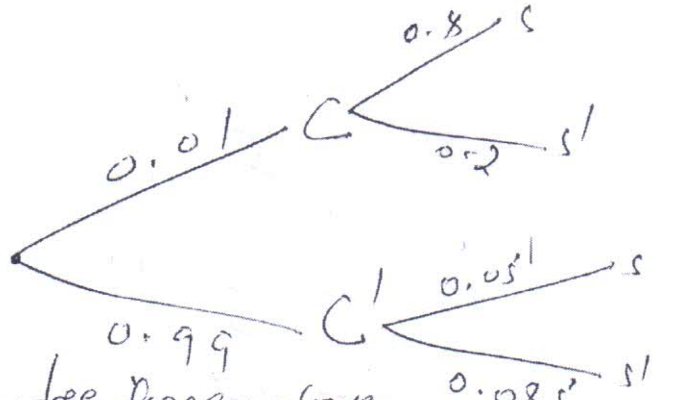
THE UNITED REPUBLIC OF TANZANIA
PRESIDENT'S OFFICE-RALG
ADVANCED MATHEMATICS 02
MARKING GUIDE

1. a) Let C be the event "the patient has a disease"

S be ~~the~~ "the screening test gives a +ve test"

<u>USING TREE DIAGRAM</u>

But $1\% = 0.01$



——— (01 mark)

from tree diagram above

(i) $P(C' \cap S) = 0.99 \times 0.05$

$= 0.0495$ ——— (02 marks)

(ii) $P(S) = P(C \cap S) + P(C' \cap S)$ ——— ($00\frac{1}{2}$ mark)

$= 0.01 \times 0.8 + 0.0495$

$= 0.0575$ ——— (02 marks)

iii/ $\dfrac{P(C \cap S)}{P(S)} = P(C/S)$

$P(C/S) = \dfrac{P(C \cap S)}{P(S)} = \dfrac{0.01 \times 0.8}{0.0575}$ — ($00\frac{1}{2}$ mark)

$= 0.139$ — (02 mark)

1. (b) Given that

$P(A') = \frac{1}{2}$, $P(B') = \frac{7}{10}$ and $P(A \cup B)' = \frac{2}{5}$

(i) $P(A \cap B') = P(B') - P(A' \cap B')$ — (00½ mark)

$= \frac{7}{10} - \frac{2}{5}$

$\therefore P(A \cap B') = \frac{3}{10}$ — (01 mark)

$= 0.3$

ii) $P(A \cap B) = P(A) - P(A \cap B')$ — (00½ mark)

$= \frac{1}{2} - \frac{3}{10} = \frac{1}{5}$

$\therefore P(A \cap B) = \frac{1}{5}$ — (01 mark)

$= 0.2$

c) Given

x	2	3	5	7	11
P(X)	0.17	0.33	0.25	x	Y

$E(X) = \frac{14}{3}$

value of $x = ?$, $y = ?$

from — (00½ mark)

$\Sigma P(x) = 1$ — (00½ mark)

$0.17 + 0.33 + 0.25 + X + Y = 1$

$X + Y = 0.25$ — ① -- (00½ mark)

From Expectation

$E(X) = \Sigma x\,P(x)$ — (00½ mark)

Pg 2

$E(x) = 2 \times 0.17 \times 3 \times 0.33 + 5 \times 0.25 + 7X + 11Y$ — (00½ mark)

$E(x) = 0.34 + 0.99 + 1.25 + 7X + 11Y$

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But

$E(x) = 14/3$

$\frac{14}{3} = 0.34 + 0.99 + 1.25 + 7x + 11y$

$7x + 11y = 2.087$ ——— (ii) ——— $00\frac{1}{2}$ mark

By solving eqn (i) and (ii)

$x = 0.166$ and $y = 0.084$ ——— 01 mark

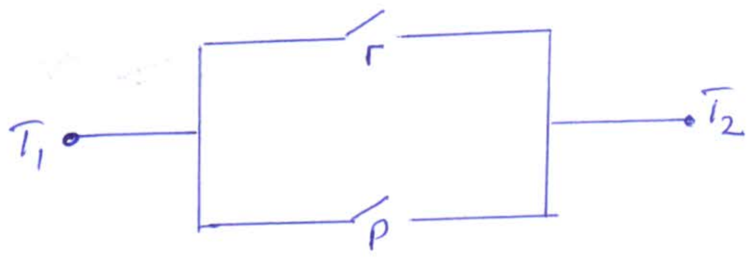
2(a) (i) Mauri is surrounded by water ——— $00\frac{1}{2}$

(ii) $r \vee [(\sim p \to q) \wedge p]$ ——— Given ——— 01

$\equiv r \vee [(p \vee q) \wedge p]$ ——— implication law

$\equiv r \vee p$ ——— Absorption law ——— 01

Simple electrical network



——— $00\frac{1}{2}$

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solution

2(b) Direct statement : $p \rightarrow (q \rightarrow \sim p)$

Contrapostive : $\sim(q \rightarrow \sim p) \rightarrow \sim p$ (01 mark)

Truth table of $\sim(q \rightarrow \sim p) \rightarrow \sim p$

p	q	$\sim p$	$q \rightarrow \sim p$	$\sim(q \rightarrow \sim p)$	$\sim(q \rightarrow \sim p) \rightarrow \sim p$
T	T	F	F	T	F
T	F	F	T	F	T
F	T	T	T	F	T
F	F	T	T	F	T
			(01) mark	(01) mark	(01) mark

(c) solution

$[(p \rightarrow \sim q) \wedge (r \rightarrow q)] \rightarrow (p \rightarrow \sim r)$ —— Given

$\equiv \sim[(\sim p \vee \sim q) \wedge (\sim r \vee q)] \vee (\sim p \vee \sim r)$ —— By definition $x \rightarrow y \equiv \sim x \vee y$ 00½

$\equiv \sim[\sim(p \wedge q) \wedge \sim(r \wedge \sim q)] \vee (\sim p \vee \sim r)$ —— Demorgan's law

$\equiv (p \wedge q) \vee (r \wedge \sim q) \vee (\sim p \vee \sim r)$ —— Demorgan's law 01

$\equiv (\sim p \vee (p \wedge q)) \vee (\sim r \vee (r \wedge \sim q))$ —— Associative law 00½

$\equiv ((\sim p \vee p) \wedge (\sim p \vee q)) \vee ((\sim r \vee r) \wedge (\sim r \vee \sim q))$ —— Distributive law

$\equiv (T \wedge (\sim p \vee q)) \vee (T \wedge (\sim r \vee \sim q))$ —— complement/negation law 01

$\equiv (\sim p \vee q) \vee (\sim r \vee \sim q)$ —— identity law

$\equiv (\sim p \vee \sim r) \vee (q \vee \sim q)$ —— Associative law

$\equiv (\sim p \vee \sim r) \vee T$ —— Identity law

$\equiv T$ —— identity law 01

Solution

2(d) Let $p \equiv$ I am illiterate

$q \equiv$ I can read

$r \equiv$ I can write ·00½

$p \rightarrow (\sim q \wedge \sim r)$, $(\sim q \wedge r)$, $\vdash \sim p$

$\therefore \left[(p \rightarrow (\sim q \wedge \sim r)) \wedge (\sim q \wedge r)\right] \longrightarrow \sim p$ 00½

						A	B	C	D	
p	q	r	$\sim p$	$\sim q$	$\sim r$	$\sim q \wedge \sim r$	$p \rightarrow A$	$\sim q \wedge r$	$B \wedge C$	$D \rightarrow \sim p$
T	T	T	F	F	F	F	F	F	F	T
T	T	F	F	F	T	F	F	F	F	T
T	F	T	F	T	F	F	F	T	F	T
T	F	F	F	T	T	T	T	F	F	T
F	T	T	T	F	F	F	T	F	F	T
F	T	F	T	F	T	F	T	F	F	T
F	F	T	T	T	F	F	T	T	T	T
F	F	F	T	T	T	T	T	F	F	T

00½ 00½ 00½ 00½ 00½

Since the final column contains all T,s, The proposition is a tautology.

Hence the argument is valid. 00½

Pg5

3 (a) Given

$$\underset{\sim}{u} = 4\underset{\sim}{i} + M\underset{\sim}{j} + 3\underset{\sim}{k}$$

$$\underset{\sim}{v} = -4\underset{\sim}{i} + 3\underset{\sim}{j}$$

$$\text{Proj}_{\underset{\sim}{u}}\underset{\sim}{v} = \frac{\underset{\sim}{u}\cdot\underset{\sim}{v}}{|\underset{\sim}{v}|} = 2/3$$ — (01) mark

$$\frac{(-4\underset{\sim}{i} + M\underset{\sim}{j} + 3\underset{\sim}{k})\cdot(-4\underset{\sim}{i} + 3\underset{\sim}{j})}{\sqrt{(-4)^2 + (3)^2}} = 2/3$$

$$\frac{16 + 3M + 0}{\sqrt{25}} = 2/3$$ — (01) mark

$$\frac{16+3m}{5} = 2/3$$

$$16 + 3m = \frac{10}{3}$$ — (01) mark

$$3m = 10/3 - 16$$

$$3m = \frac{10-48}{3}$$

$$3m = \frac{-38}{3}$$

$$M = -\frac{38}{9}$$ — ~~01~~ (01) mark

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$3M = \frac{10}{3} - 16$

$M = \frac{1}{3}\left(\frac{10}{3} - \frac{16}{1}\right) = \frac{1}{3}\left(\frac{10-48}{3}\right)$

$M = -\frac{38}{9}$.

$\therefore$ The value of M is $-\frac{38}{9}$ --- ~~(01 mark)~~

(b) Given $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are unit vectors

Then required to show that $\frac{1}{2}(\underset{\sim}{a}+\underset{\sim}{b}) = \cos\frac{\theta}{2}$

starting from:

$|\underset{\sim}{a}+\underset{\sim}{b}| = (\underset{\sim}{a}+\underset{\sim}{b})^2$.

$= |\underset{\sim}{a}|^2 + |\underset{\sim}{b}|^2 + 2|\underset{\sim}{a}||\underset{\sim}{b}|\cos\theta$ --- (01 mark)

since $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are unit vector

$|\underset{\sim}{a}| = |\underset{\sim}{b}| = 1$ --- (01 mark)

$|\underset{\sim}{a}+\underset{\sim}{b}|^2 = 2+2\cos\theta$

$|\underset{\sim}{a}+\underset{\sim}{b}|^2 = 2(1+\cos\theta)$

but

$1+\cos\theta = 2\cos^2\frac{\theta}{2}$ --- (01 mark)

$|\underset{\sim}{a}+\underset{\sim}{b}|^2 = 2\left(2\cos^2\frac{\theta}{2}\right)$

$|\underset{\sim}{a}+\underset{\sim}{b}|^2 = 4\cos^2\frac{\theta}{2}$ ——— (00½ mark)

Apply square root both sides

$\sqrt{|\underset{\sim}{a}+\underset{\sim}{b}|^2} = \sqrt{4\cos^2\frac{\theta}{2}}$

$\frac{|\underset{\sim}{a}+\underset{\sim}{b}|}{2} = \frac{2\cos\frac{\theta}{2}}{2}$

$\therefore \frac{1}{2}|\underset{\sim}{a}+\underset{\sim}{b}| = \cos\frac{\theta}{2}$ --- (01 mark)

Pg 7

c) i) Given $r(t) = (3\cos 2t)\underset{\sim}{i} + (3\sin 2t)\underset{\sim}{j}$

$r'(t)$ = velocity

$r'(t) = (-6\sin 2t)\underset{\sim}{i} + (6\cos 2t)\underset{\sim}{j}$ —— $00\frac{1}{2}$

Required to show that $r'(t) \cdot \overline{OP} = 0$

(The velocity of a particle is at right angle)

$$\begin{pmatrix} -(6\sin 2t) \\ 6\cos 2t \end{pmatrix} \cdot \begin{pmatrix} 3\cos 2t \\ 3\sin 2t \end{pmatrix} \text{ - - - - } (00\tfrac{1}{2}\text{mark})$$

$$-18\sin 2t\cos 2t + 18\cos 2t\sin 2t = 0$$

$$0 = 0 \text{ —— } (00\tfrac{1}{2}\text{mark})$$

$\therefore$ Since $r'(t) \cdot \overrightarrow{OP} = 0$. then the velocity of a particle is always at the right angle to $\overline{OP}$ —— (01mark)

ii) Speed $= |r'(t)| = \sqrt{(-6\sin 2t)^2 + (6\cos 2t)^2} = 6$ unit/sec - - - (01mark)

Speed = 6unit/sec - - - (01mark)

(iii) $r''(t) = (-12\cos 2t)i + (-12\sin t)\underset{\sim}{j}$.

at $t = 0$

$r''(t) = (-12\cos 0)\underset{\sim}{i} + (-12\sin 0)\underset{\sim}{j}$ - - - - (01mark)

$r''(0) = -12i$

$|\underset{\sim}{a}| = \sqrt{(-12)^2} = 12$ unit/sec square -- (01mark)

$\therefore$ The acceleration of a particle p at t=0 is 12unit/sec^2

Pg 8

4 @ Given $\frac{1+z^2}{1-z^2} = i$

$1+z^2 = i(1-z^2)$

$1+z^2 = i - z^2 i$

$z^2 + iz^2 = -1+i$

$\frac{z^2\cancel{(1+i)}}{\cancel{1+i}} = \frac{-1+i}{1+i}$ (0.1 mark)

$z^2 = \frac{-1+i}{1+i}$

$= \frac{-1+i}{1+i} \times \frac{(1-i)}{(1-i)}$

$= \frac{-1+2i+1}{1+1}$

$= \frac{2i}{2}$

$z^2 = i$ (0.1 mark)

Let

$z = x+iy$

$(x+iy)^2 = i$

$x^2 + 2ixy - y^2 = i$

compare two complex numbers'

Pg 9

4

(a) $x^2 - y^2 = 0$

$2ixy = i$

$x^2 - y^2 = 0 \Rightarrow x = y$

$2xy = 1 \Rightarrow 2x \cdot x = 1$ ——— (01 mark)

$= \frac{2x^2}{2} = \frac{1}{2}$

$\sqrt{x^2} = \sqrt{\frac{1}{2}}$

$x = \pm\frac{\sqrt{2}}{2}$ ——— (00½ mark)

since

$x = y$

$y = \pm\frac{\sqrt{2}}{2}$ ——— (00½ mark

$\therefore z = \sqrt{2}/2 + i\sqrt{2}/2$ or $-\sqrt{2}/2 - i\sqrt{2}/2$ ——— (01 mark)

(b) from

$2\cos\theta = z + \frac{1}{z}$ ——— (00½ mark)

$(2\cos\theta)^5 = \left(z + \frac{1}{z}\right)^5$ ——— (00½ mark

$32\cos^5\theta = z^5 + 5z^4\left(\frac{1}{z}\right) + 10z^3\left(\frac{1}{z^2}\right) + 10z^2\frac{1}{z^3}$

$+ 5z\left(\frac{1}{z^4}\right) + \frac{1}{z^5}$ ——— (00½ mark

$32\cos^5\theta = \left(z^5 + \frac{1}{z^5}\right) + 5\left(z^3 + \frac{1}{z^3}\right) + 10\left(z + \frac{1}{z}\right)$ — (00½ mark)

$32\cos^5\theta = 2\cos 5\theta + 5(2\cos 3\theta) + 10(2\cos\theta)$ — (00½ mark)

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4 (b) $32\cos^5\theta = 2\cos 5\theta + 10\cos 3\theta + 20\cos\theta$ — (00 ½ mark)

$\frac{2}{2}(16\cos^5\theta) = \frac{2}{2}(\cos 5\theta) + \frac{2}{2}(5\cos 3\theta) + \frac{2}{2}(10\cos\theta)$ — (00 ½ mark)

$16\cos^5\theta = \cos 5\theta + 5\cos 3\theta + 10\cos\theta$ — (00 ½ mark)

$\therefore 16\cos^5\theta = \cos 5\theta + 5\cos 3\theta + 10\cos\theta.$

Hence proved — (01 mark)

(c) Let $z = x+iy$ — (00 ½ mark)

$|x+iy-6i| = 2|x+iy-3|$ — (01 mark)

$|x+(y-6)i| = 2|(x-3)+iy|$

$x^2+(y-6)^2 = 4(x-3)^2+y^2$ — (01 mark)

$x^2+y^2-12y+36 = 4(x^2-6x+9+y^2)$

$3x^2+3y^2-24x+12y = 0.$

$x^2+y^2-8x+4y = 0.$ — (00 ½ mark)

$(x-4)^2-16+(y+2)^2-4 = 0$

$(x-4)+(y+2)^2 = 20$ — (01 mark)

Comparing with an equation of circle

Centre $=(4,-2)$ and $r=\sqrt{20}$ — (01 mark)

Pg 11

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5 a(i) $\frac{1+\cos x+\cos 2x}{\sin x+\sin 2x} = \cot x$

Considering L.H.S

$\frac{1+\cos x+\cos 2x}{\sin x+\sin 2x} = \frac{1+\cos x + 2\cos^2 x - 1}{\sin x + 2\sin x\cos x}$ —— (00 ½ mark

$= \frac{\cos x\,(1+2\cos x)}{\sin x\,(1+2\cos x)}$ —— (00 ½ mark

$= \frac{\cos x}{\sin x}$

$= \cot x$ —— (01 mark) Hence shown

ii/ $\cos^2(\alpha+\beta)+\cos^2(\alpha-\beta) = \cos 2\alpha\cos 2\beta + 1$

Considering L.H.S

$\cos^2(\alpha+\beta)+\cos^2(\alpha-\beta) = \frac{1}{2}\left(\cos 2(\alpha+\beta)+1\right) + \frac{1}{2}\left[\left(\cos\left(2(\alpha-\beta)\right)+1\right]$ —— (00½ mark

$= \frac{1}{2}\left[\cos(2\alpha+2\beta)+\cos(2\alpha-2\beta)+2\right]$ ------ (00½ mark).

$= \frac{1}{2}\left[2\cos 2\alpha\cos 2\beta + 2\right]$

$= \cos 2\alpha\cos 2\beta + 1$ —— (01 mark

Hence shown!

Pg 12

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5 (b) Given

$9\sin^2 x + 10\sin x\cos x - 2\cos^2 x = 1$

Dividing by $\cos^2 x$ through out gives

$9\tan^2 x + 10\tan x - 2 = \sec^2 x$

$9\tan^2 x + 10\tan x - 2 = 1 + \tan^2 x$ — (00$\frac{1}{2}$ mark

$8\tan^2 x + 10\tan x - 3 = 0$ — (01 mark

$\tan x = \frac{1}{4}$ or $\tan x = -\frac{3}{2}$ — (00$\frac{1}{2}$ mark

$x = 180°n + \tan^{-1}(\frac{1}{4})$ or $x = 180°n + \tan^{-1}(-\frac{3}{2})$ -- (01 mark

for $n = 0, \pm 1, 2, 3 - - - -$

$\therefore x = 180°n + 14.04°$ or $x = 180°n - 56.31°$ — (01 mark

(c) Given

$\tan^{-1}(2x+1) - \tan^{-1}(2x-1) = \tan^{-1}(\frac{1}{8})$

Apply tan both sides

$\tan\left(\tan^{-1}(2x+1) - \tan^{-1}(2x-1)\right) = \tan\left(\tan^{-1}\left(\frac{1}{8}\right)\right)$

$\dfrac{\tan(\tan^{-1}(2x+1)) - \tan(\tan^{-1}(2x-1))}{1 + \tan(\tan^{-1}(2x+1))\tan(\tan^{-1}(2x-1))} = \frac{1}{8}$ — (00$\frac{1}{2}$ mark — (00$\frac{1}{2}$ mark

$\dfrac{2x+1 - (2x-1)}{1 + (2x+1)(2x-1)} = \frac{1}{8}$ — 01 mark

Pg 13

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5 (c) $\frac{2}{1+4x^2-1} = \frac{1}{8}$

$\frac{2}{4x^2} = \frac{1}{8}$ —— (00 $\frac{1}{2}$ mark

$x^2 = 4$

$x = \pm 2$ —— (01 mark

$\therefore$ The value of x is 2 or -2 —— (00 $\frac{1}{2}$ mark

(d) Let $\cos 2\theta - \sin 2\theta = r\cos(2\theta + \alpha)$ —— (01 mark

$\cos 2\theta - \sin 2\theta = r\cos 2\theta \cos\alpha - r\sin 2\theta \sin\alpha$

Compare L.H.S and R.H.S'

$r\cos\alpha = 1$ —— (i)

$r\sin\alpha = 1$ —— (ii)

$\tan\alpha = 1$

$\alpha = \tan^{-1}(1) = \frac{\pi}{4}$ —— (00 $\frac{1}{2}$ mark

$r = \sqrt{1^2 + 1^2} = \sqrt{2}$

$\therefore \cos 2\theta - \sin 2\theta = \sqrt{2}\cos\left(2\theta + \frac{\pi}{4}\right)$ —— (01 mark'

Pg 14

Then'

$\sin 2\theta = \cos 2\theta - 1$

$\cos 2\theta - \sin 2\theta = 1$ —— (00 $\frac{1}{2}$ mark

5 (d) $\sqrt{2}\cos(2\theta+\pi/4) = 1$

$\cos(2\theta+\pi/4) = \frac{1}{\sqrt{2}}$ —— (00½ mark)

$2\theta+\pi/4 = 2\pi n \pm \cos^{-1}(1/\sqrt{2})$

$2\theta+\pi/4 = 2\pi n \pm \pi/4$ — (00½ mark

$2\theta+\pi/4 = 2\pi n+\pi/4$ or $2\theta+\pi/4 = 2\pi n-\pi/4$.

$\therefore\ \theta = \pi n$ or $\theta = \pi n-\pi/2$ — (01mark

where $n = 0, \pm1, \pm2, ---$

(e) Given

$\frac{1+\sin\theta}{5+3\tan\theta-4\cos\theta}$ where θ is very small

$\frac{1+\sin\theta}{5+3\tan\theta-4\cos\theta} = \frac{1+\theta}{5+3\theta-4(1-\theta^2/2)}$ — (00½ mark

$= \frac{1+\theta}{5+3\theta-4+2\theta^2}$ —— (00½ mark

$= \frac{1+\theta}{1+3\theta+2\theta^2}$

Pg 15

5 (e) $= \frac{1+a}{(1+a)(1+2a)}$ —— (01 mark)

$= \frac{1}{1+2a}$

$\therefore$ The expression involving a is $\frac{1}{1+2a}$ -- (01 mark)

6 (a)(i) Required to show $2b^2 = 9ac$

Given $ax^2+bx+c=0$

$\frac{ax^2}{a}+\frac{bx}{a}+\frac{c}{a}=\frac{0}{a}$

$x^2+\frac{bx}{a}+c/a=0$

sum of roots $= -b/a$ —— (01 mark)

product of roots $= c/a$

If one root is α the other is 2α

$\alpha+2\alpha = -b/a$ —— (00 ½ mark)

$3\alpha = -b/a$

$\alpha \times 2\alpha = 2\alpha^2$

$2\alpha^2 = c/a$ —— (i)

Pg16

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6 (a) (i) from

$$3\alpha = -b/a$$

$$\alpha = -\frac{b}{3a} \text{ ——— (ii)}$$ ——— 00½ mark

substituting eqn (ii) into eqn (i)

from $2\alpha^2 = c/a$

$$2\left(-\frac{b}{3a}\right)^2 = c/a$$

$$\frac{2b^2}{9a^2} = \frac{c}{a}$$

$$2ab^2 = 9a^2c$$

$$2b^2 = 9ac$$ ——— 01 mark

(ii) Solution

Let $P(x) = 2x^3 + ax^2 + 16x + 6$

$Q(x) = 2x + 1$

at $2x + 1 = 0 \rightarrow x = -1/2$ ——— 00½

then $P(-1/2) = 0$ since $(2x+1)$ is a factor of $P(x)$

$$\therefore 2(-1/2)^3 + a(-1/2)^2 + 16(-1/2) + 6 = 0$$

$$-1/4 + a/4 - 8 + 6 = 0$$

$$a/4 = 9/4$$

$$a = 9$$ ——— 01 mark

Pg 17

$$2x^3 + 9x^2 + 16x + 6 = (2x+1)\, f(x)$$

$$f(x) = \frac{2x^3 + 9x^2 + 16x + 6}{2x+1}$$

$$\begin{array}{r|l} & x^2 + 4x + 6 \\ \hline 2x+1 & 2x^3 + 9x^2 + 16x + 6 \\ & -(2x^3 + x^2) \\ \hline & \quad 8x^2 + 16x \\ & \quad -(8x^2 + 4x) \\ \hline & \qquad 12x + 6 \\ & \qquad 12x + 6 \\ \hline & \qquad\quad 0 \end{array}$$

00½

$$\therefore 2x^3 + 9x^2 + 16x + 6 = (2x+1)(x^2 + 4x + 6)$$

Hence the quadratic factor is $x^2 + 4x + 6$ 01

BS 18

6(b)(i) Solution

The general term of the expansion of $(ax+1/x)^n$

$$A_{r+1} = {}^nC_r(ax)^{n-r}\left(\frac{1}{x}\right)^r$$

$$= {}^nC_r a^{n-r} x^{n-r} x^{-r}$$

$$= {}^nC_r a^{n-r} x^{n-2r}$$ 00½

But $A_4 = 5/2$ the term is independent of x

$$A_4 = 5/2 x^0$$

from $A_{r+1} = {}^nC_r a^{n-r} x^{n-2r}$

at $r=3$

$$A_4 = {}^nC_3 a^{n-3} x^{n-6}$$

$$5/2 x^0 = {}^nC_3 a^{n-3} x^{n-6}$$ 00½

By comparing

$$x^0 = x^{n-6}$$

$$0 = n-6$$

$$\underline{n = 6}$$ 01 mark

and ${}^nC_3 a^{n-3} = 5/2$

Pg 19

$${}^6C_3 a^{6-3} = 5/2$$

$$20a^3 = 5/2$$ $\therefore a = 1/2,\ n = 6$

$$a^3 = 1/8$$

$$\underline{a = 1/2}$$ 01 mark

(b) (ii) Given

$\log_x 8 - \log_{x^2} 16 = 1$

$\frac{\log 8}{\log x} - \frac{\log 16}{\log x^2} = 1$ — (00½)

$\frac{\log 8}{\log x} - \frac{\log 4^2}{\log x^2} = 1$ — (00½)

$\frac{\log 8}{\log x} - \frac{2\log 4}{2\log x} = 1$

$\frac{\log 8}{\log x} - \frac{\log 4}{\log x} = 1$

$\frac{\log 8 - \log 4}{\log x} = 1$ — (00½)

$\frac{\log (8/4)}{\log x} = 1$

$\frac{\log 2}{\log x} = 1$ — (00½)

$\log_x 2 = 1$

$2 = x^1$

$x = 2$ — (01) mark

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6 (c) Let $a = 1/x$, $b = 1/y$ and $c = 1/z$

$$\begin{cases} 2a + 3b + 10z = 4 \\ 4x - 6b + 5z = 1 \\ 6a + 9b - 20z = 2 \end{cases}$$ (00 1/2 marks

$$a = \frac{\Delta x}{\Delta s} = \frac{\begin{vmatrix} 4 & 3 & 10 \\ 1 & -6 & 5 \\ 2 & 9 & -20 \end{vmatrix}}{\begin{vmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{vmatrix}} = 1/2$$

(00 1/2) mark

$a = 1/2$, but $a = 1/x$

$1/x = 1/2 \Rightarrow x = 2$. (00 1/2) mark

$$b = \frac{\begin{vmatrix} 2 & 4 & 10 \\ 4 & 1 & 5 \\ 6 & 2 & -20 \end{vmatrix}}{\begin{vmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{vmatrix}} = 1/3,$$

(00 1/2) mark

Pg 21

6 (c) $b = \frac{1}{y}$.

$\frac{1}{3} = \frac{1}{y} \Rightarrow y = 3$ (00 1/2 mark)

$$c = \frac{\begin{vmatrix} 2 & 3 & 4 \\ 4 & -6 & 1 \\ 6 & 9 & 2 \end{vmatrix}}{\begin{vmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{vmatrix}} = \frac{1}{5}$$ (00 1/2 mark)

$\Leftrightarrow c = \frac{1}{z}$

$\frac{1}{5} = \frac{1}{z} \Rightarrow z = 5$ (00 1/2 mark)

$$\therefore \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix}$$

Pg 22

6 (d) Required to express $\frac{16x}{x^4-16}$ as partial

$$\frac{16x}{x^4-16} = \frac{16}{(x^2-4)(x^2+4)}$$ —— (00 $\frac{1}{2}$ mark

$$= \frac{16}{(x-2)(x+2)(x^2+4)}$$

$$\frac{16x}{(x-2)(x+2)(x^2+4)} = \frac{A}{x-2} + \frac{B}{x+2} + \frac{Cx+D}{x^2+4}$$ —— (01 mark

$$16x = A(x+2)(x^2+4) + B(x-2)(x^2+4) + (Cx+D)(x^2-4)$$

for $x = -2$

$B = 1$ —— (00 $\frac{1}{2}$ mark)

for $x = 0$

$0 = 8A - 8B + -4D$

$D = 0$.

Pg 23

for $x = 1$

$16 = 15 - 5 - 3C$

$C = -\cancel{6}/3$ —— (00 $\frac{1}{2}$ mark)

$C = -2$

For $X = 2$

$$16(2) = A(4)(8) + B(0)(8) + (2C+D)(0)$$

$$32 = 32A$$

$$A = 1$$

$00\frac{1}{2}$ mark

from

$$\frac{16X}{(X-2)(X+2)(X^2+4)} = \frac{A}{X-2} + \frac{B}{X+2} + \frac{CX+D}{X^2+4}$$

$$= \frac{1}{X-2} + \frac{1}{X+2} - \frac{2X}{X^2+4}$$

$00\frac{1}{2}$ mark

$$\therefore \frac{16X}{X^4-16} = \frac{1}{X-2} + \frac{1}{X+2} - \frac{2X}{X^2+4}$$

01 mark

Pg 24

soln

7(a)(i) Given $x\frac{dy}{dx} - \frac{y^2}{x} = y$

$$\frac{dy}{dx} - \frac{y^2}{x^2} = \frac{y}{x}$$ 01 mark

$$\frac{dy}{dx} - \left(\frac{y}{x}\right)^2 = \frac{y}{x}$$

Let $u = y/x$ 01

$$\frac{dy}{dx} = u + x\frac{du}{dx}$$

$$u + x\frac{du}{dx} - u^2 = u$$ 01

$$x\frac{du}{dx} = u^2$$

$$\int \frac{du}{u^2} = \int \frac{dx}{x}$$

$$\frac{-1}{u} = \ln x + \ln c$$ 01

$$\frac{-1}{y/x} = \ln(cx)$$

$$\frac{-x}{y} = \ln(cx)$$

$$y = \frac{-x}{\ln(cx)}$$ 01

Pg 25

(ii) Given $y'' - 2y' - 3 = 0$

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = 3 \text{ ——————— } (00\tfrac{1}{2}\text{ mark})$$

By considering

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = 0. \text{ ——————— } (00\tfrac{1}{2}\text{ mark})$$

Let $y = e^{mx}$, $\frac{dy}{dx} = Me^{Mx}$, $\frac{d^2y}{dx^2} = M^2e^{Mx}$

$$M^2e^{mx} - 2Me^{Mx} = 0 \text{ ——————— } (01\text{ mark})$$

$$e^{Mx}(M - 2M) = 0$$

$$e^{Mx} \neq 0$$

$$M^2 - 2M = 0$$

$$M = 0,\ M = 2 \text{ ——————— } (01\text{ mark})$$

$$Y_c = Ae^{0x} + Be^{2x}$$

$$Y_c = A + Be^{2x} \text{ ——————— } (01\text{ mark})$$

For particular integral

Let $Y = kx$

$$\frac{dy}{dx} = k$$

$$\frac{d^2y}{dx^2} = 0$$

from. $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = 3$ ——————— $(00\tfrac{1}{2}\text{ mark})$

$$0 - 2k = 3$$

$$k = -\frac{3}{2}$$

7 (a)(ii) $Y = A + Be^{2X} - \frac{3}{2}X$ ———— (01/2 mark)

(b) Given $y = A\cos 2X + B\sin 2X$

$\frac{dy}{dX} = -2A\sin 2X + 2B\cos 2X$ ———— (01 mark)

$\frac{d^2y}{dX^2} = -4A\cos 2X - 4B\sin 2X$ ———— (01 mark)

$\frac{d^2y}{dX^2} = -4(A\cos 2X + B\sin 2X)$ ———— (01 mark)

but $y = A\cos 2X + B\sin 2X$

$\frac{d^2y}{dX^2} = -4y$ ———— (01 mark)

$\frac{d^2y}{dX^2} + 4y = 0$

$\therefore \frac{d^2y}{dX^2} + 4y = 0$ ———— (01 mark

Pg 27

7 (c) $\frac{dT}{dt} \propto (T - T_s)$ —— (01/2 mark)

$\frac{dT}{dt} = -k(T - T_s)$

$\frac{dT}{T - T_s} = -k\,dt$

$\int_{T_0}^{T} \frac{dT}{T - T_s} = \int_{t_0}^{t} -k\,dt'$ —— (01/2 mark)

$\left| \ln(T - T_s) \right|_{T_0}^{T} = kt$

$\ln\left(\frac{T - T_s}{T_0 - T_s}\right) = -kt$ —— (01 mark)

$T - T_s = (T_0 - T_s)e^{-kt}$ —— (01 mark)

But $T_s = 30°C$, $T_0 = 100°C$, $T(3) = 70°C$

$70 = 30 + (100 - 30)e^{-kt'}$ —— (01 mark

$K = -\frac{1}{3}\ln\frac{4}{7}$

Pg 28

$T(t) = 30 + 70e^{\frac{t}{3}\ln\frac{4}{7}}$, $t = ?$

$T(t) = 31$

7 ⓒ $\frac{1}{70} = e^{\frac{t}{3}\ln 4/7}$

$t = 22.78$

$\therefore t = 22.78$ Minutes ——— (01 mark)

8 ⓐ Given $y^2 = 4ax$ (p, q) as point of tangent

Required to show that $qy = 2a(x+p)$ is a tangent

from,

$y^2 = 4ax$

$2y\frac{dy}{dx} = 4a$

$\frac{dy}{dx} = \frac{4a}{2y}$

at point (p, q)

$\frac{dy}{dx} = \frac{4a}{2q}$ ——— (01mark

$\frac{dy}{dx} = \frac{2a}{q}$

$M = \frac{2a}{q}$

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~~8(c)~~

8(a)

$$y = m(x - x_1) + y_1$$

$$y = \frac{2a}{q}(x - p) + q$$ (00½) mark

$$qy = 2a(x - p) + q^2$$

$$qy = 2ax - 2ap + q^2$$

but (p, q) lie on $y^2 = 4ax$ (00½)

$$q^2 = 4ap$$ ~~(01/2) mark~~

$$qy = 2ax - 2ap + q^2$$

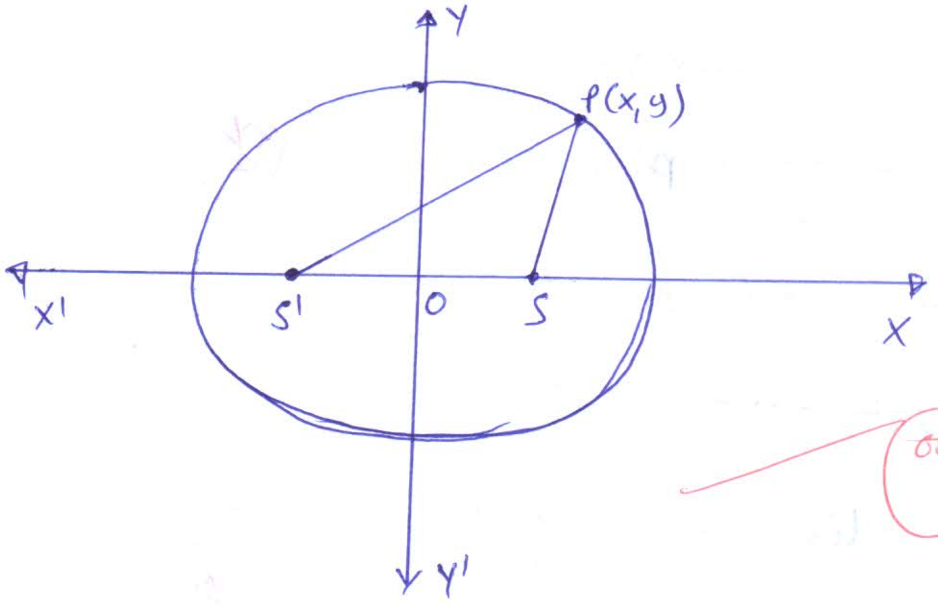
$$qy = 2ax - 2ap + 4ap$$

$$qy = 2ax + 4ap$$

$$qy = 2a(x + p)$$ (01) mark

8 (b)

Solution



00½

The path traced by a man is an ellipse having its foci at two flag posts

Let $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where $b^2 = a^2(1-e^2)$

$PS + PS' = 2a$

$10 = 2a$

$a = 5$ 00½

also $2ae = 8$

$5e = 4$

$e = 4/5$ 00½

but $b^2 = a^2(1-e^2)$

$b^2 = 5^2(1-(4/5)^2)$

$b^2 = 25\left(\frac{25-16}{25}\right)$ 00½

$b^2 = 9$

Pg 31

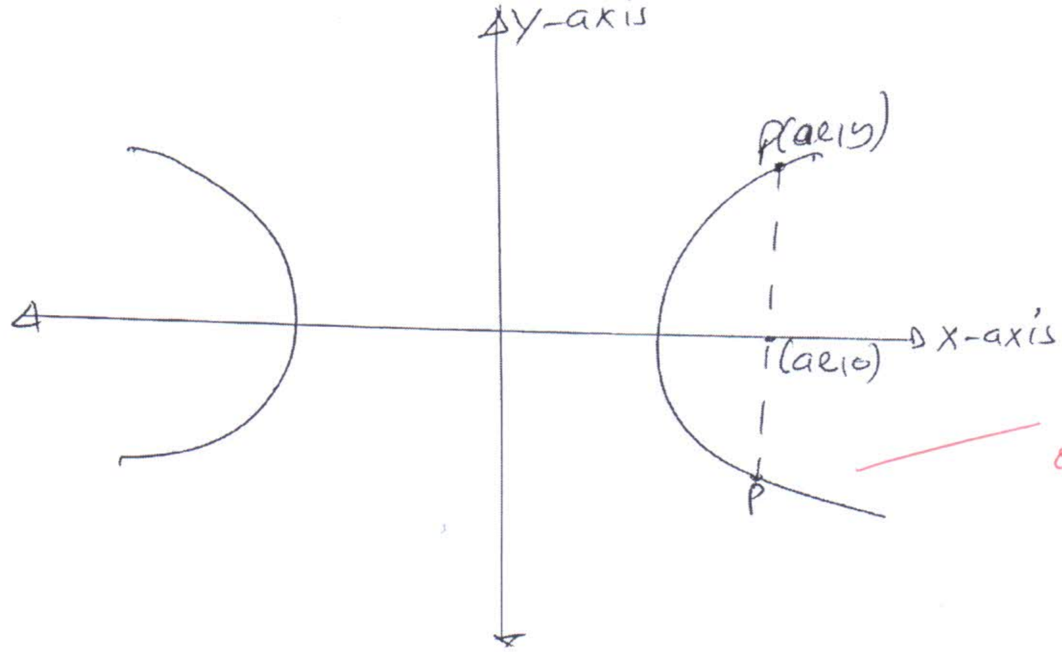
Hence the equation of the path is $\frac{x^2}{25} + \frac{y^2}{9} = 1$

01

8 (c)(i) Given $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Required length of latus rectum

consider the figure below.



00½

By substituting (ae, y) into the given equation

$$\frac{(ae)^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$e^2 - \frac{y^2}{b^2} = 1$$

01

$$\frac{y^2}{b^2} = e^2 - 1$$

but

$$e^2 - 1 = \frac{b^2}{a^2}$$

Pg 32

$$\frac{y^2}{b^2} = \frac{b^2}{a^2}$$

00½

(c) (i) $\frac{y}{b} = \frac{b}{a}$

$y = b^2/a$

$L = 2y$

$L = 2b^2/a$

$\therefore$ Length of latus rectum $(L) = \frac{2b^2}{a}$ — 01

(ii) Required to show that:

$c = \pm\sqrt{m^2a^2 - b^2}$ as condition of tangent of $y = mx + c$ to $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

from $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ — 01 mark

$\frac{x^2}{a^2} - \frac{(mx+c)^2}{b^2} = 1$

$b^2x^2 - a^2(mx+c)^2 = a^2b^2$

$b^2x^2 - a^2(m^2x^2 + 2mcx + c^2) = a^2b^2$

$b^2x^2 - a^2m^2x^2 - 2mca^2x - a^2c^2 = a^2b^2$

$(b^2 - a^2m^2)x^2 - 2mca^2x - (a^2c^2 + a^2b^2) = 0$

$(a^2m^2 - b^2)x^2 + 2mca^2x + (a^2c^2 + a^2b^2) = 0$ — 01 mark

For tangent:

$b^2 = 4ac$

8 | (c) (ii) $(2Mca^2)^2 = 4(a^2M^2 - b^2)(a^2c^2 + a^2b^2)$

$4M^2c^2a^4 = 4a^2(a^2M^2 - b^2)(c^2 + b^2)$

$M^2c^2a^2 = (a^2M^2 - b^2)(c^2 + b^2)$

$M^2c^2a^2 = (a^2M^2 - b^2)(c^2 + b^2)$ —— (01 mark

$M^2c^2a^2 = a^2M^2c^2 - b^2c^2 + a^2M^2b^2 - b^2b^2$

$a^2M^2b^2 - b^2c^2 - b^2b^2 = 0$

$(a^2M^2 - c^2 - b^2)b^2 = 0$

$a^2M^2 - c^2 - b^2 = 0$

$c^2 = a^2M^2 - b^2$

$\sqrt{c^2} = \sqrt{a^2M^2 - b^2}$

$\therefore c = \sqrt{a^2M^2 - b^2}$, Hence shown —— (01 mark

(d)(i) Given $r = \frac{4}{1+\cos\theta}$

Required to prove its cartesian equation represents a parabola

proof
~~needed to~~ change polar eqn into cartesian eqn

$r = \frac{4}{1+\cos\theta}$

$r = \frac{4}{1+x/r}$

Pg 34

00/2

8 d(i) $r = \frac{4r}{r+x}$

$r(r+x) = 4r$ 00 1/2

$r+x = 4$

but $r = \sqrt{x^2+y^2}$

$\sqrt{x^2+y^2} = 4-x$

$x^2+y^2 = (4-x)^2$ 01 mark

$x^2+y^2 = 16-8x+x^2$

$y^2 = 16-8x$

$y^2 = -8(x-2)$

$(y-0)^2 = -8(x-2)$

∴ The required eqn for translated parabola is $(y-0)^2 = -8(x-2)$. Hence proved — (01 mark

(ii) Given $r = 2+4\cos\theta$

θ	$0°$	$30°$	$60°$	$90°$	$120°$	$150°$	$180°$	$210°$	$240°$	$270°$	$300°$	$330°$	$360°$
r	6	5.5	4	2	0	-1.5	-2	-1.5	0	2	4	5.5	6

(01 mark

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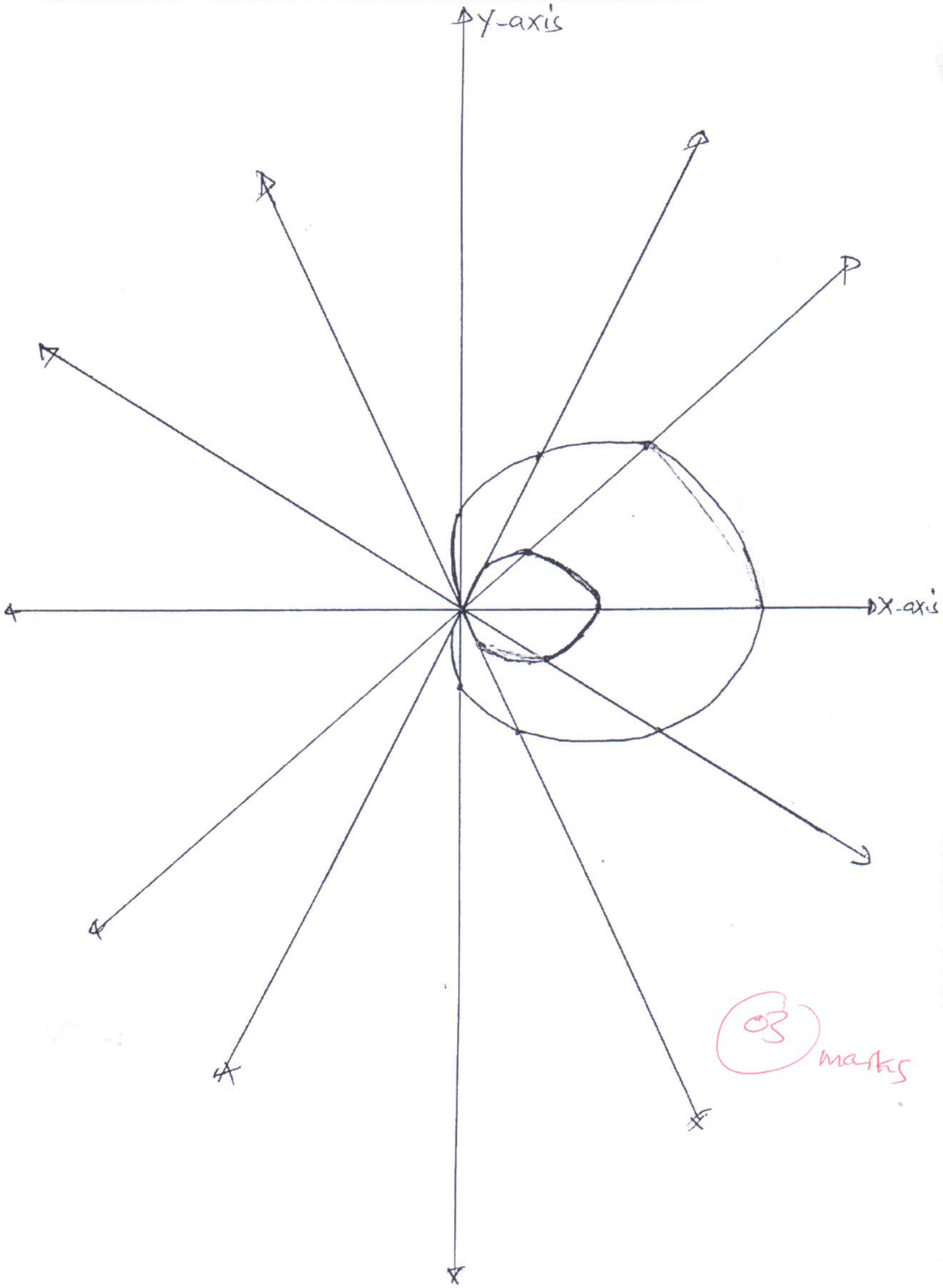

03 marks

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