water vapor particles travel away with the wind, reducing the amount of water vapor in the environment. This increases the rate of water evaporation.
- For example; when the clothes are spread out on a clothesline on a windy day, they will dry up quicker than on a still day. This is because the water vapor in the surrounding air will move away from the clothes by the wind, leading to faster water evaporation from wet clothes.

Applications of evaporation

(i) Evaporation has cooling effect

Because it takes thermal energy away from the surface. This is because the faster particles escape from the liquids surface leaving behind the slower ones and the faster ones have more energy, hence the temperature of the liquid drops

(ii) Sweating uses the cooling effect caused by evaporation

As the sweat evaporates, It takes away thermal energy from the skin to the surrounding

(iii) Evaporation coolers can significantly cool a building by simply blowing dry air over a filter saturated with water

(iv) Cooling effect from evaporation used in Refrigerators

(v) In a clothes drier, hot air is blown through the clothes allowing water to evaporate very rapidly

Vapour Pressure (VP)

- When a liquid evaporates in a closed container, the vapor formed above the liquid exerts a pressure. According to kinetic molecular theory, the molecules of the vapor are in constant motion and will hence exert a pressure just like the molecules of a gas. This pressure is called **the vapor pressure of the liquid**.

Question: How can you differentiate between gas and vapor

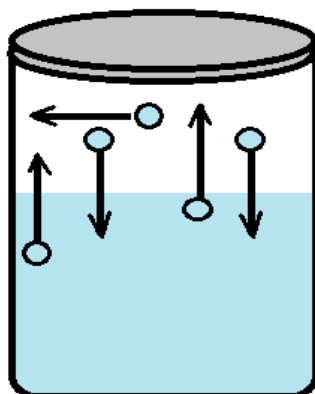
- A gas refers to a substance that has a single defined thermodynamic state at room temperature whereas vapor refers to a substance that is a mixture of two phases (ie. gaseous and liquid phase) at room temperature

Types of Vapour Pressure (VP)

- Saturated vapour pressure
- Unsaturated vapour pressure

Saturated Vapour Pressure (SVP)

- A saturated vapor is a vapor that is in contact with its own liquid within a confined space.
- When a partially filled container of liquid is sealed with a stopper, some liquid molecules at the surface evaporate into the **vapor phase**. However, the vapor molecules cannot escape from the container and so after a certain amount of time, the space above the liquid reaches a point where it cannot hold any more vapor molecules.
- Now some of the vapor molecules condense back into a liquid. The system reaches the point where **the rate of evaporation is equal to the rate of condensation** (ie,.. the number of molecules escaping from the liquid per unit time is equal to the number of molecules returning to the liquid per unit time). See **Figure** below.

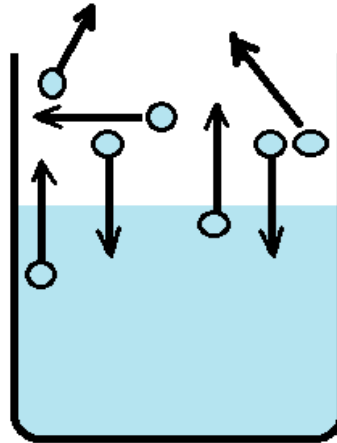


Liquid in a closed space

- This is called a dynamic equilibrium between the liquid and vapor phases. At this state of equilibrium, the vapor is said to be saturated. The pressure of the vapor when it is saturated above the liquid surface is called the saturated vapor pressure (S.V.P).

Unsaturated Vapour Pressure (U.S.VP)

- On the other hand, the unsaturated vapor is the vapor which is not in contact with its own liquid in a confine space. It is not in dynamic equilibrium with its own liquid. The rate at which the liquid evaporates is greater than the rate at which the liquid condenses. Thus, the pressure exerted by a vapor which is not in contact with its own liquid in a confined space is called unsaturated vapor pressure. See the figure below



Liquid in open space

NB:

- A substance of high vapour at room temperature is called **volatile**
- Increase in temperature of a solid or liquid, the rate evaporation or condensation increases which results to an increase in the vapor pressure

Measurement of SVP

- Saturated vapour pressure is measured by mercury barometer.
- It is given by, $SVP = (760 - x) \text{ mmHg}$
- **Where:** $760\text{mmHg} = \text{atmospheric pressure (atm)}$, $X \text{ mmHg} = \text{vapour pressure}$

Humidity

- Humidity is the measure of wetness of the atmosphere. The exact amount of water vapor in the atmosphere at a given temperature is called **Absolute humidity**. At higher temperature, the atmosphere contains more water vapor compared to water vapor present at low temperature.
- OR – Humidity – Is the amount of water vapour present in the atmosphere
- Water vapor from atmosphere condenses to form **clouds, fog, dew and frost**

Sources of Humidity

- Evaporation from rivers, lakes and oceans
- Transpiration (evaporation of plant leaves)

Dew point and its relationship to weather

- **Dew** – Is water in the form of droplets that appears on exposed objects in the morning or evening due to condensation

- The dew point is the temperature to which the air must be cooled in order for it to become saturated. It provides a measure of the actual amount of water vapor in the air – so the higher the dew point, the more moisture in the air.
- **Fog** is a cloud floating just above the ground. It is formed when water vapor in the air is cooled down to its dew point. Fog is of more effect than the mist as it can reduce visibility to less than 200m.
- **Mist** is the condensation of vapour into water droplets occurring near the ground. Mist limits visibility to about 1000m or less.
- **Hailstones** are water droplets in clouds formed due to super cooling below 0⁰ C without freezing
- **Frost** is a deposit of small white ice crystals formed on the ground or other surfaces when the temperature falls below freezing
- **Snow is formed when the dew point is below the freezing point (0⁰ C)**

NB:

- ✓ Hot Air contains more moisture (humidity) than cold air,
- ✓ Dew is formed at night because hot air comes into contact with a cold surface ,now water vapour present in it condenses on the cold surface in the form of droplets (dew drops)
- ✓ The formation of dew is more when the sky is clear and less when it is cloudy.
- ✓ DP is measured by Renault hygrometer
- ✓ DP occurs when RH of air is 100%
- ✓ Below DP clouds, dew or frost formed

Factors influencing the formation of Dew

(a) Temperature

- The warmer the air, the more humid is the air. For this case, at low temperature, air will take shorter time to be saturated. Since the air saturate quickly at low temperature than at high temperature, dew form quickly at low temperature than at high temperature. The temperature of the atmospheric air must fall below the dew point for dew to be formed

(b) Wind

- As wind increases, the rate of evaporation also increases, thus it prevents the formation of water droplets (dew)

(c) Water Vapour

- The atmospheric air must be saturated with water vapour for dew to be formed

(d) Cloud cover

- When a surface loses heat to the atmosphere, it cools, and can form dew on that surface. Cloud cover prevents the heat from escaping and thus, hinders dew formation. The absence of clouds will allow heat to escape to outer space hence promoting dew formation

Experiment to show the Presence of water vapour in the Air

Water vapor is contained in the atmosphere, which can be easily depicted using the following experiment:

Apparatus – A glass beaker and ice cubes

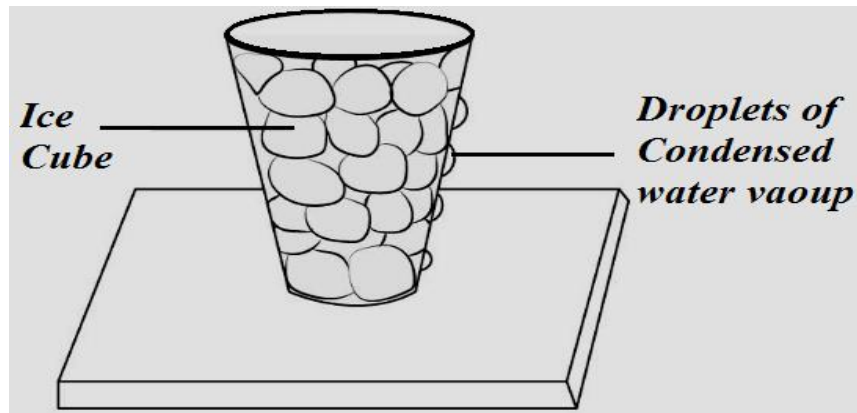
Procedure – The following procedure can be performed to show the presence of water vapour in air :

- Take the glass beaker and dry it from outside.
- Place the ice cubes in the beaker.

- Leave the ice along with the glass untouched for some minutes.

Observations;

- Water droplets appear on the outer surface of the glass, and it can be proved that it is water vapor by using a dry cobalt chloride paper that turns pink in the presence of water.



Conclusion;

- The water vapour present in the air condenses on the cold surface of the glass. This shows that air contains water vapour.

Relative Humidity (RH)

- Relative Humidity –Is the ratio of the saturated vapour pressure at the dew point to the saturated vapour pressure at the current air temperature
- **Mathematically:**

$$R. H = \frac{SVP \text{ at dew point}}{SVP \text{ at the original air temperature}} \times 100\%$$

OR

- **Relative humidity** is the ratio of the Actual vapour density to the saturated vapour density
- **Mathematically:**

$$RH = \frac{\text{actual vapour density (pressure)}}{\text{Saturated vapour density (pressure)}} \times 100\%$$

- Since absolute humidity is the density of water vapor in saturated air
Thus

$$RH = \frac{\text{actual vapour density}}{\text{absolute humidity}} \times 100\%$$

- Actual vapour density and Saturated vapour density has equal volume, therefore relative humidity can also be defined as

Absolute Humidity

- Absolute humidity – Is the mass of water vapor divided by the mass of dry air in a certain volume of air at a specific temperature, ie.,

$$\text{Absolute humidity (A. H)} = \frac{\text{mass of water vapor}}{\text{mass of dry air}} = \frac{M_1}{M_2}$$

Also can be defined as;

Specific Humidity (Humidity Ratio)

Is the ratio of water vapour to dry air in a particular mass

$$\text{Thus, Specific humidity (S.H)} = \frac{\text{mass of water vapor}}{\text{mass of dry air}} = \frac{M_v}{M_a}$$

NB;

The difference between air temperature and dew point temperature can indicate whether the Relative humidity is **LOW or HIGH**

- When the air and the dew point temperature are far apart, the Relative humidity is low
- When the air and the dew point temperature are close to the same value, the Relative humidity is high
- When the air and the dew point temperature are the same, the air is saturated and the Relative humidity is 100%

Worked examples

1. If the actual vapor density is 12 g/m³ at 20° compared to the saturation vapor density at that temperature of 20.5 g/m³, then what will be its relative humidity?

Solution:

$$\text{AVD} = 12\text{g/m}^3, \text{SVD} = 20.5\text{g/m}^3, T = 20^\circ\text{C}$$

$$\text{From: RH} = \frac{\text{actual vapour density(presure)}}{\text{Saturated vapour density(presure)}} \times 100\%$$

$$\therefore R.H = \frac{12\text{g/m}^3}{20.5\text{g/m}^3} \times 100\% = 58.5\%$$

2. A mass of air at 20°C has a relative humidity of 36%. If the air is cooled to 16°C, what will its relative humidity be? Saturation vapor pressure at 20°C and 16°C are 17.5mmHg and 13.6mmHg respectively.

Solution;

$$\text{From; Relative humidity} = \frac{\text{actual vapor pressure}}{\text{saturation vapor pressure}} \times 100\%$$

$$36\% = \frac{\text{actual vapor pressure}}{17.5} \times 100 \rightarrow \text{Vapor pressure} = 6.3\text{mmHg}$$

$$\text{Relative humidity} = \frac{\text{vapor pressure}}{\text{saturation vapor pressure}} \times 100\% = \frac{6.3}{13.6} \times 100\%$$

$$\text{Therefore; Relative humidity} = \frac{6.3}{13.6} \times 100\% = 46.3\%$$

3. Late on an autumn day, the relative humidity is 45.0% and the temperature is 20.0°C. What will the relative humidity be that evening when the temperature has dropped to 10.0°C, assuming constant water vapor density?

Given that the saturation vapor density at 20°C and 10°C are 17.2g/m³ and 9.40g/m³ respectively

$$\text{From; Relative humidity} = \frac{\text{vapor density}}{\text{saturation vapor density}} \times 100\%$$

$$45\% = \frac{\text{Vapor density}}{17.2} \times 100 \rightarrow \text{Vapor density} = 7.74\text{g/m}^3$$

$$\text{Relative humidity} = \frac{\text{vapor density}}{\text{saturation vapor density}} \times 100\% = \frac{7.74}{9.4} \times 100\%$$

$$\text{Therefore; Relative humidity} = \frac{7.74}{9.4} \times 100\% = 82.3\%$$

4. A closed vessel contains moist air at 20°C, the relative humidity being 30%. What should be the relative humidity if the vessel were cooled to 10°C? saturation vapor pressure of water at 20°C is 17.5mmHg and at 10°C is 9.2mmHg.

Solution;

$$\text{From; Relative humidity} = \frac{\text{actual vapor pressure}}{\text{saturation vapor pressure}} \times 100\%$$

$$30\% = \frac{\text{actual vapor pressure}}{17.5} \times 100 \rightarrow \text{Vapor pressure} = 5.25\text{mmHg}$$

$$\text{Relative humidity} = \frac{\text{vapor pressure}}{\text{saturation vapor pressure}} \times 100\% = \frac{5.25}{9.2} \times 100\%$$

$$\text{Therefore; Relative humidity} = \frac{5.25}{9.2} \times 100\% = 57\%$$

5. The temperature and relative humidity of air on a certain day are 20°C and 80%, respectively. Find the fraction of mass of water vapor that will condense if the temperature falls to 5°C. Saturation vapor at 20°C and 5°C are 17.5mmHg and 6.5mmHg, respectively.

Solution;

$$\text{From; Relative humidity} = \frac{\text{actual vapor pressure}}{\text{saturation vapor pressure}} \times 100\%$$

$$80\% = \frac{\text{actual vapor pressure}}{17.5} \times 100 \rightarrow \text{Vapor pressure} = 14\text{mmHg}$$

NB;

- In order to solve this question, advanced formula of ideal gas equation may be applied.

Ideal gas equation

$$PV = nRT = \frac{m}{M}RT \rightarrow m_1 = \frac{P_1VM}{RT_1}, m_2 = \frac{P_2VM}{RT_2}$$

$$T_1 = 273 + 20 = 293K, T_2 = 273 + 5 = 278K$$

$$P_1 = 14\text{mmHg}, P_2 = 6.5\text{mmHg}$$

When substituting for m_1 and m_2 ;

In the fraction of water vapor condensed = $\frac{m_1 - m_2}{m_1} = 1 - \frac{m_2}{m_1}$, we will get

$$1 - \frac{m_2}{m_1} = 1 - \left(\frac{P_2VM}{RT_2} \right) \div \left(\frac{P_1VM}{RT_1} \right) = 1 - \left(\frac{P_2VM}{RT_2} \right) \times \left(\frac{RT_1}{P_1VM} \right)$$

$$1 - \left(\frac{P_2}{T_2} \right) \times \left(\frac{T_1}{P_1} \right) = 1 - \left(\frac{T_1}{T_2} \right) \times \left(\frac{P_2}{P_1} \right) = 1 - \frac{293}{278} \times \frac{6.5}{14}$$

Therefore; the fraction of water vapor condensed = $1 - 0.49 = 0.51$

6. A mass of air has a relative humidity of 57% and a dew point of 20°C. What is the temperature of the air mass?

Solution;

$$\text{From; RH} = \frac{\text{Dew point temperature}}{\text{Original air temperature}} \times 100\% \therefore \text{RH} = \frac{20}{x} \times 100\% \rightarrow x = \frac{2000}{57} = 35.09^\circ\text{C}$$

Difference between Absolute humidity and Relative humidity

Absolute humidity	Relative humidity
Is the actual amount of water vapor present in the air	Is a percentage of the amount of moisture the air could possibly hold
It is expressed in moisture per cubic meter of air	It is expressed in percentage as the ratio of

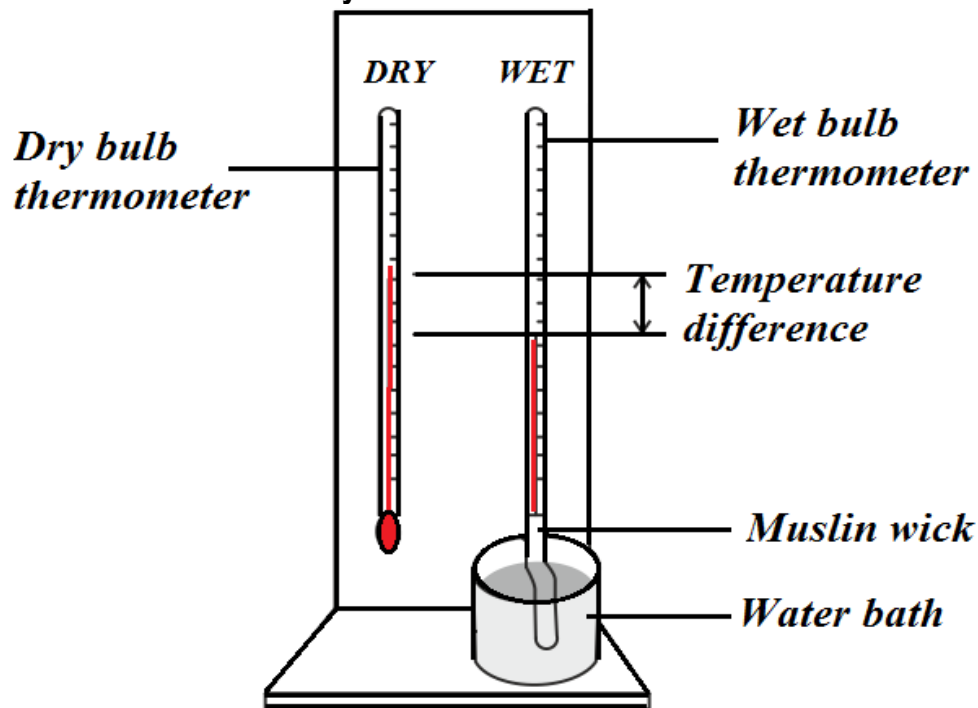
(g/m ³)	vapor pressure to saturated vapor pressure
It is totally independent of the temperature	If the temperature goes up, relative humidity goes down and vice – versa

Measurement of Relative Humidity

- It is measured by **dry and wet bulb hygrometer** and **Renault hygrometer**
- **Hygrometer** is an instrument used to measure the amount of humidity and water vapor in the atmosphere ,in soil or in confined spaces

Dry and Wet Bulb Hygrometer

- One type of instrument used to find Relative humidity is a Psychrometer.
- A Psychrometer consists of two thermometers. The bulb of one thermometer is covered with a wet cloth and is called **the wet-bulb**, whereas the bulb of the other thermometer is left bare and is referred to as **the dry-bulb**.



Wet and Dry bulb hygrometer

- Relative humidity can be found by subtracting the temperature on the wet-bulb thermometer from the temperature on the dry-bulb thermometer and using a relative humidity chart.

Dry-bulb Temp. (°C)	Relative humidity (%)																					
	Dry bulb temperature (°C) minus Wet bulb temperature (°C), [Dry temp - Wet temp]																					
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
-20	28																					
-18	40																					
-16	48	0																				
-14	55	11																				
-12	61	23																				
-10	66	33	0																			
-8	71	41	13																			
-6	73	48	20	0																		
-4	77	54	37	11																		
-2	79	58	43	20	1																	
0	81	63	45	28	11																	
2	83	67	51	36	20	6																
4	85	70	56	42	27	14																
6	86	72	59	46	35	22	10	0														
8	87	74	62	51	39	28	17	6														
10	88	76	65	54	43	33	24	13	4													
12	88	78	67	57	48	38	28	19	10	2												
14	89	79	69	60	50	41	33	25	16	8	1											
16	90	80	71	62	54	45	37	29	21	14	7	1										
18	91	81	72	64	56	48	40	33	26	19	12	6	0									
20	91	82	74	66	58	51	44	36	30	23	17	11	5	0								
22	92	83	75	68	60	53	46	40	33	27	21	15	10	4	0							
24	92	84	76	69	62	55	49	42	36	30	25	20	14	9	4	0						
26	92	85	77	70	64	57	51	45	39	34	28	23	18	13	9	5						
28	93	86	78	71	65	59	53	47	42	36	31	26	21	17	12	8	2					
30	93	86	79	72	66	61	55	49	44	39	34	29	25	20	16	12	8	4				
32	93	86	80	73	68	62	56	51	46	41	36	32	27	22	19	14	11	8	4			
34	93	86	81	74	69	63	58	52	48	43	38	34	30	26	22	18	14	11	8	5		
36	94	87	81	75	69	64	59	54	50	44	40	36	32	28	24	21	17	13	10	7	4	
38	94	87	82	76	70	66	60	55	51	46	42	38	34	30	26	23	20	16	13	10	7	
40	94	89	82	76	71	67	61	57	52	48	44	40	36	33	29	25	22	19	16	13	10	7

Relative Humidity

Worked Examples

1. The dry bulb temperature reading of a hygrometer is 40° C and the wet bulb temperature reading is 30° C. What is the RH? **(A: 48%)**

Solution;

Dry bulb reading = 40°C

Wet bulb reading = 30°C

Difference in temperature = 40° – 30° = 10° C

Therefore; from the Psychrometric table above, Relative Humidity = 48%

2. A dry bulb thermometer reads 30° C and a wet bulb thermometer reads 24° C .What is the Relative Humidity of the air

Solution:

Dry bulb reading = 30°C

Wet bulb reading = 24°C

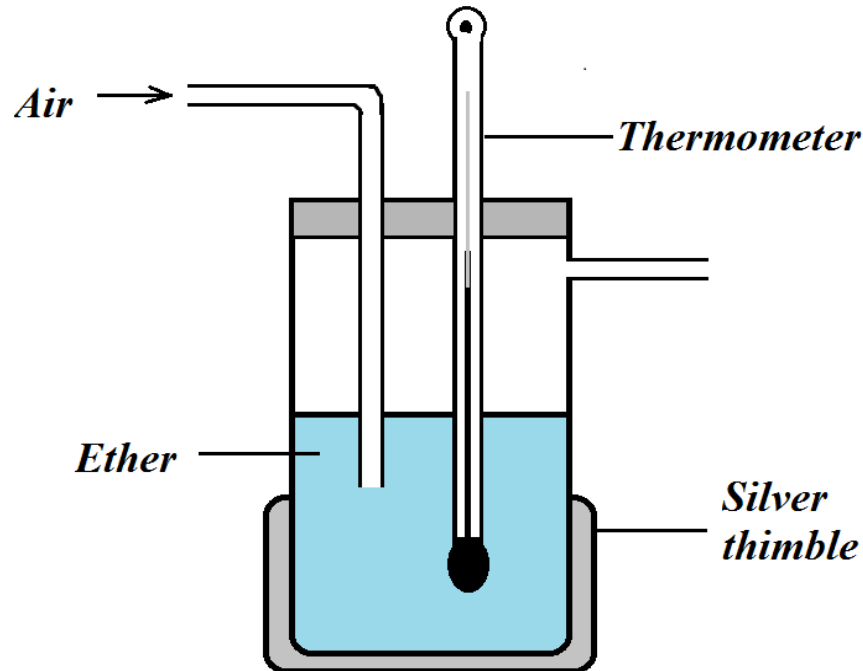
Difference in temperature = 30° – 24° = 6° C

Therefore; from the Psychrometric table above, Relative Humidity = 61%

Renault Hygrometer

- It consists of an enclosed thin silver tube containing ether and a thermometer.
- There is also a tube through which air can be pumped into the ether.

Diagram:



Regnault hygrometer

Mechanism of Renault Hygrometer

- The heat transfer from atmosphere to ether by convection in a tube, Ether evaporates result cooling of the silver tube surface. Cooling continues until air adjacent to the outside surface of the tube becomes saturated with water vapour. Some water vapour condense outside the tube to form dew

Applications of Humidity

- ✓ It is used by meteorological departments to forecast the weather
- ✓ It is used to determine the appropriate site to locate cotton
- ✓ Electrical and electronic components are usually transported and stored in a dry air
- ✓ Used in hospitals in an operating room. RH at operating room is at least 50%
- ✓ It is used in storage and transportation of food items

Class Activity – 9:1

1. The dry bulb temperature reading of a hygrometer is 22°C and the wet bulb temperature reading is 18°C. What is the RH? (**ANS: 68%**)
2. The dry bulb temperature reading of a hygrometer is 40° C and the wet bulb temperature reading is 25° C. What is the RH? (**ANS: 29%**)
3. The relative density of a place was measured at 25° C and found to be 53.6%. if the absolute humidity is 23.05g/m³, determine the actual water vapour density at this experiment

(ANS: $AVD = 12.35 \text{ g/m}^3$)

4. At a certain temperature and pressure, air can hold a maximum of 120g of water vapor. If this temperature and pressure the air is holding only 40g of water vapor, what is the relative humidity of the air? [ANS; R.H = 33.3%]
5. The air in a room has 15g of water vapor per cubic meter. However, for saturation, one cubic meter of volume requires 20g of water vapor. What is the relative humidity. [ANS R.H = 75%]
6. If the actual vapor density is 10 g/m^3 at 20°C compared to the saturation vapor density at that temperature of 17.3 g/m^3 , what is the relative humidity [ANS: 57.8%]
7. A room with dimensions of 7m x 10m x 2m, holds air that is saturated with water vapor. The saturation vapor pressure of the water vapor is 7.37kPa. If all of the water vapor in the room was condensed, what volume would the water occupy? Give your answer in m^3
[ANS; $V = 7 \times 10 \times 2 = 140 \text{m}^3$]
8. On a particular day, the relative humidity is 100% and the room temperature is 30°C . What is the dew point? The saturated vapor pressure of water at 30°C is 42400Pa [ANS; 30°C]

Definition of Some terms used in this topic

1. **Dry air.** The pure dry air is a mixture of a number of gases such as nitrogen, oxygen, carbon dioxide, hydrogen, argon, neon, helium etc. But the nitrogen and oxygen have the major portion of the combination.
2. **Moist air.** It is a mixture of dry air and water vapour. The amount of water vapour present in the air depends upon the absolute pressure and temperature of the mixture.
3. **Saturated air.** It is mixture of dry air and water vapor, when the air has diffused the maximum amount of water vapor into it. The water vapours, usually, occur in the form of superheated steam as an invisible gas. However, when the saturated air is cooled, the water vapour in the air starts condensing, and the same may be visible in the form of **moist, fog or condensation on cold surfaces.**
4. **Dry bulb temperature.** It is the temperature of air recorded by a thermometer, when it is not affected by the moisture present in the air.
5. **Wet bulb temperature.** It is the temperature of air recorded by a thermometer, when its bulb is surrounded by a wet cloth exposed to the air. Such a thermometer is called **Wet bulb thermometer.**
6. **Wet bulb depression.** It is the difference between dry bulb temperature and wet bulb temperature at any point. The wet bulb depression indicates relative humidity of the air
7. **Dew point depression.** It is the difference between the dry bulb temperature and dew point temperature of air.

Class Activity – 9

1. Define evaporation and state the factors which affect the rate of evaporation of a liquid .How does the kinetic theory account for the cooling produced in a liquid which is evaporating
2. Distinguish between : (a) snow (b) hailstones (c) mist
3. Define the following terms (a) Dew (b) Dew point (c) Specific humidity
4. What factors determine saturation of vapour pressure?
5. Explain the principles used to measure relative humidity?

6. Explain the factors that affecting evaporation

7. Explain the difference between a vapour and a gas
8. At a given pressure the thermometer of a wet bulb reads 21°C . If the Relative Humidity is 30 %, what is the temperature of the air?
9. A mass of air has a relative humidity of 57% and a dew point of 20°C . What is the temperature of the air mass?
10. Why does air cool when it rises through the atmosphere? (**ANS: As air rises, it expands because air pressure decreases with an increase in altitude. When expands, it cools adiabatically**)
11. The table gives the temperature and dew point in four towns at 12 noon.

City	Temperature ($^{\circ}\text{C}$)	Dew Point
Arusha	16	1
Morogoro	24	12
Zanzibar	3	5
Dar es salaam	28	4

- (a) In which town is the relative humidity the highest?
- (b) In which town is the relative humidity the slowest?
12. Briefly explain the reasons for the following:
- (a) When a cold bottle is brought into a warm room, it becomes misted over
- (b) Frost is more likely to occur on a clear night than on a cloudy night.
13. (a) What does the term saturation of water vapor mean?
- (b) What is the difference between vapour pressure and saturated vapour pressure
14. Differentiate between
- (a) Absolute humidity and Relative humidity
- (b) Saturated and unsaturated vapour
- (c) Evaporation and Boiling
15. Explain why dew is formed at night.
16. What are the two processes that change a liquid into a gas. The state their difference
17. A student is investigating the evaporation of water in the Laboratory. He pours 200 cm^3 of water in a 250 cm^3 beaker. He pours another 200 cm^3 of water in a 1000 cm^3 .The water in both beakers is initially at 50°C
- (a) Use the kinetic theory of matter to explain which beaker evaporates faster
- (b) List three other factors that affect evaporation of a liquid
- (c) Explain why the liquid that remains during evaporation cools
18. The actual vapor density of a region at 23°C temperature is 15 g/m^3 , if the saturation vapour density at that temperature of 21.3 g/cm^3 determine the region 's relative humidity
19. At a temperature of 30°C the mass of water vapour in town K is 22 g while dry air has a mass of 15 g. Determine the specific humidity of town K at the stated temperature
20. A mass of air at 30° holds 15 g/cm^3 of water vapor .If the saturation point of the air is 30 g/cm^3 ,calculate the relative humidity of the air
21. If the dry bulb temperature is 32°C and the wet bulb temperature is 24°C , what is the relative humidity of the air? [**ANS; R.H = 51%**]

Chapter: 10

Current Electricity

- Electric current is *the rate of flow of charges past a point or region*.
- **Ammeter** is an instrument used to measure the strength of the electric current in an electric circuit. The ammeter is connected in series in a circuit where the current is to be found. . The current flows through the positive '+' red terminal of ammeter and leaves from the negative '-' black terminal.

Mathematically:

$$\text{Electric current } (I) = \frac{\text{Quantity of charges } (Q)}{\text{Unit time}(t)}$$

- **But;** $Q = ne$, whereby n =number of electrons, e =charge of an electron
- The common SI unit of current (I) is **ampere (A)**

Electric Potential difference (P.d)/ Electric pressure

- **Is** the work done per unit charge in moving electric charge from one point to another in an electric field
- OR
- Is the difference in electrical potential between any two points
 - **Voltage** (Potential difference) is what makes electric charges move. It is the '**push**' that causes charges to move in a wire or other electrical conductor.

Mathematically:

$$P. d (V) = \frac{\text{work done}(W)}{\text{charge}(Q)}$$

- The common SI unit of P.d is **Volt (V)**

Electromotive Force (e.m.f)

- Electromotive force of a cell is the maximum potential difference between the terminals of a cell when the cell is not doing any work
- It is also called **voltage**.
- It pumps charges to flow through an electric circuit although it is not a force is just a potential
- The term **EMF** is usually applied to voltage sources like cells, batteries, dynamos, alternators, photo diodes and solar cells. These all produce electricity.
- The difference between the electromotive force and the terminal voltage is known as **LOST VOLTS**

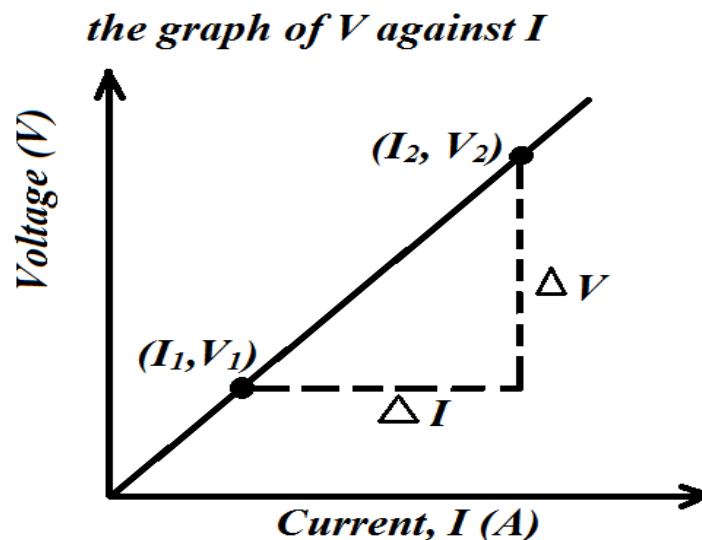
Measurement of e.m.f of a cell and p.d across a conductor

- Voltmeters are used to measure the potential difference (voltage) across a component.

- In order for a voltmeter to measure a device's voltage, **it must be connected in parallel to that device**. This is necessary because objects in parallel experience the same potential difference
- To use your voltmeter, first attach the positive input terminal lead to the positive end of the circuit. Then attach the negative input terminal lead to the negative end of the circuit. The voltmeter will turn itself on and do the measuring automatically

Ohm's Law

- It states that: **“At constant temperature and other physical factors, a current in conductor is directly proportional to the potential difference across its end”**
- According to Ohm's law, at a constant temperature, the steady current ' I ' flowing through a conductor is directly proportional to the potential difference ' V ' between the two ends of the conductor. *i. e.*, $I \propto V$. Hence, $\frac{I}{V} = \text{constant}$.
- The value of this proportionality constant is found to be $\frac{1}{R}$
- Therefore; $I = \left(\frac{1}{R}\right)V \rightarrow V = IR$
- Here, R is a constant for a given material (say Nichrome) at a given temperature and is known as the **resistance** of the material. Since, the **potential difference V** is proportional to the **current I** , the graph between V and I is a straight line for a conductor, as shown in the Figure below;



- From the graph above, **Slope(m) = $\frac{\Delta V}{\Delta I}$** , where, **Slope = Resistance**

Limitation of ohm's Law

- It does not apply to some electrolytes e.g dilute H_2SO_4
- It does not apply for conduction of electricity in gases
- It does not hold in semiconductors (diodes and transistors)

Class Activity –10:1

1. The current flowing through a lamp is 0.2A. If the lamp is switched on for one hour, What is the total electric charge that passes through the lamp? **[ANS; 720C]**
2. A current of 100 mA flows through a wire. The charge on an electron is $1.6 \times 10^{-19} \text{ C}$. Find the number of electrons passing per second through the cross-section of the conductor.

$$Q = It = ne \rightarrow n = \frac{It}{e} = 6.25 \times 10^{17}$$

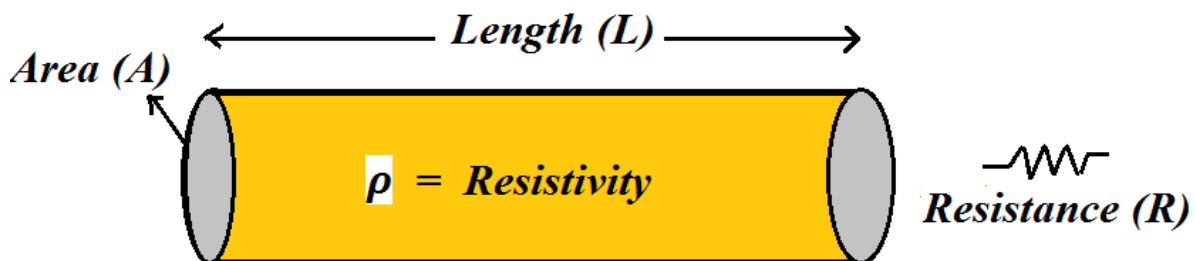
- An electric heater draws 3.5 A from a 110 V source. What is the resistance of the heating element? **(ANS: R = 31.4Ω)**
- The e.m.f of a cell is 1.5V. What is the energy provided by the cell to drive 0.5 C of charge around the circuit? **[ANS; $E = \frac{W}{Q} \rightarrow W = E \times Q = 0.5 \times 1.5 = 0.75J$]**
- A potential difference of 230 V applied across the heating coil drives a current of 10 A through it. Calculate the resistance of the coil. **[ANS; 23Ω]**
- How much current is produced by a voltage of 180 V across a 15 kΩ resistance? **[ANS; $V = IR \rightarrow I = \frac{V}{R} = \frac{180}{15000} = 0.012A$]**
- A current of 2 A is passed through a conductor of resistance 10 Ω. What is the potential difference between the ends of the conductor. **(ANS: V = 20V)**
- An electric bulb draws 0.5 A current at 3.0 V. Calculate the resistance of the filament of the bulb. **[ANS; 6Ω]**
- Four amperes of current are measured through a 24 Ω resistor connected across a voltage source. How much voltage does the source produce? **[ANS; 96V]**
- An ammeter placed in series with an electric radiator reads 0.5 amps and a voltmeter placed across it reads 230 volts. What is the resistance of the radiator? **[ANS; 460Ω]**
- A charge of 2×10^4 C flows through an electric heater. The amount of electrical energy converted into thermal energy is 5 MJ. Compute the potential difference across the ends of the heater. **[$P = IV \rightarrow V = \frac{P}{I} = \frac{Pt}{Q} = \frac{5,000,000}{2 \times 10^4} = \text{ANS; 250V}$]**
- From the following observations taken while determining the resistance of a conductor, draw the current-voltage graph and calculate the resistance of conductor. Is the conductor ohmic?

Ammeter reading I (in ampere)	Voltmeter reading V (in volt)
0.2	0.4
0.3	0.6
0.5	1.0
0.8	1.6
1.5	3.0

ANS; R = 2Ω, the conductor is ohmic

Factors affecting the Resistance of a conductor

- All *conductors* show some opposition to electrical *current*. This opposition to current is called **resistance**. There are several factors that affect the resistance of a conductor. Consider the diagram below:



$$\rho = \frac{AR}{L}$$

1. Temperature

- A cooler wire has less resistance than a warmer wire. Cooler particles have less kinetic energy, so they move more slowly. Therefore, they are less likely to collide with moving electrons in the current. Materials called superconductors have virtually no resistance when they are cooled to extremely low temperatures.
- Thus; the higher the temperature, the higher the resistance and vice versa

2. Length of the conductor

- A longer wire has more resistance than a shorter wire. Current must travel farther through a longer wire, so there are more chances for it to collide with particles of matter.
- Thus; the longer the wire the higher the resistance and vice versa **ie $R \propto L$**

3. Cross-section area

- A wide wire has less resistance than a narrow wire of the same material.
- Electricity flowing through a wire is like water flowing through a hose. More water can flow through a wide hose than a narrow hose. In a similar way, more current can flow through a wide wire than a narrow wire.
- That is the cross – sectional area of a conductor increases with decrease in the resistance
 $R \propto \frac{1}{A}$

4. Nature of material

- Resistance depends on the type of material used for making conductor
- For example, Nichrome wire has more resistance than a copper wire of the same dimensions.
- That is why Nichrome wire is used in heating element of **electric fires while** Copper wire is mostly used for **connecting wires**

We can sum up these statements (**2 and 3 above**) mathematically as follows:

- The resistance, **R** is directly proportional to the length, **L** of a wire and inversely proportional to the cross-sectional area, **A** of the wire $\rightarrow R \propto \frac{L}{A} \rightarrow R = \rho \frac{L}{A}$, whereby **ρ** , a constant of proportionality, is called the **resistivity** which depends upon the material used to make the wire.

$$\therefore \rho = \frac{RA}{L}$$

- The units of resistivity, **ρ** are Ohm-meters (**Ωm**)
- **Thus; Resistivity** is the ability of a material to oppose the flow of an electric current.
- A quantity called **conductivity** is expressed using the Greek letter sigma, **σ** is defined as the reciprocal of the resistivity, **$\sigma = \frac{1}{\rho}$** .
- The **greater the conductivity** is, the **smaller amount of resistance** in the circuit and therefore, the greater current in the circuit.

Resistivity of some materials at 20⁰ C

Material	Resistivity (Ωm)	Material	Resistivity(Ωm)
Aluminium	2.7×10^{-8}	Constantan	4.9×10^{-7}
Chromium	1.3×10^{-7}	Manganin	4.8×10^{-7}
Copper	1.68×10^{-8}	Nichrome	1.0×10^{-6}

Iron	9.71×10^{-8}	Glass	$1 \times 10^9 - 1 \times 10^{13}$
Lead	2.1×10^{-7}	Rubber	$1 \times 10^{13} - 1 \times 10^{15}$
Silver	1.6×10^{-8}	Quartz	7.5×10^{17}

Worked examples

1. What is the resistance of a copper wire of length 20m and diameter 0.08cm?

Given that the resistivity of a copper wire is, $\rho_{Cu} = 1.68 \times 10^{-8} \Omega m$

$$\text{ANS; } R_{Cu} = \frac{\rho L_{Cu}}{A_{Cu}} = \frac{1.68 \times 10^{-8} \times 20}{\pi r^2} = \frac{1.68 \times 10^{-8} \times 20}{3.14 \times 0.0004^2} = 0.67 \Omega$$

2. A copper wire is to replace an aluminium wire of length $1.20 \times 10^{-1} m$ in an electrical circuit. The copper wire has a cross-sectional area that is 80% of the aluminium wire. The resistance of the circuit must remain the same. What length of copper wire must be used? Assume the temperature remains constant at $20^\circ C$ in the circuit.

SLN; $L_{Al} = 1.2 \times 10^{-1} m$, $A_{Al} = x$, $A_{Cu} = 0.8 A_{Al}$

$$R_{Al} = \frac{\rho L_{Al}}{A_{Al}}, \quad R_{Cu} = \frac{\rho L_{Cu}}{A_{Cu}}, \quad \text{Since resistance is the same, then}$$

$$\frac{\rho L_{Al}}{A_{Al}} = \frac{\rho L_{Cu}}{A_{Cu}} \rightarrow L_{Cu} = \frac{\rho A_{Al} \times L_{Al}}{A_{Al}} \times \frac{A_{Cu}}{\rho_{Cu}} = \frac{2.7 \times 10^{-8} \times 1.2 \times 10^{-1}}{A_{Al}} \times \frac{0.8 A_{Al}}{1.68 \times 10^{-8}}$$

$$\therefore L_{Cu} = 0.154 m = 15.4 \text{ cm}$$

Class Activity – 10:2

- A wire of length 40 m and cross – sectional area 0.8 mm^2 has a resistance of 10Ω . What is the resistivity of the material of the wire? (ANS: $\rho = 2 \times 10^{-4} \Omega m$)
- A constantan wire has a length of 45cm, a diameter of 0.37 mm and resistivity of $4.9 \times 10^{-7} \Omega m$.
 - What is the resistance of the wire? [ANS; (a) $2.05 \Omega m$, (b) $I = 0.73 A$]
 - What will be the current in the wire if it is connected to a 1.5 V cell?
- A steel bar has a length of 2.3m and diameter of $2 \times 10^{-5} m$. what is resistance? (Resistivity is $10.5 \times 10^{-8} \Omega m$) (ANS: $R = 768.72 \Omega$)
- What length of a wire of cross –sectional area 0.2 mm^2 and resistivity $0.072 \mu \Omega m$ is needed to wind a coil of resistance 9Ω . (ANS: $L = 25 m$)
- The resistance of a certain wire is 12Ω . What are the resistance of another wire of the same material but with half the length and half the radius of the first wire? (ANS: $R_2 = 24 \Omega$)
- Under identical conditions, a copper wire conducts almost 60 times the current than a nichrome wire (an alloy of nickel, iron, and chromium). Which wire do you think has greater resistivity?

AN: A nichrome wire has a greater resistivity due to which it conducts less current than a copper wire. It also shows that the resistance of a nichrome wire is greater than a copper wire of the same length and the same radius.

- What do you mean by resistivity?
- A copper wire is in the form of a cylinder and has a resistance R. It is stretched till its thickness reduces by half of its initial size. Find its new resistance in terms of R.

Answer:

$$\text{Given; } R_1 = \frac{\rho L_1}{A_1} = \frac{\rho L_1}{\pi R_1^2} \text{ -----(i)}$$

NB: the volume of wire is the same before and after stretching;

$$ie, .. \quad V_1 = V_2 \rightarrow A_1 L_1 = A_2 L_2 \rightarrow L_2 = \frac{A_1 L_1}{A_2}$$

$$L_2 = \frac{A_1 L_1}{A_2} = \frac{\pi R_1^2 L_1}{\pi R_2^2} = \frac{R_1^2 L_1}{R_2^2}, \text{ but it is given that; } R_2 = \frac{R_1}{2}$$

$$L_2 = \frac{R_1^2 L_1}{R_2^2} = \frac{R_1^2 L_1}{\left(\frac{R_1}{2}\right)^2} = \frac{4R_1^2 L_1}{R_1^2} = 4L_1$$

Thus; New resistance

$$R_2 = \frac{\rho L_2}{A_2} = \frac{\rho L_2}{\pi R_2^2} = \frac{4\rho L_1}{\pi \left(\frac{R_1}{2}\right)^2} = \frac{16\rho L_1}{\pi R_1^2} = 16 \left(\frac{\rho L_1}{\pi R_1^2}\right) \text{ -----(ii)}$$

Now; substitute eqn (i) into (ii) $\therefore R_2 = 16 \left(\frac{\rho L_1}{\pi R_1^2}\right) = 16R_1$

Resistors

- Resistor is a device which offers resistance to the flow of an electric current.
- It used to control the magnitude of current and voltage according to ohms law

Types of Resistors

They are divided according to the materials used to make them and the value of resistance offered

Types of Resistors due to material used

- (i) Wire wound resistor
- (ii) Carbon resistor
- (iii) Metal film resistor
- (iv) Metal oxide film resistor

Wire Wound Resistor

- It is made by winding wires made of certain metallic alloys into spools (used to control amount of resistance)

Carbon Resistor

- It is made by mixing carbon granules with varying amount of clay and moulding them into cylinders

Metal Film Resistor

- It is made up of a stable ceramic core coated with metal alloys such as nickel chromium. It is more accuracy and more expensive than carbon resistor

Metal Oxide Film Resistor

- It is made up of a stable ceramic core coated with metal alloys such as tin oxide

Types of Resistors due to Value Offered

- ✓ Fixed resistor
- ✓ Variable resistor

Fixed Resistor

- It has a resistor value which cannot be changed. Eg. 2Ω, 3Ω, 4Ω etc.
- Most carbon resistor are fixed resistor

Variable Resistor

- It has a resistor value which can be changed by means of control.

- Example, **potentiometers, thermistors, photo resistors and rheostat**

Resistor Colour Codes

- Resistors which are used in electronic devices always painted with different colour texture called **bands**. The band represent the exactly value of a resistance. It contains fourth band with different meaning.

Color coding for Resistors

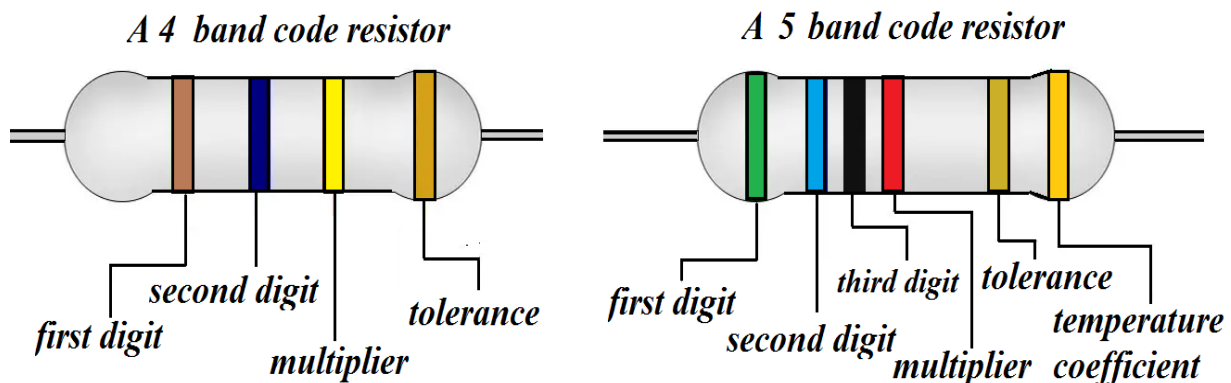
Color	Number	Multiplier
Black	0	1
Brown	1	10^1
Red	2	10^2
Orange	3	10^3
Yellow	4	10^4
Green	5	10^5
Blue	6	10^6

Color	Number	Multiplier
Violet	7	10^7
Gray	8	10^8
White	9	10^9
Gold		10^{-1}
Silver		10^{-2}
Colorless		

Tolerance colour codes

Tolerance	± 1	± 2	± 5	± 10	± 20
Colour	Brown	Red	Gold	Silver	No colour

Consider the diagram below showing the 4 and 5 resistor color code:



- ✓ First band – first digit
- ✓ Second band – second digit
- ✓ Third band (**multiplier**) – number of zero
- ✓ Fourth band (**tolerance**) – percentage accuracy

Example

From the diagram of resistor above find the exactly resistance

First band (yellow) – 4

Second band (purple) – 7

Multiplier (red) – 2 = 00 number of zero

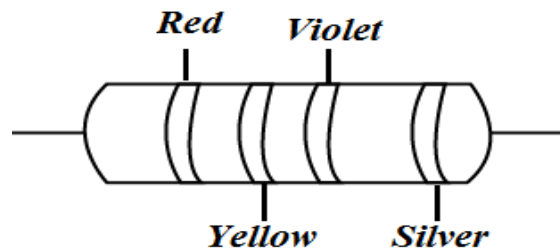
Tolerance (gold) – $\pm 5\%$

Therefore: R = $\pm 5\%$ of 4700 Ω

The actual value resistance is **$\pm 5\%$ of 4700 Ω**

Class Activity – 10:3

1. A resistor is connected across a 50 V source. What is the current in the resistor if the color code is red, orange, orange, silver? **(ANS: $I = 2.2 \text{ mA}$)**
2. The color code of a carbon resistor is given below. What is the resistance of the resistor?
[ANS: $R = 24 \times 10^7 \Omega \pm 10\%$]

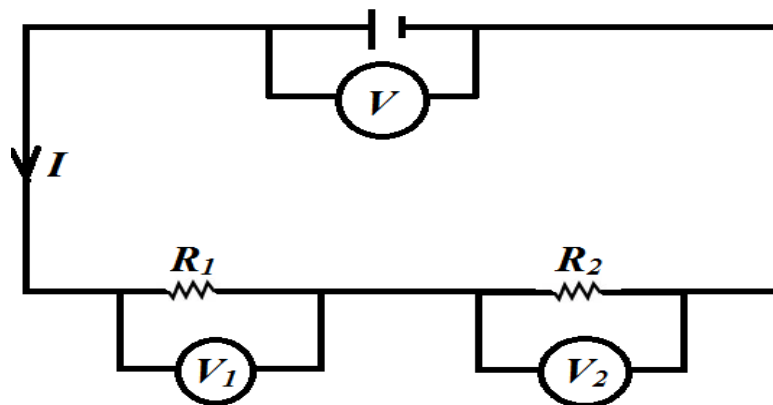


Combination of Resistors

- An electric circuit may contain a number of resistors which can be connected in different ways. For each type of circuit, we can calculate the equivalent resistance produced by a group of individual resistors.

Resistors in Series

- When two or more resistors are connected end to end, they are said to be in series. The resistors could be **simple resistors or bulbs or heating elements or other devices**. Figure below shows two resistors R_1 and R_2 connected in series.
- The amount of charge passing through resistor R_1 must also pass through resistors R_2 since the charges cannot accumulate anywhere in the circuit.
(ie,.. The same current, I , is flowing through each resistor)



- **According to Ohm's law**, if same current pass through different resistors of different values, then the potential difference across each resistor must be different. Let V_1 and V_2 be the potential difference (voltage) across each of the resistors R_1 and R_2
- But the total voltage V is equal to the sum of voltages across each resistor.

ie,.. **P.d across the battery = sum of p.d around a conducting path**
that is: $V = V_1 + V_2$ ----- (i), but **current, I** , is constant in each resistor

From ohm's law: $V = IR \rightarrow V = IR_T$

From equation (i), $V = V_1 + V_2 \rightarrow IR_T = IR_1 + IR_2$

$IR_T = I(R_1 + R_2)$ divide by I each side $\rightarrow R_T = R_1 + R_2$

∴ Total resistance (R_T) for resistors in series connection is given by

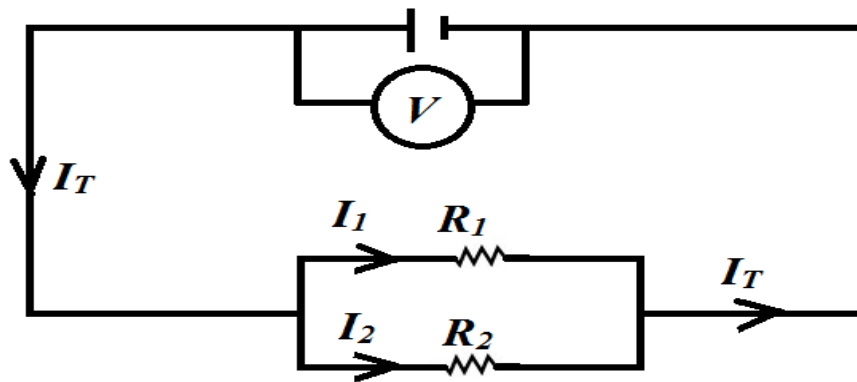
$$R_T = R_1 + R_2 + \dots + R_n$$

NB:

- ❖ As the branches increases, the voltmeter reading keeps increasing for the total volts
- ❖ The value of equivalent resistance in series connection will be greater than each individual resistance.

Resistors in Parallel

- Resistors are connected across two common points in a parallel arrangement.
- Resistors are in parallel when they are connected across the same potential difference as shown in Figure below.
- In this case, the total current I that leaves the battery is split into two separate paths. Let I_1 and I_2 be the current through the resistors R_1 and R_2 respectively. Due to the conservation of charge, total current in the circuit I is equal to sum of the currents through each of the two resistors.
- As the branches increases, the ammeter reading keeps increasing for the total current. Each branch has its own current value
- The value of equivalent resistance in parallel connection will be lesser than each individual resistance.
- Parallel connection is used in house wiring



Since: $I_T = I_1 + I_2$ (i) , but: $V =$ the same at all points round a circuit

From ohm's law: $V = IR \rightarrow I = \frac{V}{R} \rightarrow I_T = \frac{V}{R_T}$

Then, from equation (i)

$$I_T = I_1 + I_2 \rightarrow \frac{V}{R_T} = \frac{V}{R_1} + \frac{V}{R_2}$$

$$\frac{V}{R_T} = \frac{V}{R_1} + \frac{V}{R_2} \dots \dots \text{divide by } V \text{ each side}$$

$$\text{thus, } \frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} \rightarrow R_T = \frac{R_1 R_2}{R_1 + R_2}$$

∴ Total resistance (R_T) in parallel connection is given by

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}$$

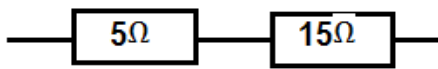
Whereby: R_n = the last resistor

N.B

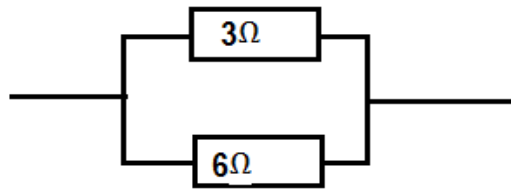
- It is advisable to connect bulbs in parallel during electrical installation so that when the bulb blows out or disconnected, the other bulbs will keep working
- If you connect in series when one bulb disconnected (blows out) will cause the other bulbs not working

Worked Examples

1. Calculate the combined resistance in:



(a)



(b)

Solution:

(a) $R_1 = 5 \Omega, R_2 = 15 \Omega$

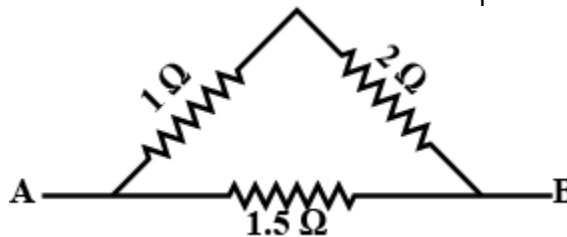
From: $R_T = R_1 + R_2 \rightarrow R_T = 5 + 15 = 20\Omega$

(b) $R_1 = 3 \Omega, R_2 = 6 \Omega, R_T = ?$

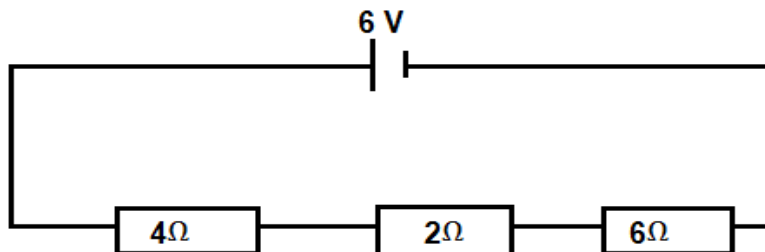
From: $R_T = \frac{R_1 R_2}{R_1 + R_2} \rightarrow R_T = \frac{3 \times 6}{3 + 6} = \frac{18}{9} = 2\Omega$

Class Activity – 10:4

1. In given figure, calculate the effective resistance between the points A and B. [ANS; 1Ω]



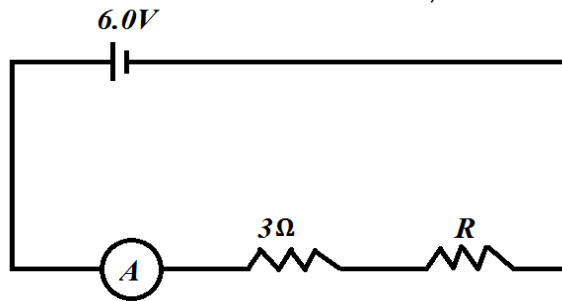
2. What is the potential difference across 2Ω resistor? (ANS: V = 1 V)



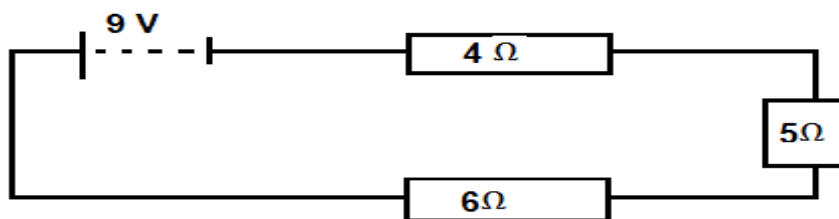
3. A wire of uniform thickness with a resistance of 27Ω is cut into three equal pieces and they are joined in parallel. Find the resistance of the parallel combination.

ANS; R of each small piece of wire = $\frac{27}{3} = 9\Omega \therefore \frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \rightarrow R_T = 3\Omega$

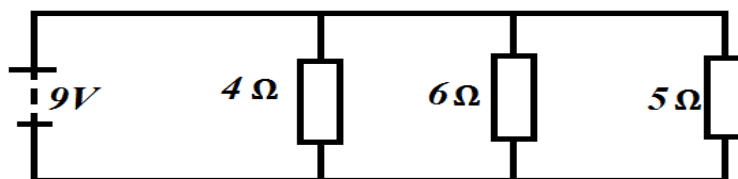
4. The figure shows a circuit. When the circuit is switched on, the ammeter reads 0.5 A.



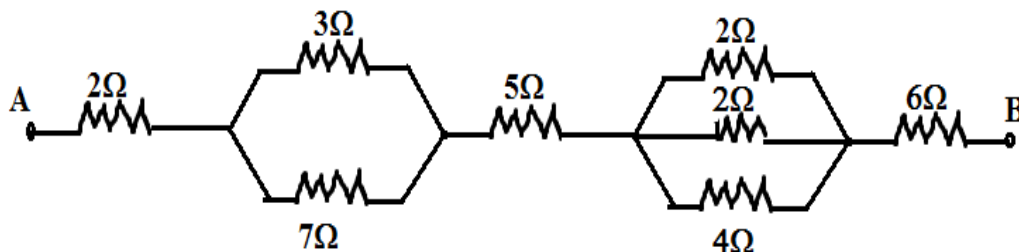
- (i) Calculate the value of the unknown resistor R . [ANS; $R = 9\Omega$]
 (ii) Calculate the charge passing through the 3Ω resistor in 120 s. [ANS; $Q = 60C$]
 (iii) Calculate the power dissipated in the 3Ω resistor. [ANS; $P = I^2R = 0.75W$]
5. The resistance of two resistors joined in series is 8Ω and in parallel is 1.5Ω . Find the value of the two resistances. [ANS; $2\Omega, 6\Omega$]
6. Consider the diagram below



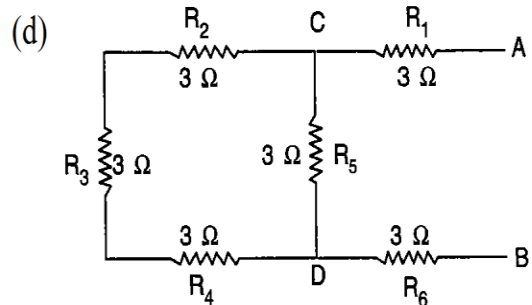
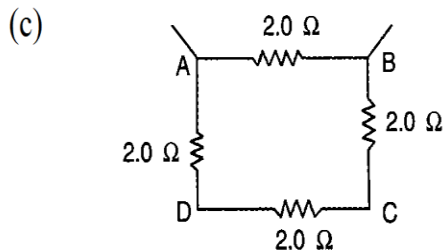
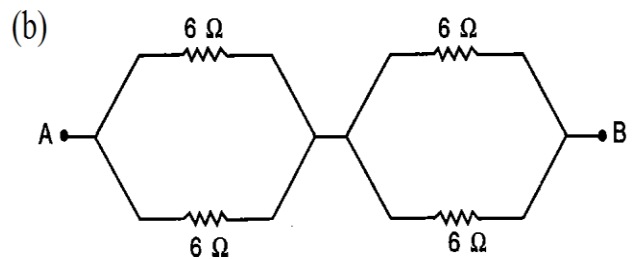
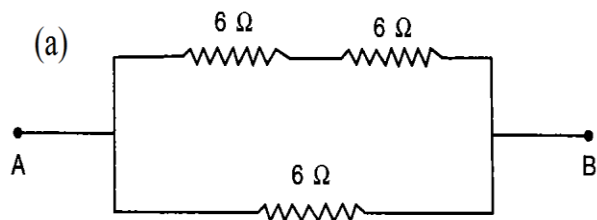
- (d) What is the total resistance of the circuit?
 (e) What current flows in the circuit?
 (f) What is the potential drop across each resistor?
 (g) What is the electric potential at point A?
 (ANS: (a) 15 ohms (b) $I = 0.6$ (c) 2V, 3.6V and 3 V (d) $V_A = 6.6V$)
7. Consider the diagram below



- (a) What is the total resistance of the circuit? (ANS: 1.62 ohms)
 (b) What total current flows in the circuit? (ANS: 5.55 A)
 (c) What current flows through each resistor? (ANS: 2.25A, 1.5A, 1.8A)
8. Find the total resistance between points A and B in the following diagram. [ANS; 15.9Ω]

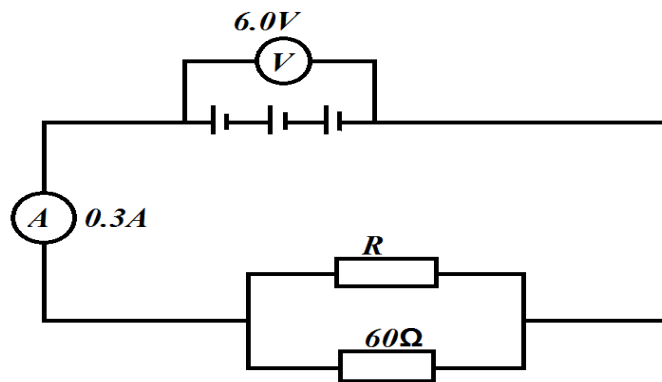


9. Calculate the equivalent resistance between the following points A and B in the following circuits. (ANS: (a) 4Ω (b) 6Ω (c) 1.5Ω (d) 8.25Ω)



10. When two resistances connected in series and parallel their equivalent resistances are 10Ω and 2.4Ω respectively. Find the individual resistances. [ANS; 4Ω and 6Ω]

11. In the figure below, the ammeter **A** reads 0.3 A . Calculate:

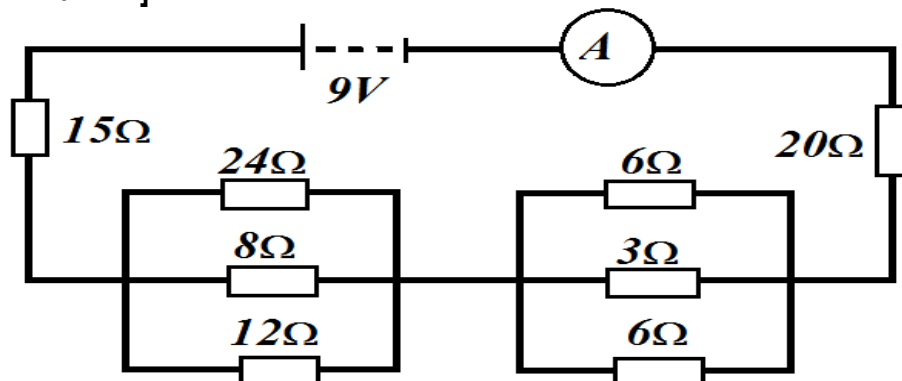


(i) the total resistance of the circuit [ANS; $R = \frac{V}{I} = \frac{6}{0.3} = 20 \Omega$]

(ii) the value of R [ANS; $\frac{1}{20} = \frac{1}{R} + \frac{1}{60} \rightarrow \frac{1}{R} = \frac{1}{20} - \frac{1}{60} = \frac{3-1}{60} = \frac{2}{60} \rightarrow R = 30 \Omega$]

(iii) the current flowing through R. [ANS; $I = 0.2 \text{ A}$]

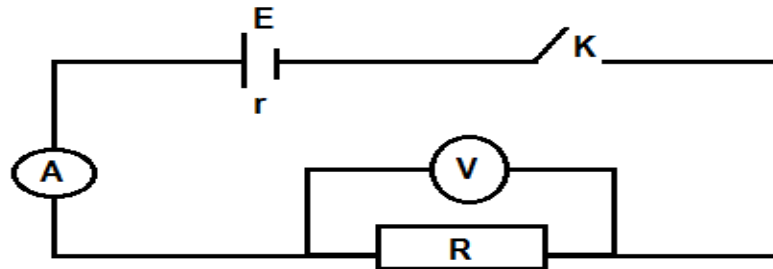
12. Determine the current reading on the ammeter in the circuit shown in the diagram below [ANS; $A = 0.22 \text{ A}$]



Internal Resistance of a Cell

- Cell has an internal resistance that opposes the flow of electric current and causes the potential drops across this resistance.
- Consider the circuit given below. The cell can be modified with an **emf 'E'** and the internal resistor with resistance **r** which is connected in series. An external load resistor with resistance **R** is also connected across the circuit
- The relationship between internal resistance (**r**) and emf (**E**) of a cell is given by

$$E = I(R + r) = IR + Ir.$$



Internal resistance of a cell

Whereby:

- Emf (**E**) = $V_T = V + V_L$, **K** = switch
- **R** = external resistance (load resistance), **r** = internal resistance
- P.d across **R** (terminal Voltage), $V = IR$
- P.d across **r** (Lost voltage), $V_L = Ir$

$$\therefore r = \frac{E - IR}{I}$$

NB:

- When the terminals of the battery are short circuited (when disconnected from their external circuit), the resistance of the circuit is the internal resistance of the cells that is:
 $E = I(R + r) = IR + Ir$, here $IR = V = 0 \quad \therefore E = Ir$
- The total e.m.f available in the cell is used up in two ways**
 - ❖ Driving the current through the cell i.e to overcome internal resistance (**r**)
 - ❖ Driving the current through the external resistance **R**
- When a cell is in closed circuit, i.e., when current is drawn from it, then the potential difference between its electrodes called the terminal voltage is always less than the **e.m.f.** **because** when charge flows in a circuit, some energy is spent in the flow of charge through the electrolyte of the cell.

Worked Examples

- The potential difference across the cell when no current flow through the circuit is 3 V. When the current $I = 0.4A$ is flowing, the terminal potential difference falls to 2.8 V. Determine the internal resistance (**r**) of the cell.

Solution:

Given: $E = 3V$, $V = 2.8 V$, $I = 0.4A$, $r = ?$

$$\text{From: } r = \frac{E - IR}{I} = \frac{E - V}{I} \quad \rightarrow \quad r = \frac{3 - 2.8}{0.4} = 0.5 \Omega$$

2. A battery is connected to a 10Ω resistor and a switch in series. A voltmeter is connected across the battery. When the switch is open (off) the voltmeter reads 1.45V . When the switch is closed the reading is 1.26V . What is the internal resistance of the battery?

Solution;

We know the emf of the cell is 1.45V and the terminal p.d. falls to 1.26V when a current flows through the 10Ω resistor, so the current through the resistor can be calculated as:

$$V = IR \rightarrow I = \frac{V}{R} = \frac{1.26}{10} = 0.126\text{A}$$

Therefore; $r = \frac{P.d}{I} = \frac{E-IR}{I} = \frac{E-V}{I} = \frac{1.45-1.26}{0.126} = 1.51\Omega$

3. A cell of e.m.f. E and internal resistance r is used to send current I in an external resistance R . Write expressions for

- the current drawn from the cell,
- the terminal voltage V of the cell, and
- the voltage drop across the internal resistance. How are E and V related?

ANS;

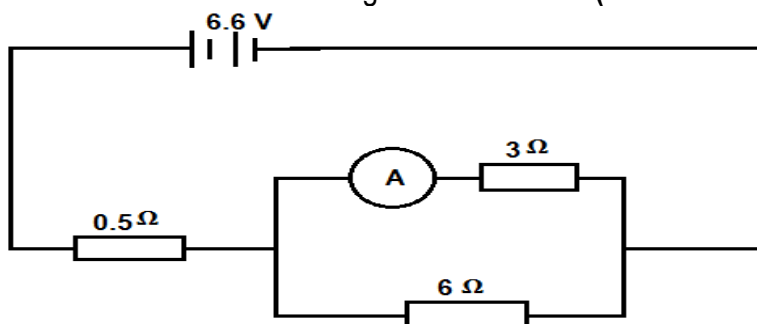
(a) Total resistance of circuit = $R + r$, current draw from the cell, $I = \frac{E}{R+r}$

(b) Terminal, voltage, $V = IR$

(c) Voltage drop, $V = Ir$. E and V are related as follow; $E = V + Ir$

Class Activity – 10:5

- What is the internal resistance of cell when there is current of 0.4A , when a battery of 6V is connected to a resistor of 13.5Ω . (ANS: $r = 1.5\Omega$)
- What is the maximum current of a battery of e.m.f 3.0V and internal resistance of 1.0Ω (ANS: $I = 3\text{A}$)
- An old cell with an emf of 1.7V has an internal resistance of 0.8Ω . How much current will initially flow if its terminals are short – circuited? (ANS: $I = 2.13\text{A}$)
- A dc source with an internal resistance of 0.11Ω is connected across a length of nichrome wire having a resistance of 20Ω . If a voltmeter across the nichrome indicates a drop of 5V , what is the emf of the source? (ANS: emf = 5.0275V)
- A cell with an emf of 1.4V and internal resistance of 0.05Ω is placed in a circuit with several resistors. The cell provides 0.52A to the circuit. What is the terminal voltage of the cell? (ANS: $V = 1.374\text{V}$)
- Explain why potential difference is always less than the e.m.f. of a cell?
- In the circuit shown below, the battery has an e.m.f of 6.6V and internal resistance of 0.3ohms . Determine the reading of the ammeter. (ANS: 2.357A)

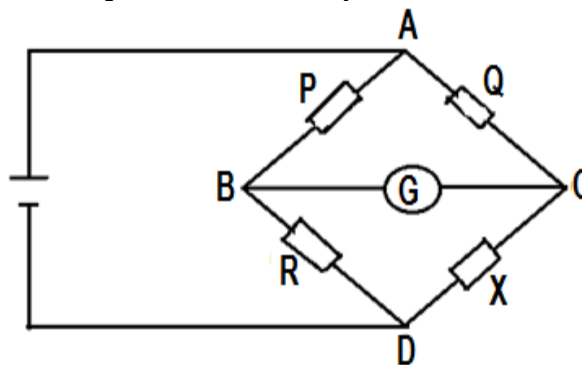


- A cell supplies a current of 0.6A through a 2Ω resistor and a current of 0.2A through a 7Ω resistor. Calculate the e.m.f of the cell and the internal resistance
- (Answer: E.m.f = 1.5V , $r = 0.5\Omega$)

10. A battery of e.m.f 15 V and internal resistance 3 ohm is connected to two resistors of resistances 3 ohm and 6 ohm in series. Find:
- (a) The current through the battery (ANS: 1.25 A)
- (b) The p.d between the terminals of the battery. (ANS: 11.25V)
11. A cell supplies a current of 0.6 A through a 2Ω coil and a current of 0.3 A through on 8Ω coil. Calculate the e.m.f and internal resistance of the cell. [ANS; 4Ω, 3.6V]
12. A complete circuit consists of a 18 V, battery and a resistor R. The terminal voltage of the circuit is 15.8V and the current is 4A. What is:
- (a) The internal resistance, r of the battery. (ANS: r = 0.55 Ω)
- (b) The resistance R of the circuit resistor. (ANS: R = 3.95 Ω)

Wheatstone bridge (Meter Bridge)

- Wheatstone bridge is an electrical bridge circuit used to measure the unknown resistance of a conductor.
- Consider the circuit in the figure below whereby X is the unknown resistor



- By adjusting one of the known resistors, usually resistor Q, we can reach a value at which the galvanometer shows no deflection (zero reading). The bridge at this point is said to be balanced and there is NO p.d across BC, thus two points have the same potential

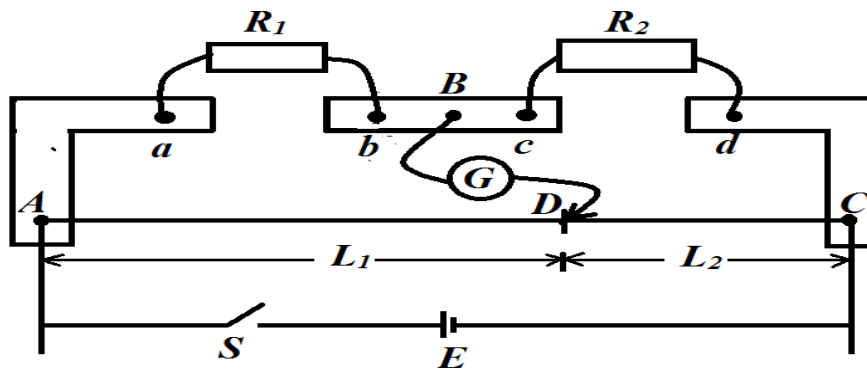
$$i. e., V_{AB} = V_{AC} = V_{BD} = V_{CD} \quad \text{OR} \quad \frac{V_{AB}}{V_{BD}} = \frac{V_{AC}}{V_{CD}}$$

- Since the current through the galvanometer is 0, P and R carry the same current I_1 . From Ohm's law:

$$\frac{V_{AB}}{V_{BD}} = \frac{I_2 P}{I_2 R} = \frac{P}{R} \quad \text{and} \quad \frac{V_{AC}}{V_{CD}} = \frac{I_1 Q}{I_1 X} = \frac{Q}{X} \quad \therefore \frac{P}{R} = \frac{Q}{X} \quad \rightarrow X = \frac{QR}{P}$$

The meter Bridge

- The metre bridge is one of the practical forms of the Wheatstone bridge used for measuring resistance
- It consists of a uniform resistance wire; AC (usually 100 cm long) stretched along side a metre scale. See the fig below
- The resistance R_1 , is connected across the gap "ab" and another resistor (R_2) is connected across the gap cd.
- A galvanometer G is connected to the terminal B and to a jockey D



- As the jockey is moved along the AC, at one position D, the galvanometer will read zero. The metre bridge is then said to be balanced. D is the balancing point and the length is the balance length
- R_1 is the resistance to be measured; R_2 is a standard resistance of a value near to R_1 . Then we can write

$$R_1 \propto L_1 \dots \dots \dots (i), \quad R_2 \propto L_2 \dots \dots \dots (ii)$$

dividing the equation we get;

$$\frac{R_1}{R_2} = \frac{L_1}{L_2} \quad \text{but } L_2 = 100 - L_1 \rightarrow R_1 = R_2 \left(\frac{L_1}{100 - L_1} \right)$$

Example

1. What is the resistance of a wire if it balances a standard resistor of 5 ohms at the 56 cm from the end of the metre bridge?

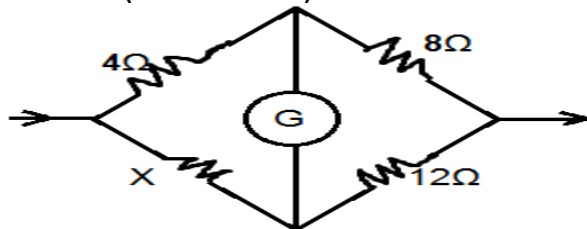
Soln:

Given: $R_2 = 5\Omega$, $L_1 = 56$ cm

From: $R_1 = R_2 \left(\frac{L_1}{100 - L_1} \right) \rightarrow R_1 = 5 \left(\frac{56}{100 - 56} \right) = 5 \times \frac{56}{44} = 7\Omega$

Class Activity – 10:6

1. The galvanometer in the bridge network shown in the fig. below, gives no deflection. What is the value of X? (ANS: $X = 6\Omega$)



Heating effect of an Electric Current

- When an electric current is passed through a conductor, the conductor becomes hot after some time and produce heat. This happens due to the conversion of some electric energy passing through the conductor into heat energy. This effect of electric current is called **heating of current**.

Factors affecting Heat Quantity

- **Resistance of a Conductor**
The higher the heat, the higher the resistance and vice versa

- **Magnitude of the Electric Current**

The higher the current, the more the heat produced

- **The time for which Current flows**

The heat produced by an electric current is proportional to the time taken by the current to pass through a conductor

Joule's Law

- It tells us the relationship between resistance, current and heat generated.
- State that **“The rate at which heat is produced in a resistor is proportional to the square of the current flowing through it, if the resistance is constant.”**

Mathematically:

$$\frac{H}{t} \propto I^2 R \quad \rightarrow \quad \frac{H}{t} = kI^2 R \quad \rightarrow \quad H = ktI^2 R, \quad \text{where } k=1$$

$$\therefore H = I^2 t R$$

But $R = \frac{V}{I}$ substitute in the formula above

$$\therefore H = ItV$$

Also, $I = \frac{V}{R}$ substitute in the formula above

$$\text{Therefore: } H(E) = t \left(\frac{V^2}{R} \right) = tIV = tRI^2$$

Worked Examples

1. An electric iron box has resistance of 30Ω and it takes a current of 10 A. Calculate the heat H developed in 1 minute.

Soln: Given: $R = 30\Omega$, $I = 10 \text{ A}$, $t = 1\text{min} = 60 \text{ seconds}$

From $E = I^2 R t$ **Therefore, $E = 10^2 \times 30 \times 60 = 180 \text{ kJ}$**

NB:

- **Whenever** an electric current passes through a conductor, electrical energy is converted to other forms of energy e.g heat, light etc (**According to the law of conservation of energy**)

Electric Power

- Electric power is the rate at which electrical energy is transferred by an electric circuit.
- **Electrical energy and Power** are closely related concepts. Electrical power is the amount of electrical current that flows through a circuit due to a certain voltage difference across it. Electrical power is also the rate at which electrical energy is delivered or consumed by a device or a system.
- **Electrical power** is measured in **watts (W)**, which are equivalent to joules per second (**J/s**). Mathematically, we can write:

$$\text{Electric Power} = \frac{\text{electric energy dissipated}}{\text{time taken}} = \frac{\text{energy transfer}}{\text{time taken}} \rightarrow P = \frac{I^2 R t}{t} = I^2 R$$

$$\text{From Ohm's law; } V = IR \rightarrow R = \frac{V}{I},$$

$$\text{Therefore; } P = I^2 R = I^2 \times \frac{V}{I} = IV \quad \text{Also; when using, } V = IR \rightarrow I = \frac{V}{R}$$

$$P = I^2 R = \left(\frac{V}{R} \right)^2 \times R = \frac{V^2}{R}$$

$$\therefore P = I^2R \quad \text{Or} \quad P = IV \quad \text{Or} \quad P = \frac{V^2}{R}$$

Worked Examples;

- An electric bulb is rated 60 W, 240 V. Determine:
 - The resistance of the filament
 - The current flowing through the bulb when it is connected to the mains supply

Soln:

Given: P = 60W, V = 240 V

(a) From $P = IV = \frac{V^2}{R}$

$$P = \frac{V^2}{R} \rightarrow R = \frac{V^2}{P} = \frac{240^2}{60} = 960\Omega \quad \therefore R = 960\Omega$$

(b) From ohms law: $V = IR$

$$\therefore I = \frac{V}{R} = \frac{240}{960} = 0.25 \text{ A}$$

Alternatively;

By using power formulae $P = IV \quad \therefore I = \frac{P}{V} = \frac{60}{240} = 0.25 \text{ A}$

Class Activity – 10:7

- If a different hair dryer is plugged into a 120-volt circuit that carries 10 amps of current. What is the power of the other hair dryer? **[ANS; 1200W]**
- An electric kettle draws a current of 10A when connected to the 230V mains supply. If all the energy produced in 5 minutes is used to heat 2kg of water. Calculate
 - The power of the kettle
 - The energy produced in 5 minutes
 - The rise in temperature (Specific heat capacity of water = $4200 \text{ Jkg}^{-1}\text{K}^{-1}$)**ANS (i) 2.3 kW (ii) 690 kJ (iii) 82.14 K**
- Calculate the energy consumed in electrical units when a 75 W fan is used for 8 hours daily for one month (30 days). **[ANS; E = Pt = 75x30x8 = 18kWh]**
- An electric motor powered by a 240 V mains supply requires a current of 30A to lift a load of mass 3 tonnes at the rate of 5 m per minute. Calculate:
 - The power input **(ANS: P_{IN} = 7200 W)**
 - The Power output **(ANS: P_{OUT} = 2500 W)**
 - The overall efficiency of the machine **(ANS: Eff = 34.72%)**
- What is the maximum number of 100 W bulbs which can be safely connected from a 240 V source supplying a current of 5 A? **(A: n =12 bulbs)**

Applications of heating effect of electric current

- Electric iron, electric heaters, electric toasters are some of the home appliances that utilize the heating effect of current. In these appliances, the heating elements are made of **nichrome**, an alloy of nickel and chromium. Nichrome has a high specific resistance and can be heated to very high temperatures without oxidation.
- It is used in electric bulbs to produce light. It consists of a tungsten filament (melting point $3380 \text{ }^\circ\text{C}$) kept inside a glass bulb and heated to incandescence by current. In incandescent

electric lamps only about 5% of electrical energy is converted into **light and the rest is wasted as heat**. Electric discharge lamps, electric welding and electric arc also utilize the heating effect of current

- It is used in electric fuse. Electric fuse uses the heating effect to safeguard residential wiring and electrical gadgets.

Electrical appliances

- ✓ Electrical appliance is the device which uses heating element to produce heat energy.
- ✓ Nichrome wire is among of the heating elements due to its high melting point.
- ✓ The common electrical appliances includes **Heaters, Electric iron, Bulbs, kettles, Cookers, fridges, Televisions and Air conditioner**

Power Rating of Electrical Appliances

- The power rating on our home appliances shows how much energy is needed to power the device. This rating helps consumers decide which appliance to buy on how much energy it will use. It also shows the maximum power the appliance can safely handle, which the cable and plug must also be able to handle.
- The label on a device shows the power rating. For instance, if the label reads 1850-2000W, it means the device needs this much power in one second. A mobile phone charger has a power rating of 5-25 watts, meaning it draws a maximum of 25 watts from the mains electricity supply. On the other hand, an electric kettle has a power rating of 3 kilowatts, which is 3000 joules per second - 120 times the power consumed by a phone charger!
- With the same amount of energy, you could boil a kettle for 1 minute or charge your phone for 2 hours (120 minutes)!
- The best way to compare the cost of running different appliances is to look at their **power consumption**, which is measure of how much power they use in **Watts**. The following list points out typical values for the wattage of some devices you would find in your home so that you can compare them.

Power Ratings of common Electrical Appliances

Electrical appliance	power ratings at 240V
Immersion heater	2000W (2KW)
Electric heater	2000W (2KW)
Electric iron	1000W (1KW)
Electric cattle	2500W (2.5KW)
Color TV	300W
Refrigerator	120W
Light bulb	25W - 150W

NB:

- If the mains supply falls below 240 V, the rating of the appliance would drop
- Similarly when the power supply rises ,the rating would rise and it would damage the appliance due to overheating
- **Power rating** is the maximum power that can be used to operate an electrical device

Measurement of Electrical Power & Billing of electrical energy

- Electric utilities measure the electricity consumption of their customers with **meters** that are usually located on the outside of the customer's property where the power line enters the property.

- Power companies like TANESCO usually measure the electrical energy in **kilowatt hours (KWh)**
- $1\text{kWh} = (1\text{kW} \times 1\text{hour}) \text{ J} = (1000\text{W} \times 60 \times 60) = 3600000\text{J} = 3600\text{KJ} = 3.6\text{MJ}$
- Therefore, **the unit of kWh** is used to determine the cost of electrical energy consumed by electricity users

Worked examples

1. Assuming the electric consumption per day to be 12 kWh and the rate of electricity to be Tsh 357.2 per unit, find how much money is to be paid in a month of 30 days?

ANS;

Electric energy consumed per day = 12 kWh

Electric energy consumed for 30 days = $12 \times 30 = 360 \text{ kWh}$

But; 1 kWh = 1 unit, thus; $360 \text{ kWh} = 360 \text{ units}$

Given; 1 unit cost = Tsh 357.2, So 360 units cost = $360 \times 357.2 = \text{Tsh } 128,592$

So money to be paid for 30 days = Tsh 128,592.

2. In a premise 5 bulbs each of 100 W, 2 fans each of 60 W, 2 Air Conditioners each of 1.5 kW are used for 5h per day. Find:
 - (c) total power consumed per day,
 - (d) total power consumed in 30 days,
 - (e) total electrical energy consumed in 30 days,
 - (f) the cost of electricity at the rate of Tsh 360 per unit.

Answer;

(a) Power consumed by 5 bulbs = $5 \times 100 = 500 \text{ W}$

Power consumed by 2 fans = $2 \times 60 = 120 \text{ W}$

Power consumed by 2 A.Cs = $2 \times 1.5 = 3 \text{ kW}$ or $3 \times 1000 = 3000 \text{ W}$

Thus; total power consumed per day = $500 + 120 + 3000 = 3620 \text{ W}$

(b) Power consumed for 30 days = $30 \times 3620 = 108600 \text{ W}$ or **108.6 kW**

(c) Electrical energy consumed in 30 days = Power \times time
 $= 108.6 \times 5 = 543 \text{ kWh}$

(d) Given; the cost of 1 unit = Tsh 360
 therefore; the Cost of 543 units = $360 \times 543 = \text{Tsh } 195,480$

Class Activity – 10:8

1. An electric cooker has a coil of resistance 5000Ω . If is operated on a 250 V mains supply for 1 hour, how much heat energy does it produce? **(ANS: E = 45kJ)**
2. A television set rated 200W is switched on for 5Hours every day. How much energy does it consume in 30 days **ANS: E = $1.08 \times 10^5\text{kJ}$**
3. A house has five rooms, each with a 60W, 240V bulb. If the bulbs are switched on 7:00p.m to 10:30p.m determine the power consumed by bulbs per day. **ANS: P = 1.05kWh**
4. A bulb rated 120 V, 75 W burns continuously for two days. Given that the cost of one unit (1kW) is 320 Tsh. Determine the:
 - (a) Total electrical energy consumed
 - (b) Total power **[ANS; (a) 3.6kWh (b) 1152Th]**

Electrical Installation of a House

- Domestic electricity is supplied by two cables, **live (L)** and **Neutral (N)**, the third cable is **Earth(E)** used to provide extra safety

Live Cable (L)

- A live wire is one that conducts electricity.
- It is responsible for carrying the high voltage and delivering it to the appliances. It's also known as phase wire or hot wire.
- The live cable is 240V relative to the neutral. The current in the live cable alternates 60 times a second (60 Hz). It is represented by **brown or red colour**
- The potential difference between the **LIVE** and the **NEUTRAL** wire is
 $240\text{ V} - 0\text{ V} = 240\text{ V}$

Neutral Cable (N)

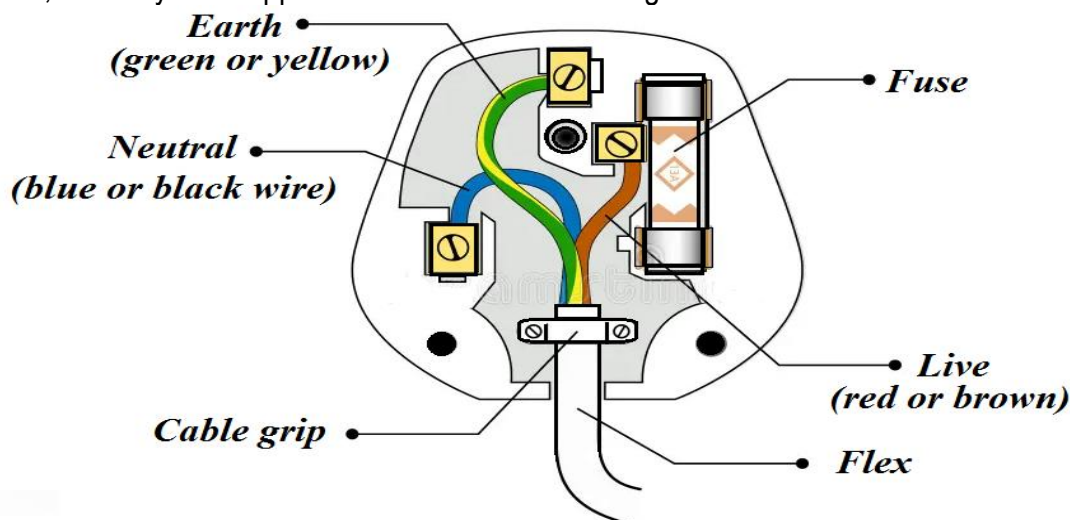
- The Neutral cable is earthed at the power station. This is to ensure current at neutral cable remains almost **zero potential ($V \approx 0V$)** so it cannot give an electric shock on touching.
- This wire completes the circuit by providing a path for the current to return to the power source.
- It represented by **blue or black colour**

Earth Cable (E)

- The primary purpose of earthing is to protect people from electrical shock.
- The earth wire connects the metallic body of electric appliances to the earth, allowing any leakage of electric current to be transported to the ground. It is represented by **yellow or green color**

Three Pin Plug

- It consists of all three cables which are **Live** cable, **Neutral** cable and **Earth** cable with a fuse connected to live cable,
- The fuse should not be connected to the neutral cable, **because** if a fault occurs and the fuse blows, the body of an appliance remains live and can give an electric shock to the user.

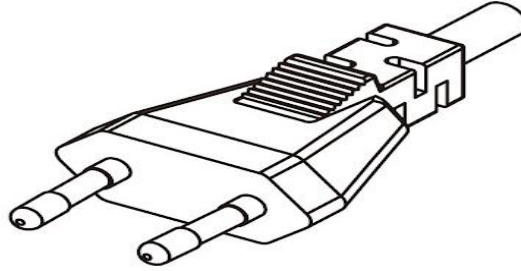


NB:

- ✓ The earth pin usually longer than the other two, this enables it to open the protective cover of the socket. It is also ensures that, an appliance is earthed before being connected to power source
- ✓ Switch must be off when you push the plug into the socket

Two Pin Plug

- It consists of only two cables which are live cable and Neutral cable.
- An appliance using a two – pin plug its body is not connected to earth (see the fig below)



Fuses

- A Fuse Is a safety device used to protect an electric circuit against excess of current.

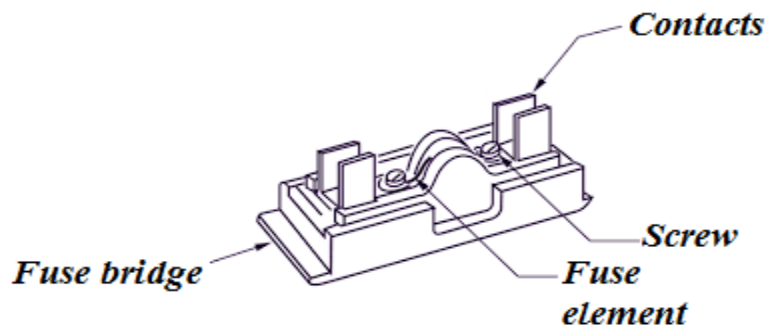
OR

- A fuse is a short piece of special wire which melts when more than a rated amount of current passes through it
- It is made of a thin copper wire covered with tin or a lead – tin alloy
- It works as a circuit breaker or stabilizer which protects the device from damage

Types of Fuses

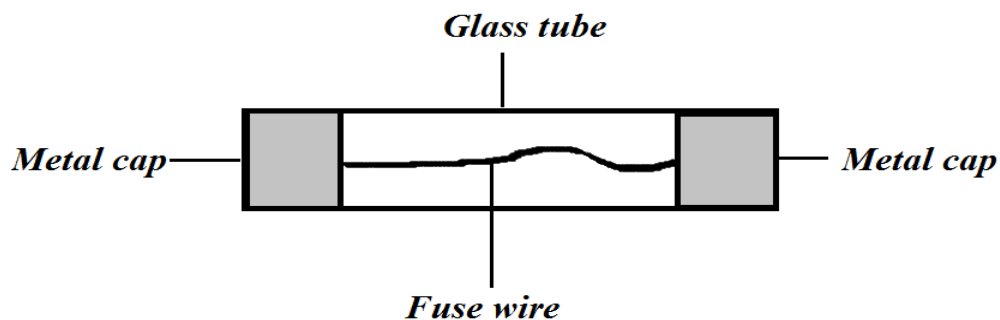
(a) Rewireable fuses

- In this kind of fuse, the fuse element is carried in a removable fuse link made of porcelain or other insulating material



(b) Cartridge fuses

- It consists of a porcelain tube with metal end caps to which the fuse element is attached



Mechanisms:

- The fuse works on the principle of the **heating effect of an electric current**
- It is always connected in series with the **electrical circuit to protect from over current in the running cables**
- When the excessive current or heat is generated due to heavy current flows in the circuit, the fuse melts down due to the low melting point of the element and it opens (breaks) the circuit.
- The excessive flow may lead to the breakdown of wire and stops the flow of current

- Then the fuse can be replaced with the new one with suitable ratings

Applications of fuses

Fuses are used in:-

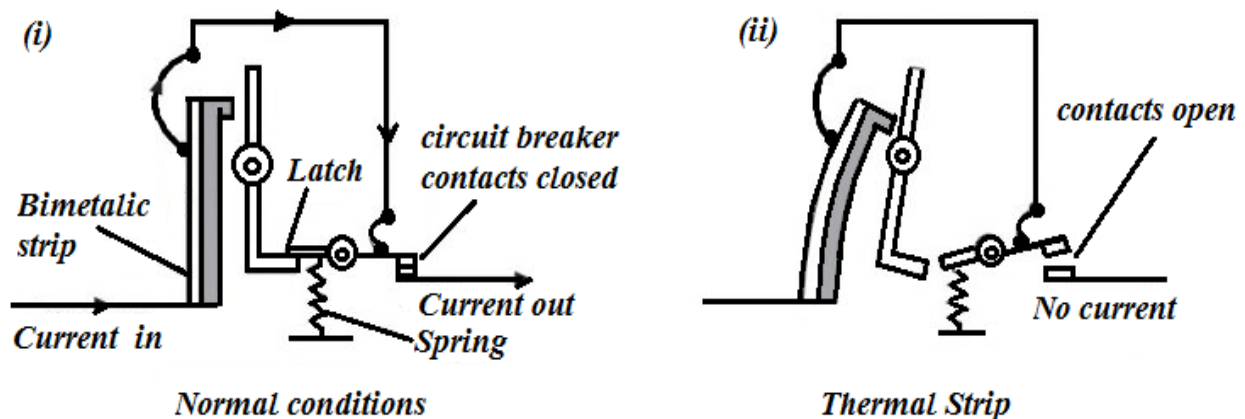
- Electrical appliances (devices)
- Automobiles such as car, trucks and other vehicles
- Scanners, portable electronics, hard disk drives
- Fuses in capacitors, transformers, power converters, motor starters, power transformers

What happen if we don't use fuse?

- If we don't use fuses, electrical faults occur in the wiring and it burns the wire and electrical appliances and this may start fire at home.
- Also the lives of television, computers, radios and other home appliances may put at risk

Circuit Breaker

- Fuses work on the principal of a large current melting the fuse wire. In some cases faults can occur in which the current may not be large enough to melt the fuse but enough to seriously harm the user of the electrical appliance. For protection against faults of this nature **a circuit-breaker is used in place of a fuse.**
- Therefore, Circuit Breaker Is a type of switch that cuts off the flow of electric current when the current exceeds a specific value. (See the fig. below)



Mechanism of Circuit Breakers

- When current exceeds, it increases the temperature and bimetallic strip bends to push latch mechanism, enables the spring to cut off current (see the figure (ii) above)

Circuit-breakers offer the following advantages compared to fuses.

- Circuit-breakers respond quicker than fuses.
- Circuit-breakers are more reliable.
- Circuit-breakers are more sensitive.
- Unlike fuses which only operate once and need to be replaced a circuit-breaker can be reset.

What is Earthing?

- If there is a fault in your electrical installation you could get an electric shock if you touch a live metal part. **This is because the electricity may use your body as a path from the live part to the earth part.**
- **Earthing is used to protect you from an electric shock.** It does this by providing a path (a protective conductor) for a fault current to flow to earth. It also causes the protective device (either a circuit-breaker or fuse) to switch off the electric current to the circuit that has the fault.

- **For example, if a cooker has a fault**, the fault current flows to earth through the protective (earthing) conductors. A protective device (fuse or circuit-breaker) in the consumer unit switches off the electrical supply to the cooker. The cooker is now safe from causing an electric shock to anyone who touches it.

Class Activity – 10:9

1. The metal case of an electric appliance is earthed. Explain the reason.
2. You have a choice of the following fuses 1A, 3 A, 5 A, 9 A, 13 A and 30 A. Select the best fuse for (a) a 240 V , 7.2 kW electric cooker (b) a 240 V ,2kW electric iron

ANS: (a) The best fuse is a 30 A fuse (b) The best fuse is 9 A fuse

Domestic Wiring Circuit

- **Electrical wiring (A wiring system)** is a network of wires that connect various accessories for the distribution of electrical energy from the **supplier metre board** to a variety of **electrical appliances** such as lamps, fans, and other domestic appliances via regulating and safety devices.
- The power company connects power to the house up to the consumer unit where the house wiring starts
- Consumer unit is the place where the main switch, main fuse and distribution board are placed in a single box or unit
- From the consumer unit , the cables branch into the various parts of the house

Types of Domestic Wiring Circuit

- Ring main circuit
- Lighting circuit

Ring Main Circuit

- This is a cable which begins and ends at the consumer unit. Its three cables (Live, Neutral and Earth) are forming ring around part of the house. Its fuse is of 30A fuse

Lighting Circuit

- In this circuit the first lamp connected from the customer unit, in turn is connected to the second lamp and so on.

Types of Lighting Circuit

- Loop – in lighting circuit
- Junction box lighting circuit

Loop In Lighting Circuit

- All three cables from consumer unit run to each ceiling roses, one after another. From each rose another set of cables runs to the switch which operates the light

Junction Box Lighting Circuit

- All three cables from consumer unit run to one junction box to another, where one cable runs to the light and another run to the switch for that light.

Repairing Electrical Appliances Faults

- **Multimeter** and **Live mains lead indicator** are devices important when checking electrical appliances faults.

Multimeter

- Multimeter is the single meter for measuring current (both a.c and d.c), voltage and resistance

Live Mains Lead Indicator (Testor)

- Is an electronic device used for testing the flow of electric current

Repairing of Faults Procedures

If electrical appliance fails to work the following procedure should be done

- Check by using **live mains lead indicator** if there is power or not
- Check the cable from the socket to the appliance
- If no fault is detected , open the plug and check the fuse
- Check each cable for continuity by using a **Multimeter**
- If cables are working properly, then check the fault is in the element by using a Multimeter
- If element is in fault, replace element as repair may not be possible
- If the element is not in fault , look for loose connections, these should be made firm and/or cleaned of rust and other dirt

Sources of Faults in domestic system

- When fuse blows or melt
- Wire cutting
- Wire joining
- Socket getting dirty
- Switches breaking

Class Activity – 9:10

1. Sarah complains that one of her wall sockets is not functioning properly. She checked the live and neutral terminals using a domestic electric tester. Both indicate the presence of electricity, but the appliance does not work. Give a possible explanation.

ANS;

If the circuit is not faulty, the neutral wire should not indicate the presence of electricity in it. The live wire and the neutral may be in contact at some point in the circuit, which in turn keeps the two wires at the same potential.

Since there is no potential difference, the bulb will not work, when connected.

2. **An electrician is carrying out some repairs in your house. He wants to replace a fuse by a piece of wire. Would you agree? Give reasons for your response.**

3. It is highly unlikely that a bulb connected to the Earth terminal and the neutral wire on a wall socket would light up. Explain.

ANS;

Since both ends are at zero potential, there is no potential difference between the two. So, there would be no flow of current through the bulb for it to light up.

Cells

A Cell is a device used to cause a flow of electric current in a conductor

Types of electrochemical Cells

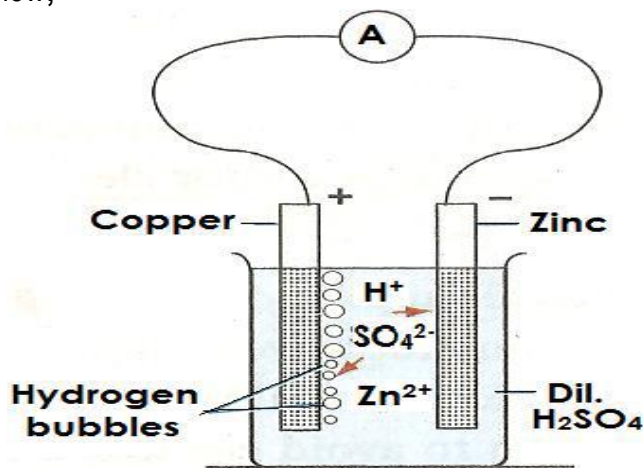
- Primary cell
- Secondary cell

Primary Cell

- Is a cell which produces current as a result of irreversible chemical changes taking place within the cell
- Examples of primary cells are **A Simple cell**, **Leclanché cell** and **Dry cell**

Simple Cell

- Made from copper as anode, zinc as cathode and Dilute sulphuric acid as electrolyte
- See the figure below;



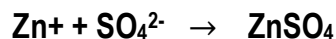
The processes that occur when the cell is in operation

- The dilute sulphuric acid separates into sulphate ions (SO_4^{2-}) and hydrogen ions (H^+):



At anode

- Zinc metal dissolved into solution to form zinc cation which reacts with sulphate anion to form zinc sulphate



At cathode:

- Hydrogen cation discharges to liberate hydrogen gas (bubbles)



Defects of a Simple Cell

- A Simple cell has two defects which cause the current to drop quickly when the cell is being used.
- These defects are **local action** and **polarization**

Local Action

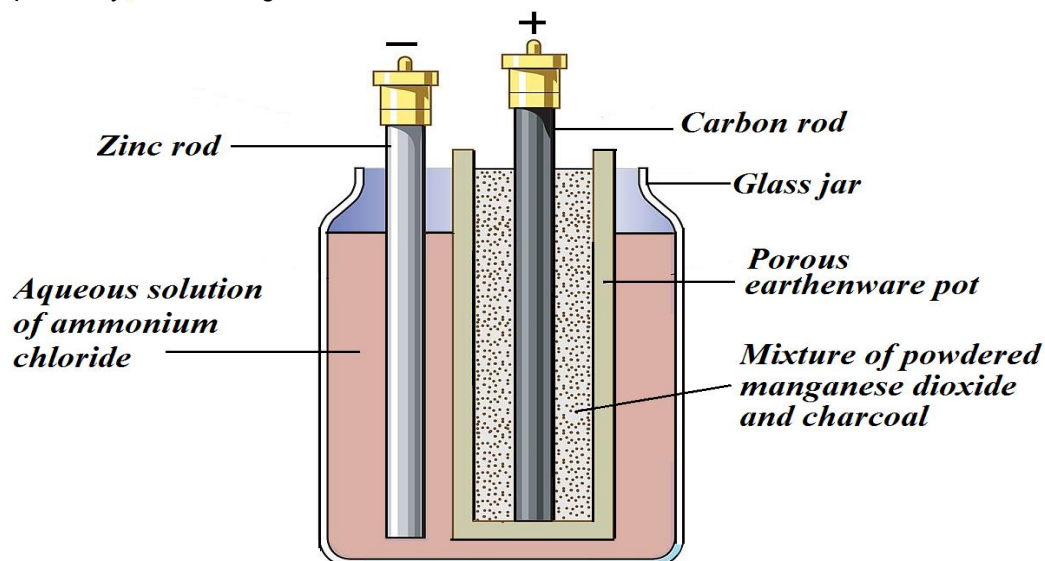
- Is caused by the presence of small impurities in the zinc electrode
- Local action can be minimized by coating the surface of zinc with mercury (Amalgamating)

Polarization

- Is the formation of a layer of hydrogen bubbles on the copper plate
- The hydrogen insulates the copper plate thus increasing the internal resistance of the cell, this lowers the current
- Polarization can be minimized by adding a chemical (**depolarizer**). Which reacts with hydrogen bubbles to form water
- Example of depolarizer is potassium dichromate

Leclanché Cell

- A **Leclanche cell** consists of a glass vessel which is filled with ammonium chloride solution. Ammonium chloride solution acts as an electrolyte. In it, there stands a zinc rod and porous pot containing a carbon rod, which is packed around with a mixture of manganese dioxide and powdered carbon. The carbon and zinc rods act as positive and negative electrodes respectively. See the figure below

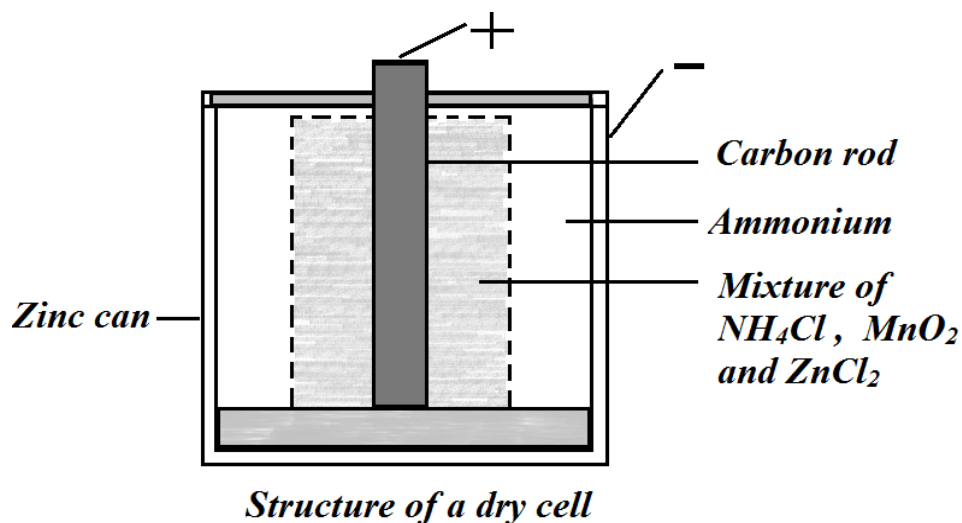


Dry Cell

- Dry cell is a modified Leclanche cell in which the ammonium chloride solution is replaced with ammonium chloride jelly

It consists of;

- **Carbon Rod.** The center of a zinc-carbon battery is a rod of pure carbon in the form of graphite. The carbon rod is covered in a mixture of carbon powder and manganese dioxide.
- **The purpose of the carbon rod** is simply to allow the flow of electrons. The carbon powder will increase the electrical conductivity.
- **Zinc Container.** The outer case is of zinc metal. The zinc metal will oxidize, causing it to donate two electrons for each zinc atom. These electrons will flow through the electrolyte into the carbon rod to produce an electrical current.
- This zinc layer will get thinner as the zinc oxidizes and the battery will no longer be able to conduct electricity once the zinc sleeve is completely gone.
- **Electrolyte.** The carbon rod is surrounded by an electrolytic paste of ammonium chloride (NH_4Cl) and Manganese oxide. The electrolyte is a chemical medium that allows the flow of electrical charge.
- **The Manganese(iv) oxide** acts as depolarizer (**See the fig below**)



Uses of Dry Cell

- It is used to operate on radios, electronic calculators and other small electrical devices

Advantages of Dry Cell

- It is portable
- The chemicals for its production are relatively cheap
- It has a relatively high e.m.f
- It is able to recover from its polarized

Disadvantage of dry cell

- Cannot be recharged thus disposed after use

Secondary Cell

- Secondary cell is the cell which can be recharged after running down.
- Eg **lead acid cell** and **nickel ferrous cell**. Also called **accumulators**

Advantages of secondary cells

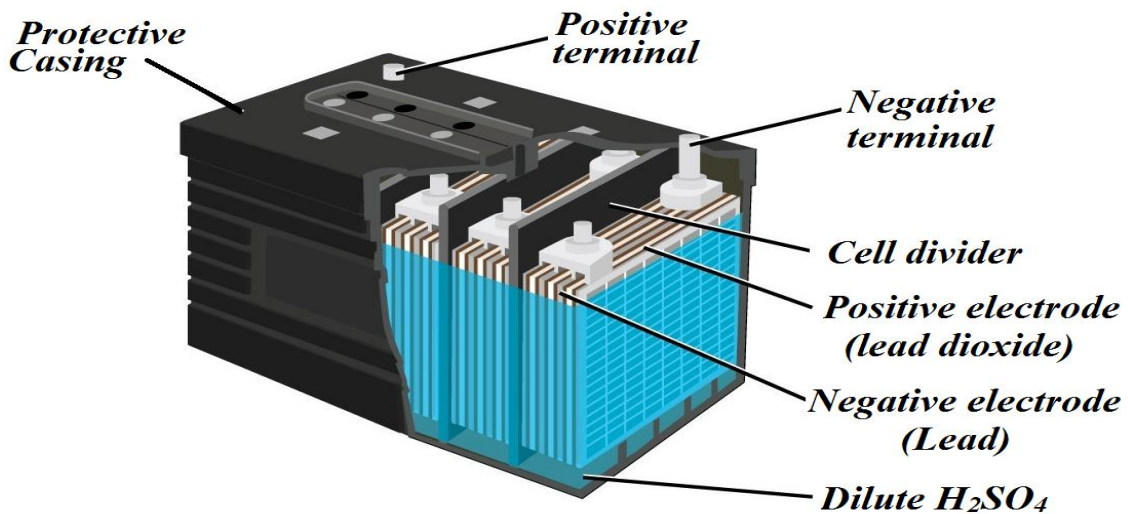
- Last longer than primary cell
- Can supply large current (low internal resistance) for a long time
- Can be recharged

Disadvantage of secondary cells

- Heavy and cumbersome to carry
- The electrolyte is corrosive
- It produces gases which may explode if ignited
- It cannot produce large currents in cold weathers (**Lowering the temperature causes chemical reactions to proceed more slowly**)

Lead Acid battery

- It consists of two plates of lead immersed in dilute sulphuric acid .These are Anode of lead(IV) oxide and cathode of spongy lead
- An electrode is separated by insulator called **separator**.
- Cathode joining together to form negative terminal while anode joining together to form positive terminal. (see the fig. below)



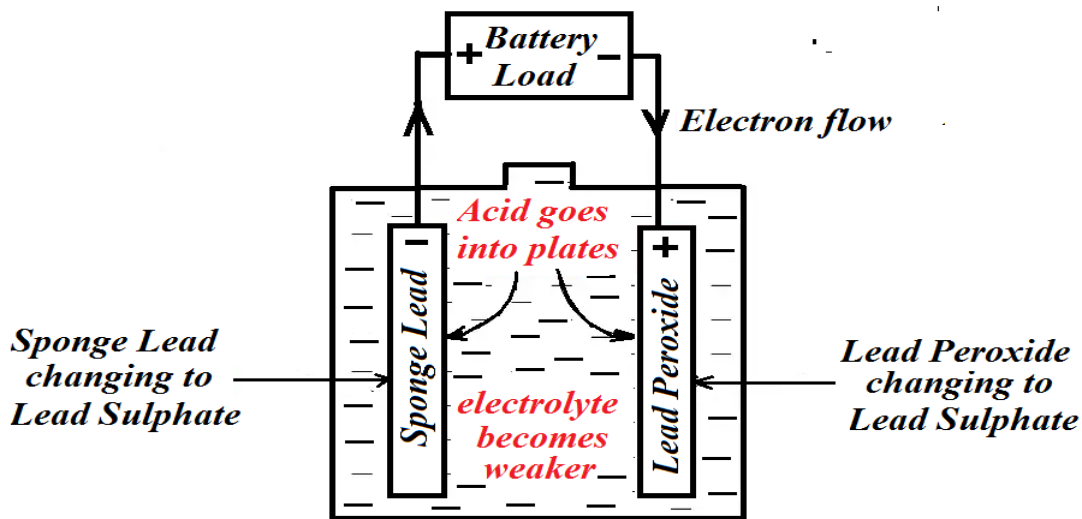
The lead - acid accumulator

The main components of lead-acid batteries include plates, separators, electrolytes, and battery cover.

- **Polar plates:** Both the positive and negative electrodes are made of special alloy grids which are coated with active materials. The plates store and release electricity during charging and discharging to ensure the reliability of the battery's capacity and performance.
- **Separator:** A separator is an isolation medium placed between the positive and negative electrodes of the battery to prevent the battery's positive and negative poles from directly contacting and short-circuiting.
- **Electrolyte:** The electrolyte of a lead-acid battery is dilute sulfuric acid prepared with distilled water. The electrolyte plays a role in transmitting ions between the positive and negative electrodes during charge and discharge. Therefore, the electrolyte must be free of impurities.
- **Container (battery case cover):** The electrolyte and the electrode plate are in the container. The container mainly plays a supporting role, and at the same time prevents internal materials from spilling or external materials from entering the internal structure to pollute the battery.

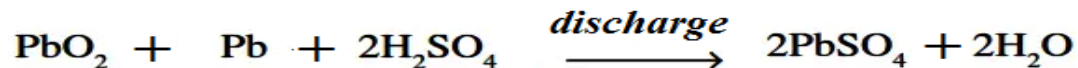
Charging and Discharging of Lead – Acid Battery

- In a lead-acid battery, two types of lead are acted upon electro-chemically by an electrolytic solution of diluted sulfuric acid (H_2SO_4). The positive plate consists of lead peroxide (PbO_2), and the negative plate is sponge lead (Pb), shown in Figure below.

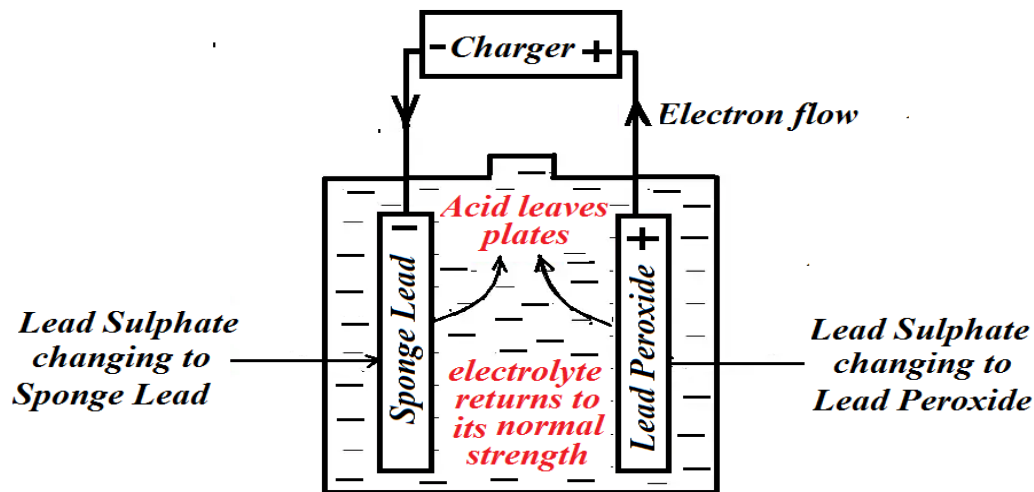
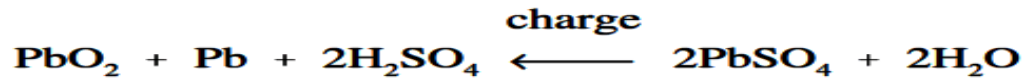


(a) Chemical Action during Discharging

- When a lead-acid battery is discharged, the electrolyte divides into H₂ and SO₄ combine with some of the oxygen that is formed on the positive plate to produce water (H₂O), and thereby reduces the amount of acid in the electrolyte. The sulfate (SO₄) combines with the lead (Pb) of both plates, forming lead sulphate (PbSO₄), as shown in Equation.



- As a lead-acid battery is charged in the reverse direction, the action described in the discharge is reversed. The lead sulphate (PbSO₄) is driven out and back into the electrolyte (H₂SO₄). The return of acid to the electrolyte will reduce the sulphate in the plates and increase the specific gravity. This will continue to happen until all of the acid is driven from the plates and back into the electrolyte, as shown in below Equation and Figure (b).



(b) Chemical Action during Charging

NB;

- As a lead-acid battery charge nears completion, hydrogen (H₂) gas is liberated at the negative plate, and oxygen (O₂) gas is liberated at the positive plate. This action occurs since the charging current is usually greater than the current necessary to reduce the remaining amount of lead sulphate on the plates. The excess current ionizes the water (H₂O) in the electrolyte. Since hydrogen is highly explosive, it is necessary to provide adequate ventilation to the battery whenever charging is in progress. Also, no smoking, electric sparks, or open flames are allowed near a charging battery.

Taking Care of Accumulators

- Cells should be charged regularly and should never left discharged
- The acid level should be maintained by adding distilled water when necessary (never add acid)
- The terminal should be kept clean and greased
- Rough handling should be avoided
- The cells should not be short – circuited , Example, if you connect two terminals to each other
- The rate specified by manufacturer should not be exceeded during charging

Uses of Accumulators

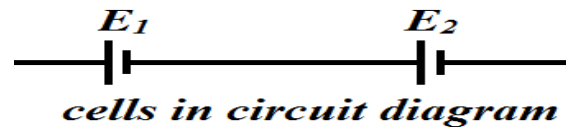
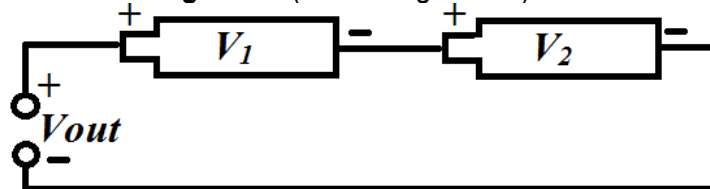
- Used to provide power in motor vehicles
- Used to provide energy to power domestic appliances such as radio

- Used together with solar panels to convert solar energy to electrical energy
- They are used to store electrical energy

Arrangement of Cells

Series Arrangement of Cells

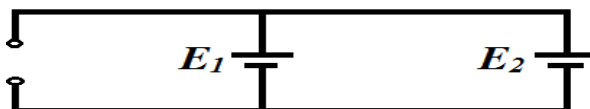
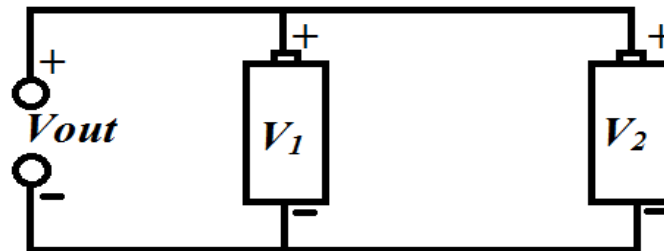
- In this series arrangement the positive terminal of one cell is connected to the negative terminal of another cell
- Electric current is the same at each cell
- Total voltage across the cells is equal to the sum of the voltage of the individual cells, **that is why torch light uses this arrangement** (see the fig. below)



- The total voltage, for cells arrangement from the figure above is then given as; $E_T = E_1 + E_2 + E_3 + \dots + E_n \rightarrow E_T = nE$
- Also; the sum of individual internal resistance, r equals total internal resistance, r_T i.e., $r_T = r_1 + r_2 + r_3 + \dots + r_n \rightarrow r_T = nr$

Parallel Arrangement of Cells

- In parallel arrangement, all the positive terminals of the cells are connected together and negative terminals of the cells are connected together. (see the fig below)



NB:

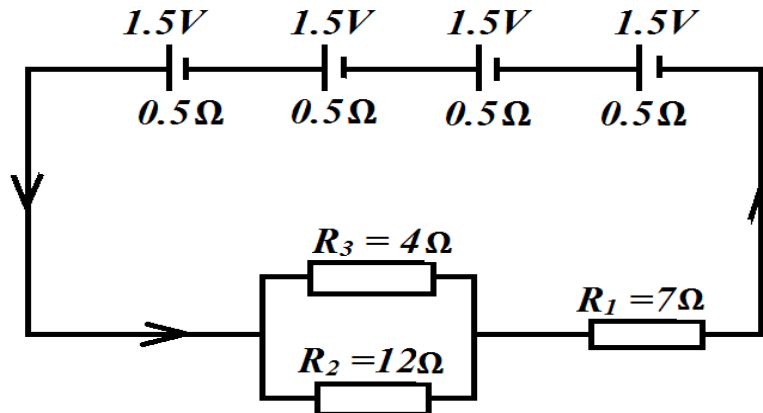
- The total voltage of all of the cells is the same as that of a single cell
i.e., $E_1 = E_2 = E_3 = E_T$
- Total current across the cells is equal to the sum of the current across each cell, that is **why lead acid accumulator uses this arrangement**

- The internal resistance of the battery is different from that of a single cell. For this case, the internal resistance of two cells in parallel connection as shown above the total internal resistance can be obtained as follow

$$\frac{1}{r_T} = \frac{1}{r_1} + \frac{1}{r_2} + \dots + \frac{1}{r_n}, \quad \text{but; for given two cells } r_T = \frac{r_1 r_2}{r_1 + r_2}$$

Class Activity – 10: 11

- A battery of 4 cell, each of e.m.f. 1.5 volt and internal resistance 0.5 Ω is connected to three resistances as shown in the figure. Calculate:



- The total resistance of the circuit. [ANS; $R = R_T + r_T = 10 + 2 = 12\Omega$]
 - The current through the cell. [ANS; $I = \frac{E}{R_T + r_T} = \frac{6}{10 + 2} = 0.5A$]
 - The current through each resistance. [ANS; 0.375A, 0.125A, 0.5A]
 - The p.d. across each resistance. [ANS; 1.5V, 1.5V, 3.5V]
- You have a battery with an EMF of 5 V and an unknown internal resistance. You hook the battery up to a circuit with one 3Ω resistor and measure the current through the resistor to be 1.5 A. What is the internal resistance of the batter and how much power is the battery's resistance dissipating.

ANS;

- From; $E = I(R + r) = IR + Ir \rightarrow r = \frac{P.d}{I} = \frac{E - IR}{I} = \frac{5 - 1.5 \times 3}{1.5} = 0.33\Omega$

- Therefore; the power dissipated by the resistor, $P = IV = I(Ir) = I^2 r$

$$P = 1.5 \times 1.5^2 \times 0.33 = 0.74W$$

- A battery has an emf of 12 V and connected to a resistor of 3 Ω. The current in the circuit is 3.5 A. Calculate

- Terminal voltage and the internal resistance of the battery
- Power delivered by the battery and Power delivered to the resistor

ANS;

- Terminal voltage, $V = IR = 3 \times 3.5 = 10.5A$, $r = \frac{P.d}{I} = \frac{E - IR}{I} = \frac{12 - 10.5}{3.5} = 0.43\Omega$

- Power delivered by the battery, $P = IV = IE = 12 \times 3.5 = 42W$

Power delivered to the resistor, $P = IV = I(IR) = I^2 R = 3.5^2 \times 3 = 36.75W$

The remaining power = (42 – 36.75) = 5.25 W is delivered to the internal resistance and cannot be used to do useful work. (it is equal to $I^2 r$).

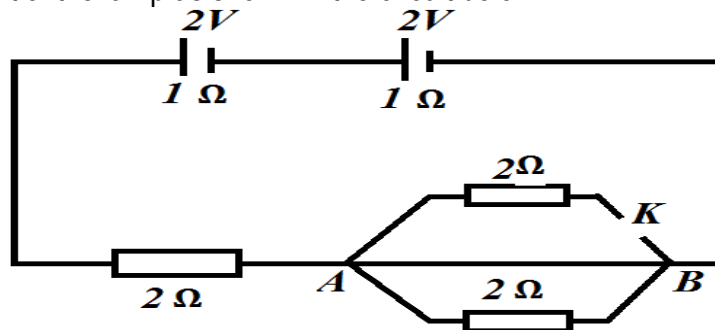
- Semitractor trucks use four large 12-V batteries. The starter system requires 24 V, while normal operation of the truck's other electrical components utilizes 12 V. How could the four batteries be connected to produce

- (i) 24 V? (i) 12 V?

- (b) Why is 24 V better than 12 V for starting the truck's engine (a very heavy load)?
5. Explain Why lead acid accumulators are used in car battery rather than dry cells
 6. You are provided with a car battery, a switch and two car headlights. Draw a possible circuit diagram for the arrangement that will allow the driver to switch on the two lights simultaneously.
 7. Eight dry cells can be arranged to produce a total emf of 12V, just like a car battery.
 - (a) Find an emf of an individual cell. **[ANS: 1.5V]**
 - (b) Explain why it is possible to start a car with the lead – acid accumulator, but not with the eight dry cells in series?

Self Assessment –10

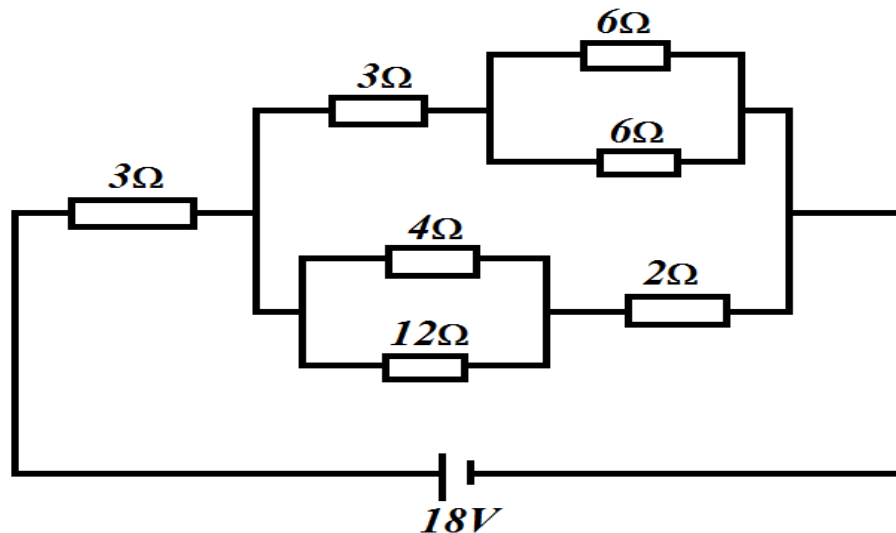
1. A current of 1.5 A flows in a wire .Find the total charge passed in 20 seconds(**Q = 30 C**)
2. A charge of 3600 C passes through an electric lamp in 3.0 minutes .What is the current in the lamp (**ANS: I = 20 A**)
3. How many electrons pass through a lamp in 10 seconds, if the current is a 125 mA and the charge of one electron is 1.6×10^{-19} C? (**ANS: n = 7.813 x 10¹⁸ electrons**)
4. Electron in hydrogen atom revolves around the nucleus with frequency 6.0×10^{-4} per second .Calculate the current in the orbit .Given that charge on an electron = 1.6×10^{-19} coulomb. (**ANS: I = 9.6 x 10⁻⁵ A**)
5. If a container of surface area 2.1 m^2 is to be coated with silver about 0.1 mm thick ,calculate the time it will take if a current of 4 amperes has to flow
6. A nichrome wire of radius 0.35 mm has a resistivity of $1.5 \times 10^{-6} \Omega \text{m}$.Given that the wire has a length of 80 cm .Calculate
 - (a) its resistance
 - (b) Conductance
 - (c) Conductivity
7. Explain the factors which determine the resistance of a conductor
8. When resistors are connected in series ,which of the following is the same for all the resistors
 - (a) potential difference
 - (b) current
9. Find the resistance of a wire of 1100 cm long , 0.2 mm diameter and of resistivity $1.57 \times 10^{-6} \Omega \text{ m}$ (**ANS: R = 546.7 Ω**)
10. Calculate the energy dissipated by a resistor of 12Ω in 4 seconds if a voltage of 6 V is applied. (**ANS: H = 12 J**)
11. Calculate the heat lost by a wire of resistance 16Ω when a current of 30 A flows through it in 1 second. (**ANS: H = 14 400 J**)
12. A battery consisting of two cells of 2 V and internal resistance of 1Ω each is sending a current through the filament of the lamp as shown in the circuit below



Calculate :

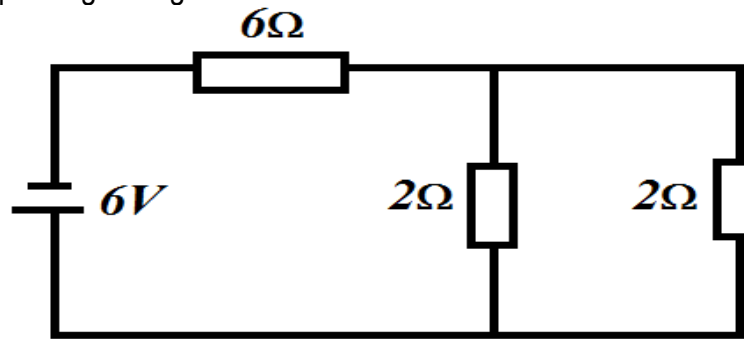
- (a) The current in the circuit when
 - (i) Switch K is open **(ANS: I = 0.67 A)**
 - (ii) Switch K is closed **(ANS: I = 0.8 A)**
- (b) the potential difference across the battery of two cells when K is closed (**P.d = 2.4 V**)

13. Define resistance and state its SI units. When is the resistance of a conductor said to be one ohm? A current of 2 A is observed to flow through a conductor when a potential difference of 50V is applied between its ends. Calculate the resistance of the conductor.
14. Distinguish between primary and secondary cells
15. Find the current in the 12 Ω resistor (**ANS: I = 0.429 A**)



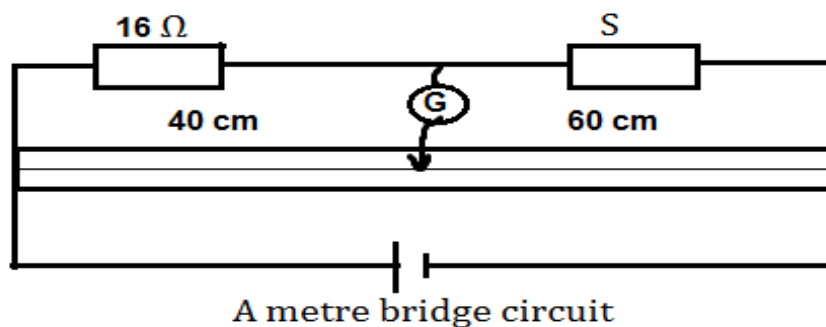
16. State the factors which determine the resistance of a conductor. Define Resistivity and give its SI unit. Find the length of constantan wire of diameter 1cm needed to make a resistor of 3 Ω . Take the resistivity of constantan as $4.9 \times 10^{-7} \Omega m$
17. Two wires A and B are made of the same material .A has half the length and twice the diameter of B .What is the ratio of the resistance of B to that of A?
18. A battery consists of three accumulators in series, each having an e.m.f of 2 V. A second battery consists of four dry cells also in series, each having an e.m.f of 1.5 V. What is the e.m.f of each battery? Why could you get a bigger current from the battery of accumulators.
(**ANS: 6 V each**)
19. Each of the two new dry cells has an e.m.f of 1.5 volts and internal resistance of 1.0 ohm.The two cells are connected to a 10 ohm resistor. Find the current and heat developed per second on the 10 ohm resistor
20. A fine wire has a resistance of 4.0 Ω/m .When a coil made from this wire is connected to a 50 V supply a current of 25 mA flows
(a) what is the length of wire making this coil?
(b) **Determine the resistivity of this wire if its diameter is 0.35 mm**
21. Show that if two resistors are joined in parallel and in series, the effective resistance R is given by: $R = \frac{R_1 R_2}{R_1 + R_2}$ and $R = R_1 + R_2$ respectively, where R_1 and R_2 are the separate resistances
22. Three resistors of resistances 10 Ω , 25 Ω and 50 Ω respectively, are to be connected between two points A and B. What will be the resistance between A and B if the three resistors are connected (a) In series (b) In parallel
23. Three conductors of resistances 10 Ω ,15 Ω and 25 Ω are joined in series across a 100V supply, Find (a) The total resistance (b) The current in the circuit
(c) The potential difference across each conductor

24. Calculate the current passing through the 6Ω resistor in the circuit shown in the figure below



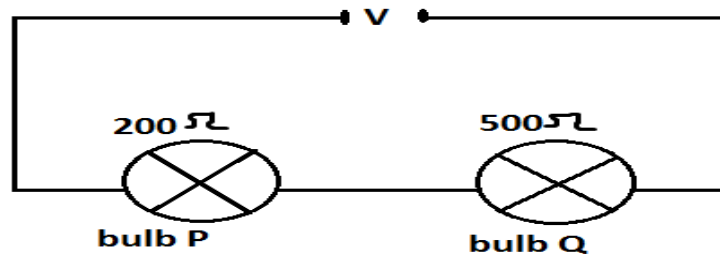
25. State the main facts about cells connected
(a) in series and (b) in parallel. Three cells each of e.m.f. 1.5V and internal resistance 0.6Ω are connected in parallel. The group of cells is then connected across a conductor of resistance 1Ω . Calculate the current in the circuit.
26. A cell has an e.m.f of 1.5 V , and an internal resistance of $1\ \Omega$, and is connected to two resistances of $2\ \Omega$ and $3\ \Omega$ in series. Find the current flowing and the potential difference across the ends of each resistance. **(ANS: 0.25A , 0.5 V , 0.75 V)**
27. Two cells each having an e.m.f of 1.5 V and an internal resistance of 1Ω are connected to a resistance of $4\ \Omega$. What is the current in this resistance if the cells are connected in parallel? **(ANS: $I = 0.33\text{ A}$ or $1/3\text{ A}$)**
28. Two resistors of resistances 3Ω and 5Ω respectively, are connected in the gaps of a metre bridge. At what point on the wire of the bridge will a centre-zero galvanometer show no deflection?
29. The P.d across the terminals of a cell is 3.0 V when it is not connected to a circuit and no current is flowing. When the cell is connected to a circuit and a current of 0.37 A is flowing the terminal P.d falls to 2.8 V . What is the internal resistance of the cell?. **(ANS: $r = 0.54\ \Omega$)**
30. Two resistors of resistance $30\ \Omega$ and $80\ \Omega$ are connected in parallel. Calculate their equivalent resistance
31. A cell pushes a current of 2.0 A through a $0.6\ \Omega$ resistor. When the same cell is connected to a $1\ \Omega$ resistor, the current that flows is now 1.2 A . Calculate:
(a) the internal resistance of the cell
(b) The e.m.f of the cell
32. State joule's law of heating. Describe an experiment to show that the heat developed in a conductor by the passage of an electric current depends on the magnitude of the current.
33. Give joule's formula for the quantity of electrical energy generated in a wire carrying a current and define all symbols used. A current of 2A is passed through a conductor of resistance 10Ω for 5 minutes. Calculate the quantity of heat dissipated in the conductor
34. In an experiment to find the resistance of a resistor R using the wheat stone bridge, the balance point was found to be at the 35 cm mark on a 100 cm nichrome wire. If the value of the resistance needed to balance the bridge on the other side was $30\ \Omega$, calculate the value of the resistance of the resistor R . **(ANS: $R = 16.2\ \Omega$)**
35. .When is an electrical conductor said to dissipate one joule? The current in an electrical appliance operating from a 240V supply is 5A . How much energy is used up in operating it for 20 minutes?
36. The resistance of a heating coil of an electrical hot water system is 100 ohm . If the coil operates from a 240V supply, calculate the rate at which the coil consumes electrical energy.
37. What is the resistance of a wire if it balances a standard resistor of 2 ohms at the 56 cm from the end of the metre bridge? **(ANS: $R = 2.54\text{ cm}$)**

38. Find the value of the unknown resistor S in the balanced wheat stone bridge circuit in the figure below



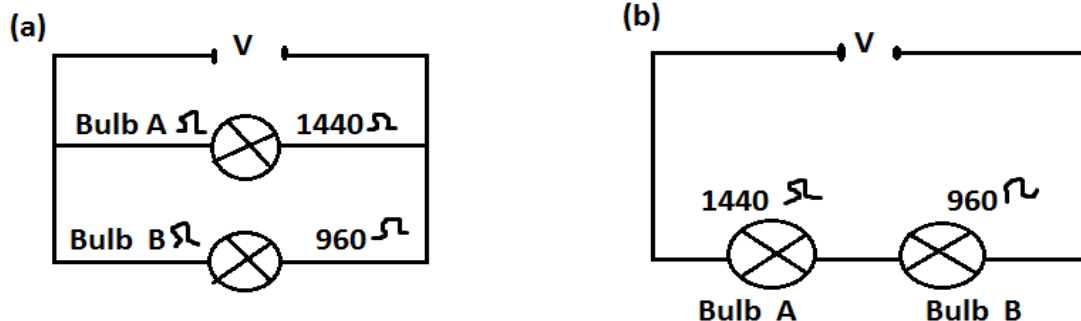
39. (a) A domestic electric bulb is marked 240 V, 100 W. Calculate
- The maximum current it can take
 - The resistance of the filament of the bulb
- (b) If the bulb in (a) above is switched on for 8 hours, find the total amount of energy dissipated in the filament
40. (a) The heating coil of an aluminium electric kettle is labeled 240 V, 4 Kw. Water of mass 3 kg at a temperature of 25° C is poured into the kettle. If the kettle is connected to a 240 V supply when its temperature is 25° C, Calculate the time taken for the water to reach its boiling point
- (b) In what further time will 10% of the water in the kettle evaporates away? (Mass of kettle = 0.5 kg, specific heat capacity of aluminium = 900 J/kg⁻¹K⁻¹ , Specific latent heat of vaporization of water = 2.268 x 10⁶ J/kg⁻¹K⁻¹)
41. If electrical energy is charged at the rate of Tsh 100 per kilowatt – hour , Calculate the cost of using :
- a 60 W light bulb for 8 hours
 - a 1 kW electric iron for 1.5 hours
 - a 6000 W electric cooker for 2 hours
42. An electric iron consumes 2.592 MJ of energy in 1 hour when connected to the mains power supply of 240 V. Calculate the current through the filament in the electric iron. **(ANS: I = 3 A)**
43. State the properties and functions of a fuse. How does a fuse in the lighting circuit differ from that used in the heating or power circuit?
44. (a) What is the importance of using a fuse in an electrical appliance
- (b) A refrigerator is marked 250 V, 400 W. Calculate the maximum current that can flow through it?
- (c) Discuss what might happen to the refrigerator if it is connected to:
- a 230 V supply
 - a 110 V supply
45. (a) State and explain the causes of electrical short – circuit.
- (b) Explain briefly why cables in a lighting circuit are different from those in a power circuit
- (c) Fuse wires are labeled 2 A, 3 A, 5 A ,13 A and 14 A .Which of these is most suitable for:
- a 220 V ,2.8 kW electric iron?
 - a 240 V, 400 kW refrigerator?
 - a 220 V, 3 kW water heater?
46. State the defects of a simple cell and explain how these defects may be minimized?
47. (a) What is the difference between a dry and a wet Leclanche cell
- (b) Explain why a Leclanche cell is not suitable for use as a continuous sources of energy.
48. State four household electrical appliances where electrical energy is converted into heat
49. A washing machine is marked 240 V, 3Kw .What does this mean? Hence calculate the electrical energy used up by this machine in 1 hour
50. Calculate the (a) current through (b) resistance of the filament of
- a bulb rated at 240 V , 60W

- (b) an electrical kettle rated at 2 KW, 240 V
51. The filament of the bulb is made of tungsten and the bulb contains a mixture of argon and nitrogen at low pressure
- (a) what is the purpose of the presence of the gases inside the bulb
- (b) why is tungsten a suitable material for the filament
52. Electrical heaters are said to be environmentally friendlier than the heating devices which use firewood or charcoal .Explain this statement
53. Starting from electrical work done $W = ItV$, show that electrical power (P) generated in a conductor is given by V^2/R ,where the symbols have the usual meaning
54. Three cells each of e.m.f 1.5 v and internal resistance 0.6Ω joined in series to form a battery and connected across a 0.5Ω resistor .Calculate
- (a) the current (b) The P.d between the terminals of the cell
55. Which bulb in the figure below is the brightest? Explain your answer

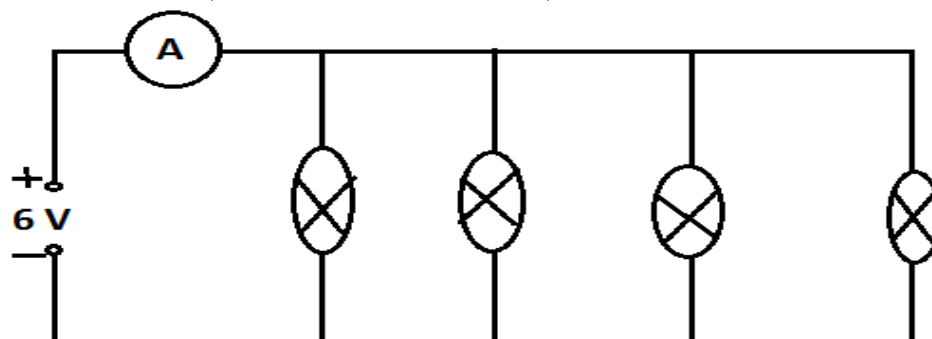


56. (a) State Ohm's law and state two of its limitations
- (b) Determine the internal resistance of a cell and the value of R given that the p.d of the cell in open circuit is 1.5 v, when connected to a 10Ω resistor its p.d becomes 1.0 v, but when connected to a resistor of $R \Omega$ the p.d falls to 0.5 v (ANS: $r = 5 \Omega, R = 2.5 \Omega$)
57. The power rating of an electric bulb is '60 W, 240 V'
- (a) Calculate the current through the filament and the resistance of the filament (ANS: $R = 960 \Omega, I = 0.25 \text{ A}$)
- (b) By comparison with the answers to (a) above determine the current and the resistance of the filament of a '120 W,240 V' bulb. Explain your answer (A: $I = 0.5 \text{ A}, R = 480 \Omega, I \propto \frac{1}{R}$)
58. An electric bulb is labeled ' 40 W,240 v'. Calculate
- (a) the resistance of the filament used in the bulb(ANS: $R = 1\ 440 \Omega$)
- (b) The current through the filament when the bulb works normally ($I = 0.167 \text{ A}$)
59. A 3 kW immersion heater is used to heat water .Calculate the electrical energy converted into heat energy in 40 minutes (ANS: $E.E = 7.2 \text{ MJ}$)
60. A current of 2 A is passed through a resistor of 20 ohms for 1.0 hour .Calculate the electrical energy converted into heat energy in the resistor (ANS: $E.E = 2.88 \times 10^5 \text{ J}$)
61. Why does a bird safely perch on a high potential electric wire? (ANS: When a bird is perched on a single wire, its two feet are at the same electrical potential, so the electrons in the wires have no motivation to travel through the bird's body)
62. A torch bulb is labeled 2.5 V, 0.3 A. Calculate the power of the bulb(ANS: $P = 0.75 \text{ W}$)
63. What is the terminal p.d for a cell of emf 2 V and internal resistance 1 ohm when it is connected to a 9 ohm resistor?
64. (a) Explain the terms Live, Neutral and Earth as applied in domestic electrical appliances
- (b) What are the color codes used at present in domestic electrical appliances

- (c) An electric stove is rated 1000 W, 250 V. Electricity is charged at shs. 45/= per kilowatt – hour, and the stove is used for 30 minutes per day
- (i) How much will the cost be in the month of January?
- (ii) What is the maximum current that flows through the element without destroying it?
65. When two resistors are connected in series, the total resistance is 25 ohm .If they are connected in parallel ,the total resistance is 6 ohm. Find the resistance of each
(ANS: 15 ohm and 10 ohm)
66. The emf of a cell is 12 V and its internal resistance is 2 ohm .Find the current and the terminal potential difference across the cell , if it is connected to 4 ohm external resistor
67. Briefly, explain what the fuse is
68. Select the best fuse for the following
- (a) a refrigerator rated 250 V, 400 W **(ANS: I = 1.6A, the best is 2 A)**
- (b) the electric cooker rated 240 V, 7.2kW **(ANS: I = 30 A, the best is 30A)**
- (c) the electric iron rated 240V, 2 kW **(ANS: I = 8.3A, the best is 10 A)**
69. The ratings of a bulb is 60 W, 240 V. Due to a power outage, the voltage drops down to 200 V .Find the new power of the bulb .What would you notice in the bulb?
70. The ratings of an iron is 1200 W, 240 V. Find the current and the energy used up in an hour
71. In the two circuits (a) and (b) shown in the figure below which bulb, A or B is brighter? **(ANS: From $P = \frac{V^2}{R} = I^2 R$, in parallel V is the same while in series I is the same. This gives that in fig (a) Bulb B has more power so it is more brighter but in fig (b) Bulb A has more power than B so it is more brighter than B)**

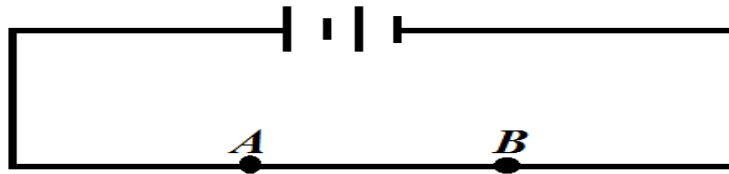


72. In the circuit shown below ,each bulb is rated at ' 6 V, 3 W'



- (a) Calculate the current through each bulb, when the bulbs are working normally
- (b) How many coulombs of charge pass in 6 seconds through each bulb?
- (c) What would the ammeter read when all the bulbs are working normally
- (d) Calculate the electrical power delivered by the battery

73. Two cells, each of 1.5 V are used to drive a current through a wire AB of resistance 90 ohms (see the figure below)



- (a) Calculate the current in the circuit
 (b) What would be the difference, if any, to the current, if the two cells are connected in parallel.
74. A carbon resistor has a value of 20 mega ohms $\pm 5\%$.What is the colour code for this resistor?
75. A carbon resistor's ABC bands represent yellow, blue and brown colors respectively. Determine the (a) resistance (b) Conductance
76. The p.d across the terminals of a cell is a 1.5 V where there is no current in the cell .Where is a current of 0.50 A in the circuit the p.d falls to 1.3 V
 (a) What is the e.m.f of the cell?
 (b) What is the terminal voltage at the cell?
 (c) Calculate the internal resistance of the cell
77. Explain the term internal resistance of a cell .How does it arise?
78. The e.m.f of a cell is given by the expression $E = I(R + r)$. Explain the meaning of each term in the expression

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