MARKING GUIDE
PRENECTA 2024.

1 (a) $y = -0.137$ (03 marks)

(b) $\Delta = 9.01 \times 10^{09}$ (03 marks)

(c)(i) $|A| = -1.3$ (02 marks)

$$A^{-1} = \begin{bmatrix} 27.3 & -1.0 & -0.3 \\ -39.9 & 1.7 & 0.6 \\ 19.6 & -0.7 & -0.3 \end{bmatrix}$$ (02 marks)

2 (a) $x + 1 = 0$

$x = -1$

$ax^3 + bx^2 - 13x + 6 = 18$

$a(-1)^3 + b(-1)^2 - 13(-1) + 6 = 18$

$-a + b + 13 + 6 = 18$

$-a + b = 18 - 19$

$-a + b = -1$

Also $2x - 1 = 0$

$x = 1/2$ (01 marks)

$ax^3 + bx^2 - 13x + 6 =$

$a\left(\frac{1}{2}\right)^3 + b\left(\frac{1}{2}\right)^2 - 13\left(\frac{1}{2}\right) + 6 =$

$\frac{a}{8} + \frac{b}{4} - \frac{13}{2} + 6 = 0$

$a + 2b - 52 + 48 = 0$ (01 marks)

$a + 2b = 4$ ——— (ii)

$$\begin{cases} -a + b = -1 \\ a + 2b = 4 \end{cases}$$

$a = 2$

$b = 1$ (01 marks)

2 (b)

$$x - y \overline{)\, x^3 + 0x^2y + 0xy^2 - y^3 \;\big|\; x^2 + xy + y^2}$$

$-x^3 - x^2y$

$x^2y + 0xy^2$ (01 marks)

$-x^2y - xy^2$

$xy^2 - y^3$

$xy^2 - y^3$

$- \quad - \quad - \quad -$

$\frac{P(x)}{Q(x)} = x^2 + xy + y^2$ (01 mark

2(c) $f(x) = x^3 - 4x$

$y = x^3 - 4x$

$\frac{dy}{dx} = 3x^2 - 4$

$0 = 3x^2 - 4$

$x^2 = \frac{4}{3} \Rightarrow x = \pm\sqrt{\frac{4}{3}}$ (01 marks)

$x = 1.15$ or $x = -1.15$

$\frac{d^2y}{dx^2} = 6x$

At $x = 1.15$

$\frac{d^2y}{dx^2} = +ve$ Minimum

At $x = -1.15$

$\frac{d^2y}{dx^2} = -ve$ maximum (01 marks)

$y = x^3 - 4x$

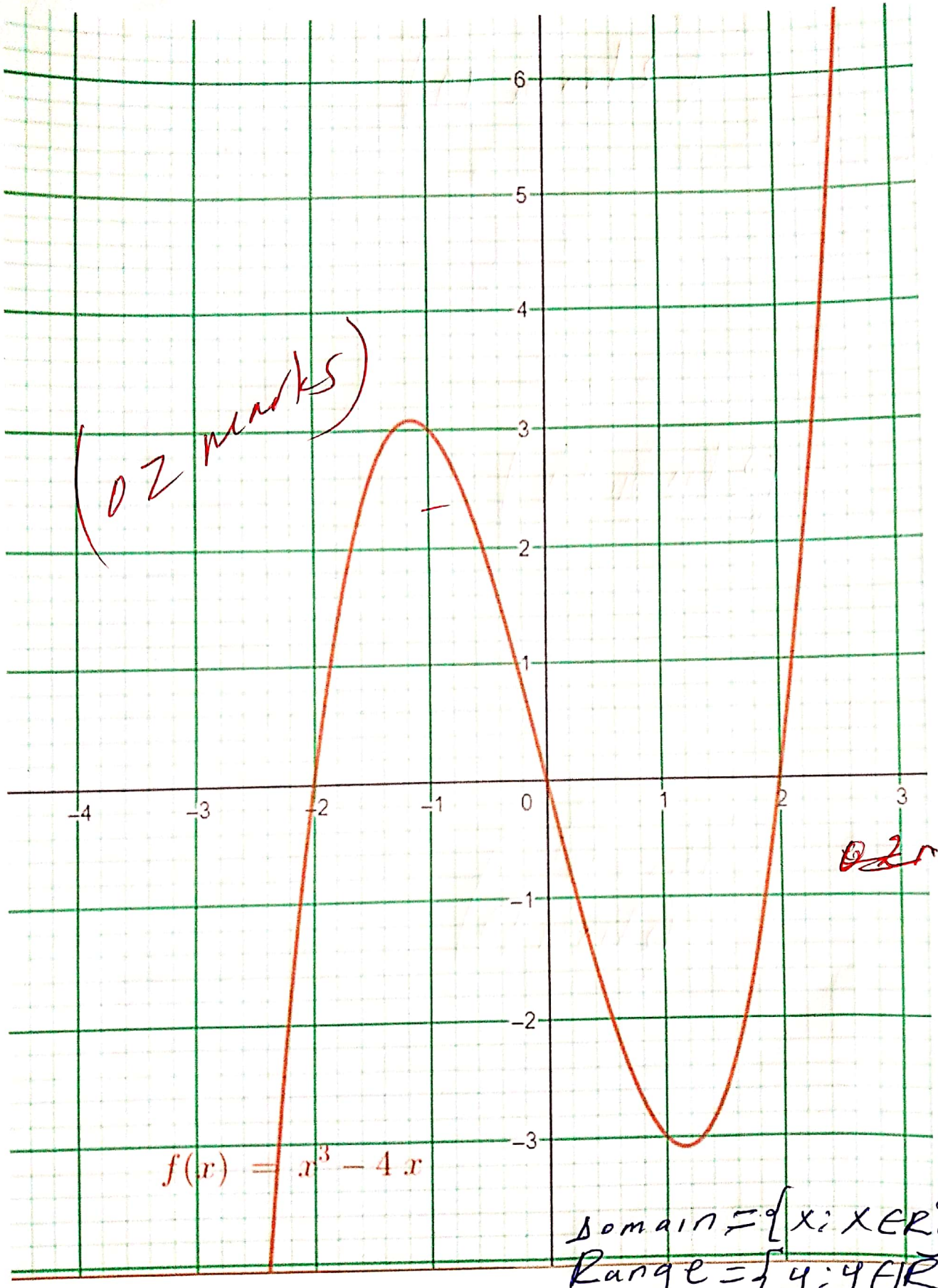
$y = (1.15)^3 - 4(1.15) = -3.079$

Minimum point $(1.15, -3.1)$

$y = (-1.15)^3 - 4(-1.15) = 3.079$

≈ 3.1

Maximum point $(-1.15, 3.1)$ (01 marks)



(02 marks)

Domain = {x : x ∈ R}

Range = {y : y ∈ R}

(01 marks)

x-axis intercept

$x^3 - 4x = 0$

$x(x^2 - 4) = 0$

$x = 0$ or $x = -2$, or $x = 2$

$x = 2$

3(a) $Q(x) = x^3 - 7x^2 + Bx - 8$

$x = \alpha_1$, $x = \alpha_2$, $x = \alpha_3$

$(x - \alpha_1)(x - \alpha_2)(x - \alpha_3) = 0$

$(x^2 - \alpha_2 x - \alpha_1 x + \alpha_1\alpha_2)(x - \alpha_3) = 0$

$x^3 - (\alpha_1 + \alpha_2)x^2 + \alpha_1\alpha_2 x - \alpha_3 x^2 + \alpha_2\alpha_3 x + \alpha_1\alpha_3 x - \alpha_1\alpha_2\alpha_3 = 0$ (01 marks)

$x^3 - (\alpha_1 + \alpha_2 + \alpha_3)x^2 + (\alpha_1\alpha_2 + \alpha_1\alpha_3 + \alpha_2\alpha_3)x - \alpha_1\alpha_2\alpha_3 = 0$

Compare

$\alpha_1 + \alpha_2 + \alpha_3 = 7$

$a + ar + ar^2 = 7$

$\alpha_1\alpha_2\alpha_3 = 8$

$a \cdot ar \cdot ar^2 = 8$

$(ar)^3 = 8$

$ar = 2$ (01 marks)

$G_1 + G_1 r + G_1 r \cdot r = 7$

$G_1 + 2 + 2r = 7$ (01 mark)

$G_1 + 2r = 5$

$G_1 r = 2$

$G_1 = \frac{2}{r}$

$\frac{2}{r} + 2r = 5$

$2 + 2r^2 = 5r$ (01 marks)

$2r^2 - 5r + 2 = 0.$

$r = 2$ or $r = \frac{1}{2}$

When $r = 2$

$G_1 = \frac{2}{2} = 1$

$G_2 = G_1 r = 2$

$G_3 = G_1 r^2 = 4$

When $r = \frac{1}{2}$

$G_1 = 4$

$G_2 = 2$ (01 marks)

$G_3 = 1$

$B = G_1 G_2 + G_1 G_3 + G_2 G_3$

$= 1 \times 2 + 2 \times 4 + 1 \times 4$

$= 2 + 8 + 4$

$B = 14$ (01 marks)

(iii) $y = x^3 - 7x^2 + 14x - 8$

$\frac{dy}{dx} = 3x^2 - 14x + 14$ (01 marks)

$3x^2 - 14x + 14 = 0$

$x = 3.22$ or $x = 1.45$. (01 marks)

The abscissa at the turning points are 1.45 and 3.22.

(b) $\frac{P_1 h_1 d_1}{J_1} = \frac{P_2 h_2 d_2}{J_2}$

~~$P_1 = 6, H_1 = 8, S.$~~

$P_1 = 6, h_1 = 8, d_1 = 10, J_1 = 1$

$P_2 = 2, h_2 = 12, d_2 = ?, J_2 = 1$ (01 mark)

$\frac{6 \times 8 \times 10}{1} = 2 \times 12 \times d_2$ (01 marks)

$480 = 24 d_2$

$d_2 = 20$ (01 marks)

It will take 20 days.

4(a) $f(x) = -x^3 + 3$

$f(x+h) = -(x+h)^3 + 3$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{-(x+h)^3 - (-x^3+3)}{h}$$

$$= \lim_{h \to 0} \frac{-(x+h)^3 + x^3 - 3}{h}$$ (01 mark)

$$= \lim_{h \to 0} \frac{-((x+h)(x^2+2xh+h^2)) + x^3 - 3}{h}$$

$$= \lim_{h \to 0} \frac{-(x^3+2x^2h+xh^2+x^2h+2xh^2+h^3) + x^3 - 3}{h}$$ (01 mark)

$$= \lim_{h \to 0} \frac{-x^3 - 3x^2h - xh^2 - h^3 + x^3 - 3 + 3}{h}$$

$$= \lim_{h \to 0} \frac{(-3x^2 - xh - h^2)h}{h}$$

$$= -3x^2 - 0.$$

$$\frac{dy}{dx} = -3x^2$$ (01 mark)

4 (b) $x = (t^2-1)^2$

$\frac{dx}{dt} = 2(t^2-1)\cdot 2t$ (01 mark)

$\frac{dx}{dt} = 4t(t^2-1)$

$\frac{dx}{dt} = 4t^3 - 4t$

$y = t^3$

$\frac{dy}{dt} = 3t^2$ (01 mark)

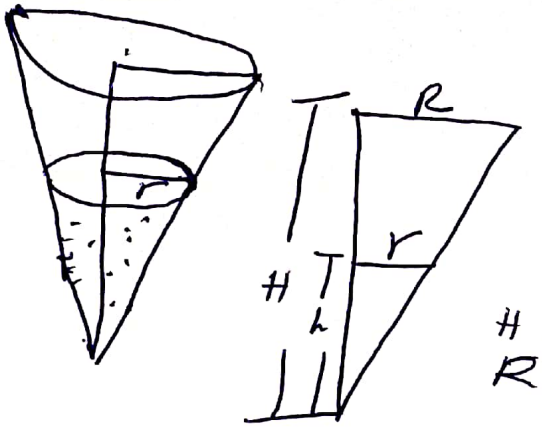
$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$

$= \frac{3t^2}{4t^3-4t}$

$\frac{dy}{dx} = \frac{3t}{4t^2-4}$ (01 mark)

4 (a) $\frac{dv}{dt} = 9\,m^3/min$

$\left.\frac{dh}{dt}\right|_{h=6M} = ?$



$H = 10M$
$R = 5$ (0/ mark)

$\frac{H}{h} = \frac{R}{r}$ | $r = \frac{5}{10}h$

$\frac{10}{h} = \frac{5}{r}$ | $r = \frac{h}{2}$.

$V = \frac{1}{3}\pi r^2 h$. (0/ mark)

$V = \frac{1}{3}\pi \left(\frac{h}{2}\right)^2 h$

$V = \frac{1}{12}\pi h^3$

$\frac{dv}{dh} = \frac{3}{12}\pi h^2$

$\frac{dv}{dh} = \frac{1}{4}\pi h^2$ (0/ mark)

$dh/dt = \frac{dh}{dt} \cdot \frac{dv}{dv}$

$= dh/dv \cdot \frac{dv}{dt}$

$= \frac{4}{\pi h^2} \cdot 9$

$\frac{dh}{dt} = \frac{36}{\pi \cdot 6^2} = \frac{1}{\pi}$ M/min (0/ mark)

Water level is rising at rate of $\frac{1}{\pi}$ M/min

$y = \frac{5x^3}{3} - \frac{4x^2}{2} + C$

$y = \frac{5}{3}x^3 - 2x^2 + C$ (01 mark)

At $x = 1$, $y = 2$

$2 = \frac{5}{3} - 2 + C$

$C = -\frac{5}{3} + 4 = \frac{7}{3}$

$f(x) = \frac{5}{3}x^3 - 2x^2 - \frac{7}{3}$

$f(x) = \frac{5}{3}x^3 - 2x^2 - \frac{7}{3}$ (01 mark)

5 (c)

x	-1	0	1	2	3	4	5
y	-6	0	0	0	6	24	60

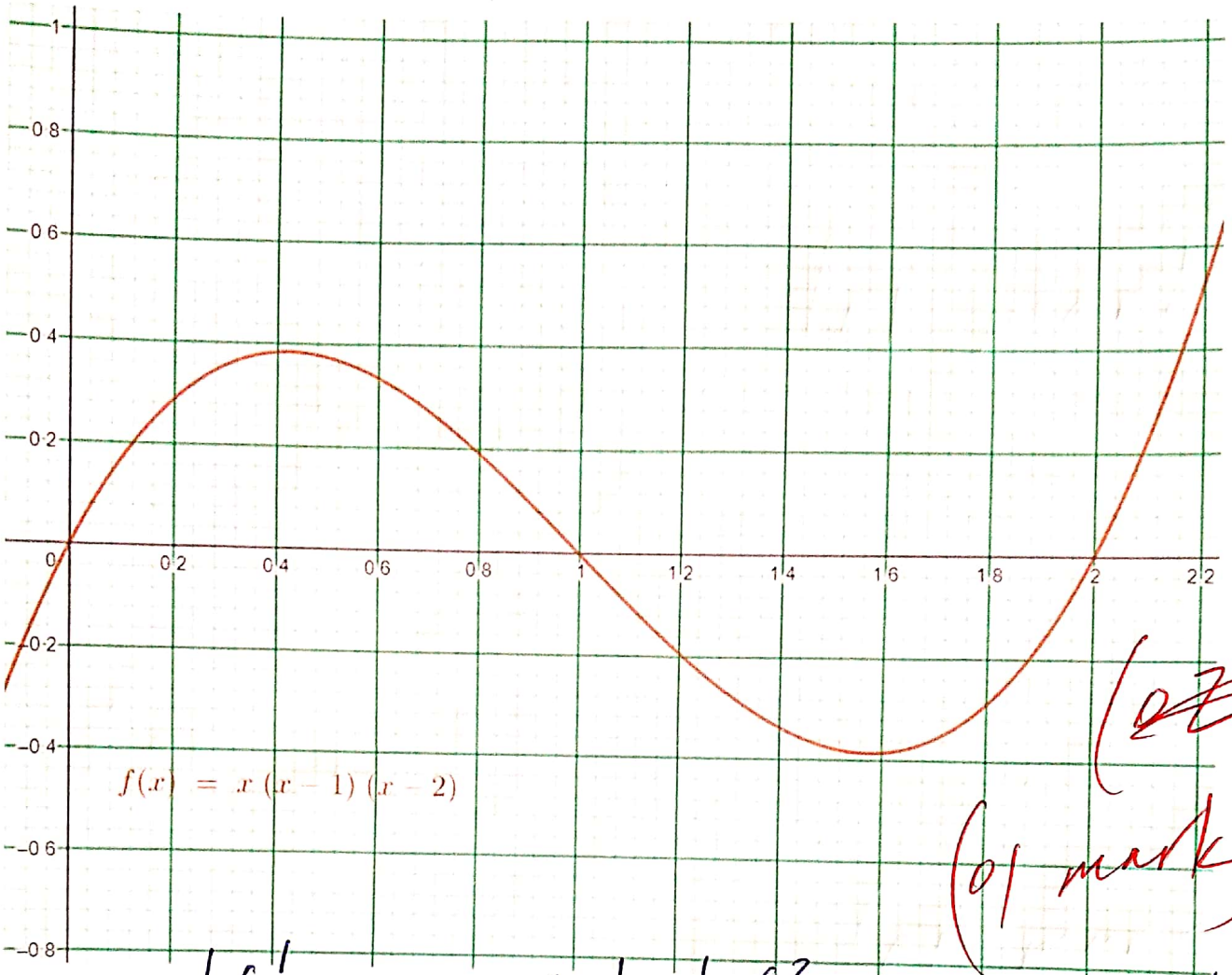
(01 mark)

$y = x(x-1)(x-2)$

$y = 0$

$x(x-1)(x-2) = 0$

$x = 0, x = 1, x = 2$



(0.2 mark)

(0.1 mark)

$$A = \left| \int_0^1 x(x-1)(x-2)\,dx \right| + \left| \int_1^2 x(x-1)(x-2)\,dx \right|$$

$$= \left| \int_0^1 (x^3 - 3x^2 + 2x)\,dx \right| + \left| \int_1^2 (x^3 - 3x^2 + 2x)\,dx \right|$$

$$= \frac{x^4}{4} - x^3 + x^2 \Big|_0^1 + \left| \frac{x^4}{4} - x^3 + x^2 \right|_1^2$$

(0.1 mark)

$$= 0.25 + |-0.25|$$

$$= 0.5$$

(0.1 mark)

Area is 0.5 square units.

x	f	d	u	u^2	fu	fu^2
1	2	-2	-2	4	-4	8
2	6	-1	-1	1	-6	6
3	5	0	0	0	0	0
4	4	1	1	1	4	4
5	3	2	2	4	6	12
	20					30

(05 marks)

$A = 3, \quad c = 1$

$$\text{Mean} = A + c\frac{\Sigma fu}{N}$$

$$= 3 + \frac{1(0)}{20}$$

$$= 3 + 0$$

$$\text{Mean} = 3$$

(02 marks)

$$\sigma^2 = c^2\left(\frac{1}{N}\Sigma fu^2 - \left(\frac{1}{N}\Sigma fu\right)^2\right)$$

$$= 1^2\left(\frac{30}{20} - 0\right)$$

$$= 3/2$$

$$\sigma = \sqrt{3/2} = 1.22$$

(03 marks)

7(a) UNIVERSAL

$$\text{Ways} = \frac{9!}{(7\times3)} = \frac{362880}{21} = 17{,}280$$

OR

Position	1st	2nd	3rd	4th	5th	6th	7th	8th	9th
chances	6	5	4	4	3	2	3	2	1

Ways $= 6\times5\times4\times4\times3\times2\times3\times2\times1$

$= 17{,}280$

There are 17,280 ways. (03 mark)

7(b) $n(A) = 32$, $n(B) = 28$, $n(A\cap B) = 15$

$P(A\cup B) = n(A) + n(B) - n(A\cap B)$

$= 32 + 28 - 15$

$n(A\cup B) = 45$ (01 mark)

$P(A) = \frac{32}{45}$, $P(B) = \frac{28}{45}$

$P(A\cap B) = \frac{15}{45} = \frac{1}{3}$

(i) $P(A/B) = \frac{P(A\cap B)}{P(B)}$

$= \frac{1/3}{28/45} = \frac{15}{28}$ (01 1/2 mark)

(ii) $P(B/A) = \frac{P(A \cap B)}{P(A)}$

$= \frac{1/3}{32/45}$

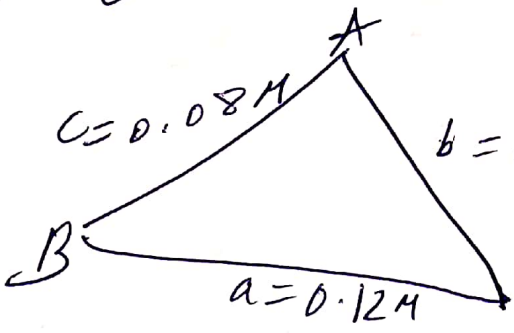
$= 15/32$

$P(B/A) = 15/32$ (01½ mark)

7 (c) Total selection $= {}^{9}C_4 \times {}^{4}C_3$

$= 504$ (03 mark)

8(a). Consider



$a^2 = b^2 + c^2 - 2bc\cos A$ (01 mark)

$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{0.06^2 + 0.08^2 - 0.12^2}{2 \times 0.06 \times 0.08}$

$\cos A = -0.4583$

$A = 117.28°$ (01 mark)

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin 117.28^\circ}{0.12} = \frac{\sin B}{0.06}$$

$$B = \sin^{-1}\left(\frac{0.06 \sin 117.28^\circ}{0.12}\right)$$ (01 mark)

$$B = 26.38$$

$$A + B + C = 180^\circ$$

$$117.28^\circ + 26.38^\circ + C = 180^\circ$$

$$C = 36.34^\circ$$

Largest angle − smallest angle

$117.28^\circ - 26.38^\circ = 90.9^\circ$ (01 mark)

8 (b) Consider L.H.S

$$\sec 2\theta + \tan 2\theta$$

$$= \frac{1}{\cos 2\theta} + \frac{\sin 2\theta}{\cos 2\theta}$$ (01 mark)

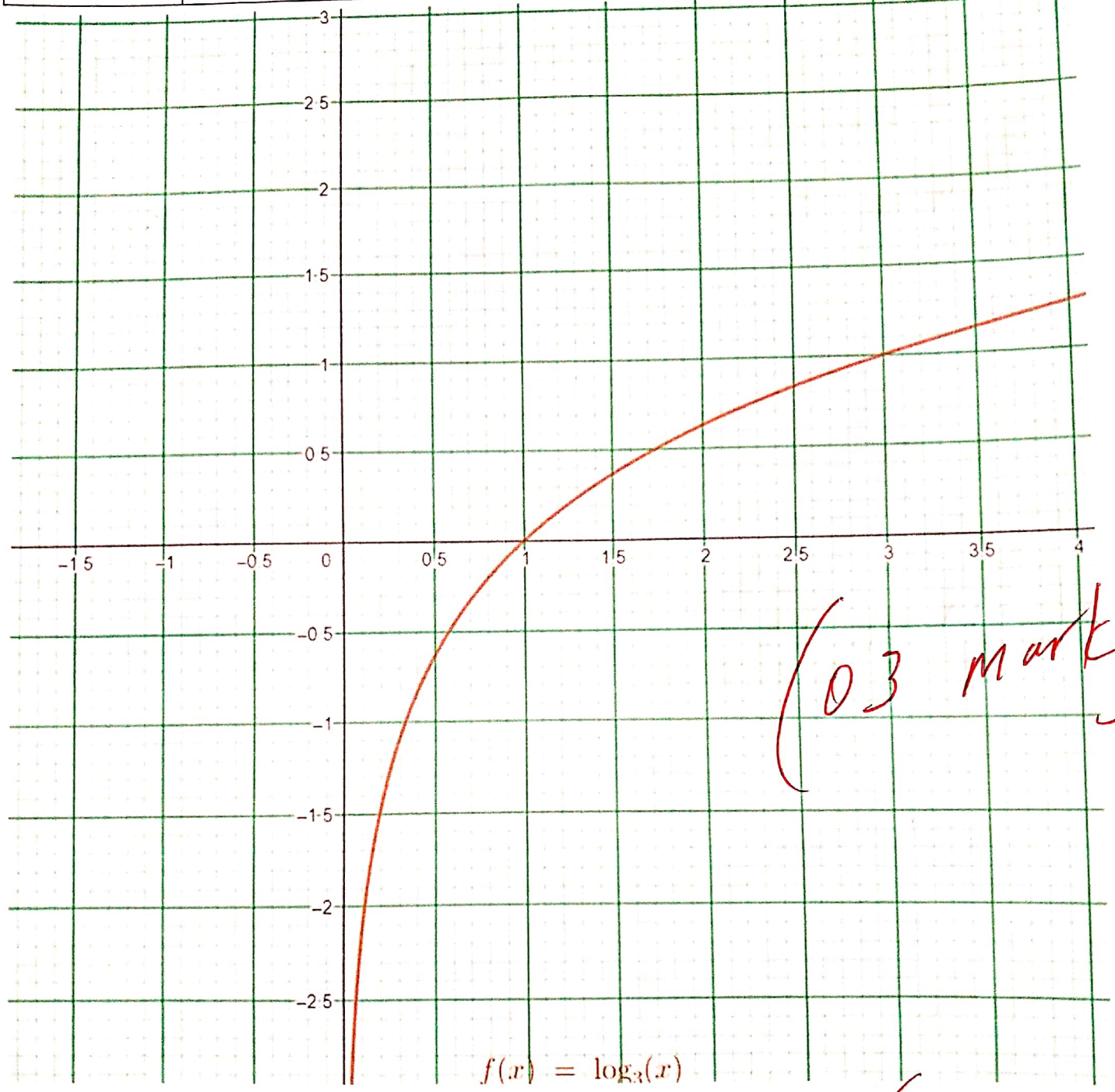
$$= \frac{1 + \sin 2\theta}{\cos 2\theta} = \frac{\cos^2\theta + \sin^2\theta + 2\cos\theta\sin\theta}{\cos^2\theta - \sin^2\theta}$$

$$= \frac{(\cos\theta + \sin\theta)^2}{(\cos\theta + \sin\theta)(\cos\theta - \sin\theta)}$$ (01 mark)

$$= \frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta}.$$ (01 mark)

9(b)

x	1	2	3	4	5
$\log_3 x$	0	0.6	1	1.3	1.5



(03 mark)

Domain $= \{x : x > 0\}$ (00½ mark)

Range $= \{y : y \in \mathbb{R}\}$. (00½ mark)

8 (c) $\cos 2\theta = \cos\theta$

$\cos^2\theta - \sin^2\theta = \cos\theta$

$\cos^2\theta - (1-\cos^2\theta) = \cos\theta$

$2\cos^2\theta - 1 = \cos\theta$

$2\cos^2\theta - \cos\theta - 1 = 0$ (01 mark)

$\cos\theta = \frac{1 \pm \sqrt{1+4\times 2\times 1}}{2\times 2}$

$\cos\theta = \frac{1 \pm 3}{4}$

$\cos\theta = 1$ or $\cos\theta = \frac{-1}{2}$ (01 mark)

When $\cos\theta = 1$

$\theta = \cos^{-1} 1$

$\theta = 0^\circ$

$\theta = 360^\circ - 0^\circ$

$\theta = \{0^\circ, 360^\circ\}$

When $\cos\theta = \frac{-1}{2}$

$\theta = \cos^{-1}\frac{1}{2}$

$\theta = 120^\circ$.

$\theta = 180^\circ + 60^\circ$

$\theta = 240^\circ$ (01 mark)

$\therefore \theta = \{0^\circ, 120^\circ, 240^\circ, 360^\circ\}$

9 (a) $7^{2x+5} = 3^{4x}$

$\ln 7^{2x+5} = \ln 3^{4x}$

$(2x+5)\ln 7 = 4x \ln 3$

$2x\ln 7 + 5\ln 7 = 4x \ln 3$ (01 mark)

$4x\ln 3 - 2x\ln 7 = 5\ln 7$

$x(4\ln 3 - 2\ln 7) = 5\ln 7$

$x = \frac{5\ln 7}{2\ln 9 - 2\ln 7}$

$= \frac{5\ln 7}{2(\ln 9 - \ln 7)}$ (01 mark)

$= \frac{5}{2}\frac{\ln 7}{\ln(9/7)}$

$x = \frac{5}{2}\frac{\ln 7}{\ln 9/7}$ (01 mark)

9(c) Given

$P_0 = 1000$, $P_T = ?$

$k = 10\% = 0.1$

$t = 7$

$P = P_0 e^{kt}$

$P = 1000 e^{0.1 \times 7}$

$P = 2013$

(03 mark)

The population after 7 years will be 2013

10@ $A = \begin{bmatrix} 1 & 2 & 4 \\ 6 & 1 & 0 \\ 4 & -1 & 3 \end{bmatrix}$

$A^{-1} = ?$

$|A| = 1\begin{vmatrix} 1 & 0 \\ -1 & 3 \end{vmatrix} - 2\begin{vmatrix} 6 & 0 \\ 4 & 3 \end{vmatrix} + 4\begin{vmatrix} 6 & 1 \\ 4 & -1 \end{vmatrix}$

$= 3 - 36 - 40$

$|A| = -73.$

(01 mark)

$$M = \begin{bmatrix} \begin{vmatrix} 1 & 0 \\ -1 & 3 \end{vmatrix} & \begin{vmatrix} 6 & 0 \\ 4 & 3 \end{vmatrix} & \begin{vmatrix} 6 & 1 \\ 4 & -1 \end{vmatrix} \\ \begin{vmatrix} 2 & 4 \\ -1 & 3 \end{vmatrix} & \begin{vmatrix} 1 & 4 \\ 4 & 3 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 4 & -4 \end{vmatrix} \\ \begin{vmatrix} 2 & 4 \\ 1 & 0 \end{vmatrix} & \begin{vmatrix} 1 & 4 \\ 6 & 0 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 6 & 1 \end{vmatrix} \end{bmatrix}$$

$$M = \begin{bmatrix} 3 & 18 & -10 \\ 10 & -13 & -9 \\ -4 & -24 & -11 \end{bmatrix}$$ (01 mark)

$$Cof(A) = \begin{bmatrix} 3 & -18 & -10 \\ -10 & -13 & 9 \\ -4 & 24 & -11 \end{bmatrix}$$

$$Adj(A) = \begin{bmatrix} 3 & -10 & -4 \\ -18 & -13 & 24 \\ -10 & 9 & -11 \end{bmatrix}$$ (01 mark)

$$A^{-1} = \frac{1}{-73} \begin{bmatrix} 3 & -10 & -4 \\ -18 & -13 & 24 \\ -10 & 9 & -11 \end{bmatrix}$$

$$AA^{-1} = \begin{pmatrix} 1 & 2 & 4 \\ 6 & 1 & 0 \\ 4 & -1 & 3 \end{pmatrix} \cdot \frac{-1}{73} \begin{pmatrix} 3 & 10 & -4 \\ -18 & -13 & 24 \\ -10 & 9 & -11 \end{pmatrix}$$

$$AA^{-1} = \frac{-1}{73} \begin{pmatrix} 1 & 2 & 4 \\ 6 & 1 & 0 \\ 4 & -1 & 3 \end{pmatrix} \begin{pmatrix} 3 & -10 & -4 \\ -18 & -13 & 24 \\ -10 & 9 & -11 \end{pmatrix}$$

$$AA^{-1} = \frac{-1}{73} \begin{pmatrix} -73 & 0 & 0 \\ 0 & -73 & 0 \\ 0 & 0 & -73 \end{pmatrix}$$

$$AA^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

0/mark

$$AA^{-1} = I$$

Hence shown

10 (b)

	Beans	Rice	Maize	Cost
x	20	20	20	1200/=
y	10	30	0	900/=
	340	540	280	

(00 1/2 mark)

Inequalities

$20x + 10y \leq 340$

$2x + y \leq 34$ (00 1/2 mark)

$20x + 30y \leq 540$

$2x + 3y \leq 54$ (00 1/2 mark)

$20x \leq 280$

$x \leq 14$ (00 1/2 mark)

$x \geq 0$

$y \geq 0$. (00 1/2 mark)

Objective function

$f = 1200x + 900y$ (00 1/2 mark)

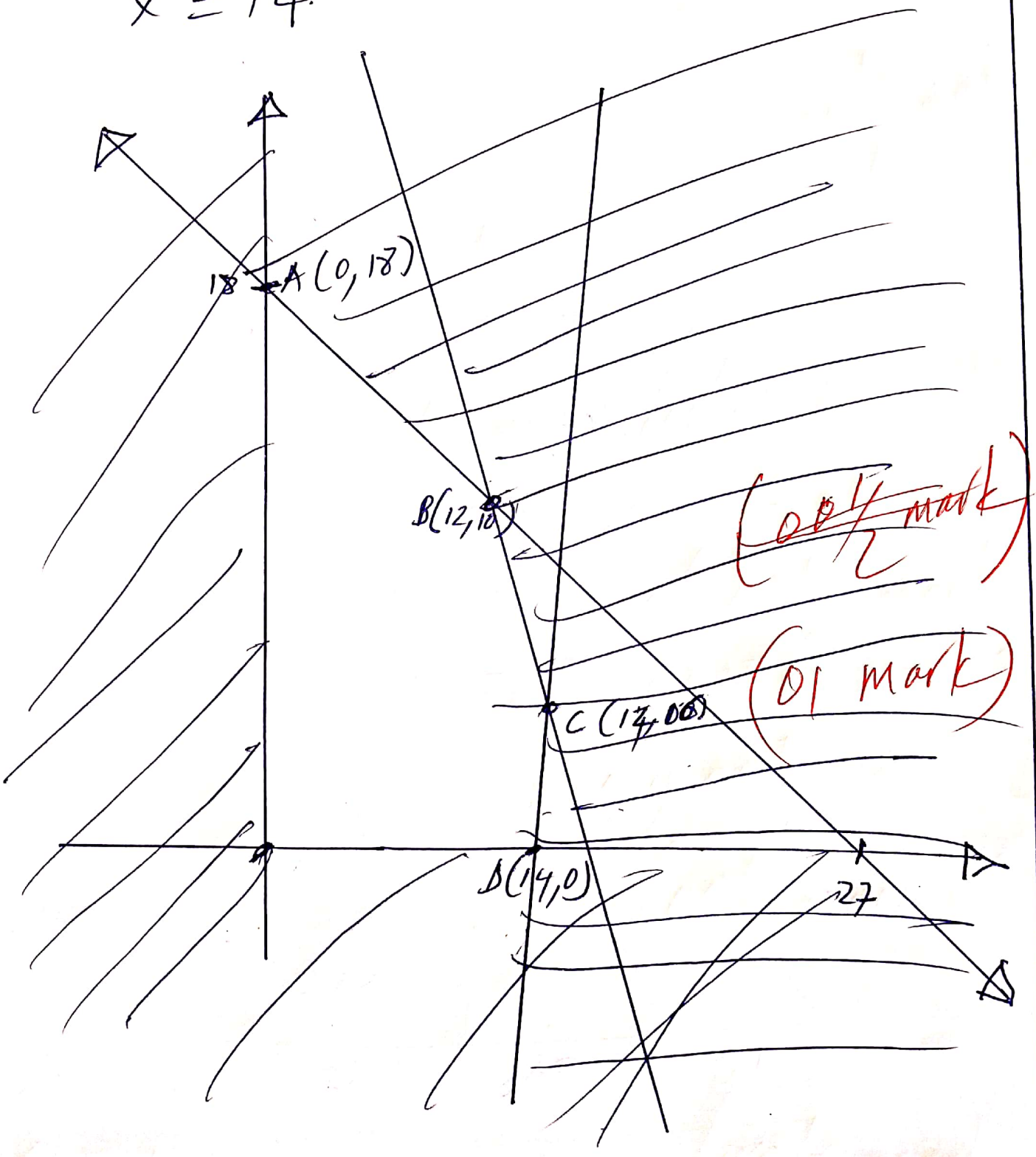
$2x + y = 34$

x	0	17
y	34	0

$2x + 3y = 54$

x	0	27
y	18	0

$x = 14$



(~~00 1/2 mark~~)

(01 mark)

POINTS	VALUE $f = 1200x + 900y$
A(0,18)	16,200/=
B(12,10	28,800/=
C(14,6)	22,200/=
D(14,0)	16,800/=

(00½ mar

- The retaile should supple 12 packetes of A and 10 packages of B to realize the maximum sale. (00½ mark