PRE NECTA 2024 NJOMBE

ADVANCED MATHEMATICS 1

1. (a) -2.4646 $(2\ marks)$

(b) $\bar{x} = 9.5329$ $(2\ marks)$

$\sigma^2 = 28.6468$ $(2\ marks)$

(c) (i) 110100 $(2\ marks)$

(ii) 34 $(2\ marks)$

2 $(a)(i)$ $4\cosh^2 x = 3\sinh x + p$

$$4(1+\sinh^2 x) = 3\sinh x + p$$

$$4 + 4\sinh^2 x = 3\sinh x + p$$

$$4\sinh^2 x - 3\sinh x + 4 - p = 0$$

$$\sinh x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\sinh x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(4)(4-p)}}{2(4)} \qquad (1\ mark)$$

$$\sinh x = \frac{3 \pm \sqrt{9 - 16(4-p)}}{8}$$

For real solution

$$9 - 16(4-p) \geq 0$$

$$16p \geq 55$$

$$p \geq \frac{55}{16} \qquad (1\ mark)$$

2 $(a)(i)$ $If \quad p = 4$

$$\sinh x = \frac{3 \pm \sqrt{9 - 16(4 - p)}}{8}$$

$$\sinh x = \frac{3 \pm \sqrt{9 - 16(4 - 4)}}{8}$$

$$\sinh x = \frac{3 \pm \sqrt{9}}{8} \qquad (1 \quad mark)$$

$$\sinh x = \frac{3 \pm 3}{8}$$

$$\sinh x = 0 \quad or \quad \sinh x = \frac{6}{8} = \frac{3}{4}$$

$$x = \ln\left(0 + \sqrt{0^2 + 1}\right) or \qquad x = \ln\left(\frac{3}{4} + \sqrt{\frac{9}{16} + 1}\right)$$

$$x = \ln 1 \quad or \quad x = \ln 2 \qquad (1 \quad mark)$$

2 $(b)(i)$ $\sinh^{-1} 7 = k \sinh^{-1} 1$

$$\ln\left(7 + \sqrt{1 + 7^2}\right) = k \ln\left(1 + \sqrt{1 + 1^2}\right)$$

$$\ln\left(7 + \sqrt{1 + 49}\right) = k \ln\left(1 + \sqrt{1 + 1}\right)$$

$$\ln\left(7 + \sqrt{50}\right) = k \ln\left(1 + \sqrt{2}\right)$$

$$\ln\left(7 + \sqrt{50}\right) = \ln\left(1 + \sqrt{2}\right)^k \qquad (1 mark)$$

$$7 + \sqrt{50} = e^{\ln\left(1 + \sqrt{2}\right)^k}$$

$$7 + 5\sqrt{2} = \left(1 + \sqrt{2}\right)^k$$

$$\left(1 + \sqrt{2}\right)^3 = \left(1 + \sqrt{2}\right)^k$$

$$k = 3 \qquad (1 mark)$$

2. $(b)(ii)$ $y=\ln\left[\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right]$

$$e^{y}=\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)..............(i)$$

$$e^{-y}=\frac{1}{\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)}.............(ii)$$

$eqn(i)-eqn(i)$

$$e^{x}-e^{-y}=\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)-\frac{1}{\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)}$$

$$e^{x}-e^{-y}=\frac{\tan\frac{\pi}{4}+\tan\frac{x}{2}}{1-\tan\frac{\pi}{4}\tan\frac{x}{2}}-\frac{1}{\frac{\tan\frac{\pi}{4}+\tan\frac{x}{2}}{1-\tan\frac{\pi}{4}\tan\frac{x}{2}}}$$

$$e^{x}-e^{-y}=\frac{\tan\frac{\pi}{4}+\tan\frac{x}{2}}{1-\tan\frac{\pi}{4}\tan\frac{x}{2}}-\frac{1-\tan\frac{\pi}{4}\tan\frac{x}{2}}{\tan\frac{\pi}{4}+\tan\frac{x}{2}}$$ $(1mark)$

$$e^{x}-e^{-y}=\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}-\frac{1-\tan\frac{x}{2}}{1+\tan\frac{x}{2}}$$

$$e^{x}-e^{-y}=\frac{1+2\tan\frac{x}{2}+\tan^{2}\left(\frac{x}{2}\right)-\left(1-2\tan\frac{x}{2}+\tan^{2}\left(\frac{x}{2}\right)\right)}{\left(1-\tan\frac{x}{2}\right)\left(1+\tan\frac{x}{2}\right)}$$

$$e^{x}-e^{-y}=\frac{4\tan\frac{x}{2}}{1-\tan^{2}\frac{x}{2}}\Rightarrow\frac{e^{y}-e^{-y}}{2}=\frac{2\tan\frac{x}{2}}{1-\tan^{2}\frac{x}{2}}$$ $(1mark)$

$\sinh x=\tanh x$

2 (c) $y = \tanh^{-1}\left(3e^{2x}\right)$

$$\tanh y = 3e^{2x}$$

$$\sec h^2 y \frac{dy}{dx} = 6e^{2x}$$

$$\frac{dy}{dx} = \frac{6e^{2x}}{1 - \tanh^2 y} \qquad \left(\frac{1}{2} marks\right)$$

$$\frac{dy}{dx} = \frac{6e^{2x}}{1 - \left(3e^{2x}\right)^2}$$

$$\frac{dy}{dx} = \frac{6e^{2x}}{1 - 9e^{4x}}$$

$$\frac{dy}{dx} = \frac{3}{\frac{1}{2}e^{-2x} - \frac{1}{2}\left(9e^{2x}\right)} \qquad \left(\frac{1}{2} marks\right)$$

$$\frac{dy}{dx} = \frac{-3}{\frac{1}{2}\left(9e^{2x} - e^{2x}\right)}$$

$$\frac{dy}{dx} = \frac{-3}{\frac{1}{2}\left(9\cosh 2x + 9\sinh 2x - \cosh 2x + \sinh 2x\right)} \left(\frac{1}{2} marks\right)$$

$$\frac{dy}{dx} = \frac{-3}{5\cosh 2x + 4\sinh 2x} \quad hence \;\; shown \qquad \left(\frac{1}{2} marks\right)$$

3 (a) $f = ax + by + c$

$5550 = 4500a + 2500b + c$

$5450 = 4000a + 3000b + c$ $(1\ mark)$

$4400 = 500a + 3000b + c$

$$\begin{cases} 5550 = 4500a + 2500b + c \\ 5450 = 4000a + 3000b + c \\ 4400 = 500a + 3000b + c \end{cases}$$ $(1\ mark)$

$a = 0.3 \quad b = 0.1 \quad and \quad c = 3950$

$f(x,\ y) = \dfrac{3}{10}x + \dfrac{1}{10}y + 3950$ $(1\ mark)$

(b) $let \quad x\ be \quad no\ of \quad personal\ computer$

$y\ be \quad no\ of \quad portable\ \text{mod}el$

$f(x, y) = 4500x + 5000y$ $\left(00\dfrac{1}{2}mark\right)$

$x + y \leq 250$ $\left(00\dfrac{1}{2}mark\right)$

$25000x + 40000y \leq 7000000$

$25x + 40y \leq 7000$ $\left(00\dfrac{1}{2}mark\right)$

$x \geq 0$

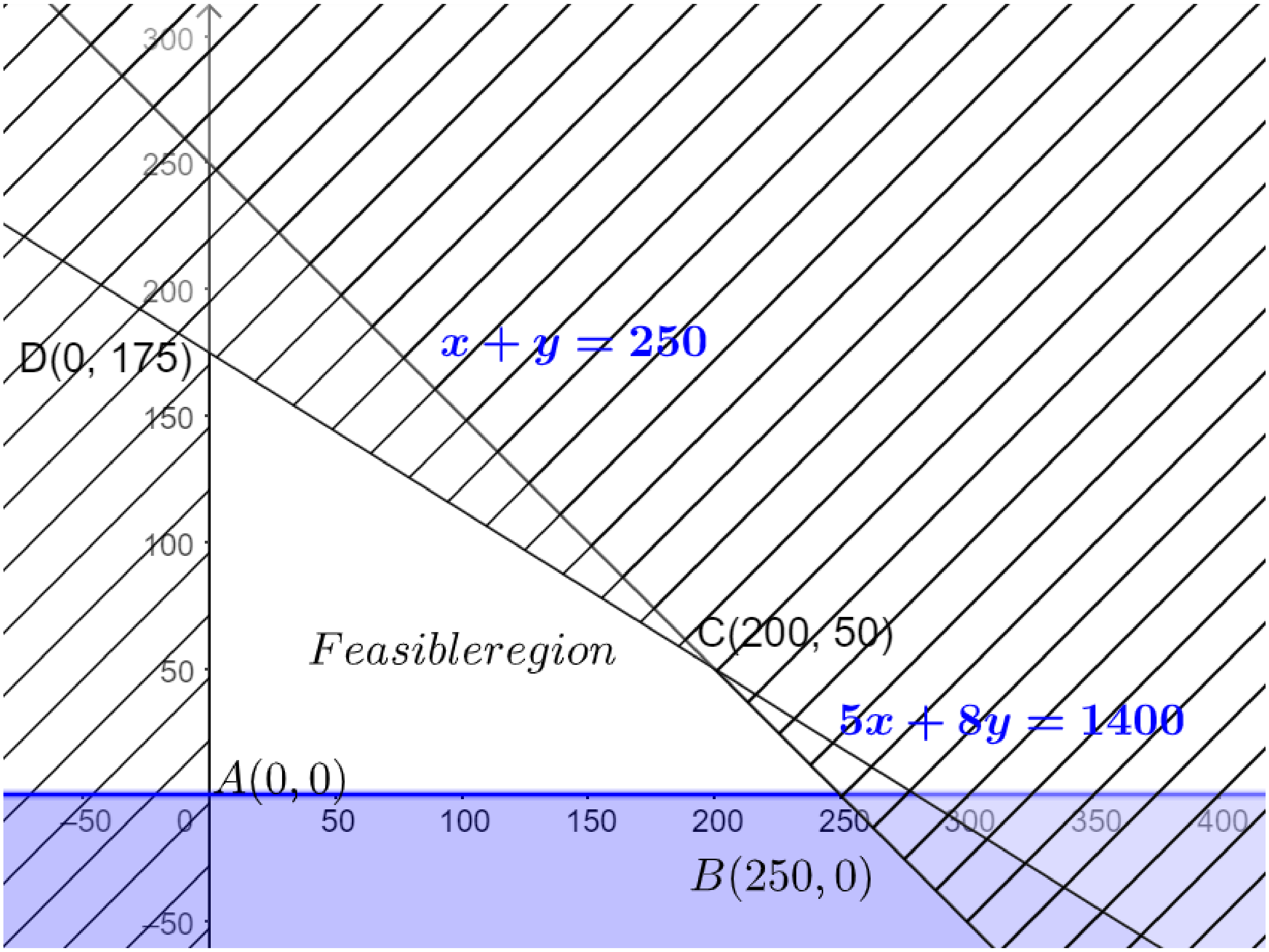
$y \geq 0$ $\left(00\dfrac{1}{2}mark\right)$

$x + y = 250$

x	0	250
y	250	0

$25x + 40y = 7000$

x	0	24
y	175	0



$(04\ marks)$

Corner point	$f(x,\ y) = 45000x + 5000y$
$(0,\ 0)$	0
$(250,\ 0)$	$11{,}250{,}000/=$
$(200,50)$	$9{,}250{,}000/=$
$(0,175)$	$875{,}000/=$

$(01mark)$

an $(1mark)$

4 (a) $n = 45$

$\bar{x} = 34\ kg$

$$\bar{x} = \frac{\sum x}{n}$$

$\sum x_1 = n\bar{x}$

$\sum x_1 = 45 \times 34 kg$

$\sum x_1 = 1575$ $(01 mark)$

When the teacher is added

$$\bar{x} = 45 + \frac{500}{1000}$$

$\bar{x} = 45 + 0.5 = 45.5 kg$ $(01 mark)$

$\sum x_2 = n\bar{x}$

$\sum x_2 = 45.5 \times 36$

$\sum x_2 = 1638$ $(01 mark)$

Weight of teacher x

$x = \sum x_2 - \sum x_1$

$x = 1638 - 1575$

$x = 63\ kg$ $(01 mark)$

4 (b) (i) $12+30+m+50+25+30+n=228$

$m+n+147=228$

$m+n=228-147$

$m+n=81.$ $(01 mark)$

$$median = L+\left(\frac{\frac{N}{2}-\sum f_b}{n_w}\right)i$$

$median = 47.5, \quad L=39.5, \quad N=228,$

$\sum f_b = 12+30+m$

$\sum f_b = 42+m, \quad f_w = 50$

$$47.5 = 39.5+\left(\frac{\frac{228}{2}-(42+m)}{50}\right)\times 10$$ $(01 mark)$

$$8=\left(\frac{114-42-m}{5}\right)$$

$40=72-m$

$m=32 \Rightarrow m+n=81 \Rightarrow 32+n=81 \Rightarrow n=49$ $(01 mark)$

x	f	fx	$x-49$	$\lvert x-49\rvert$	$f\lvert x-49\rvert$
14.5	12	174	-34.5	34.5	414
24.5	30	735	-24.5	24.5	735
34.5	32	1104	-14.5	14.5	464
44.5	50	2225	-4.5	4.5	225
54.5	25	1362.5	5.5	5.5	137.5
64.5	30	1935	15.5	15.5	465
74.5	49	3650.5	25.5	25.5	1249.5
$\sum$	228	11186	--	--	3690

$(02 \ marks)$

$$\bar{d} = \frac{\sum f\lvert x-\bar{x}\rvert}{N}$$

$$\bar{d} = \frac{3690}{228} = 16.18$$ $(01 mark)$

5 (a)

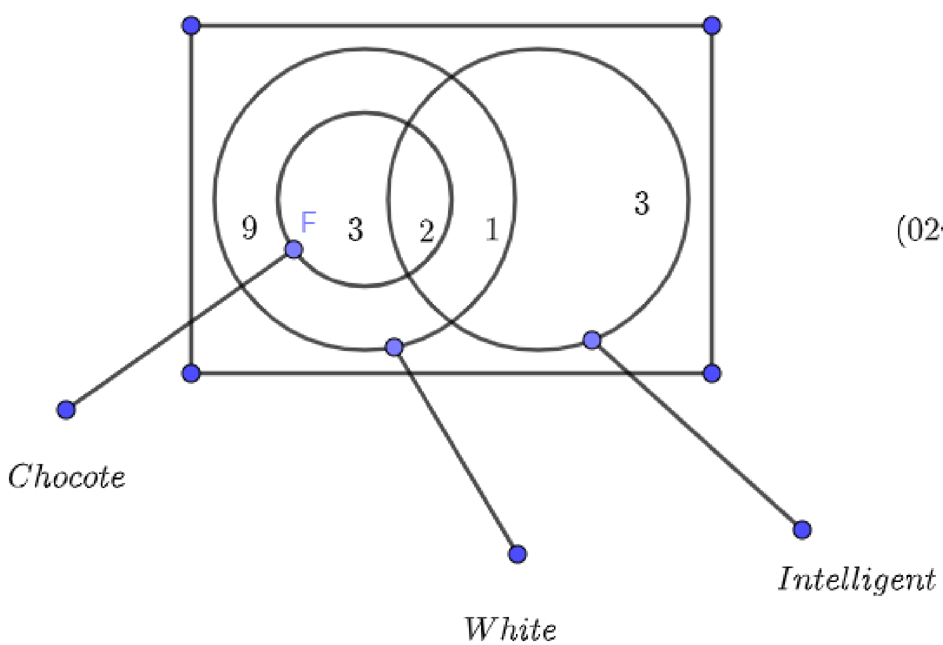
$$[A-(A\cap B')]\cap(A\cap B) \qquad \text{Given}$$
$$\equiv\left[A\cap(A\cap B')'\right]\cap(A\cap B) \qquad \text{Definition}$$
$$\equiv[A\cap(A'\cup B)]\cap(A\cap B) \qquad \text{De Morgan}$$
$$\equiv[(A\cap A')\cup(A\cap B)]\cap(A\cap B) \qquad \text{Distributive}$$
$$\equiv[\phi\cup(A\cap B)]\cap(A\cap B) \qquad \text{Complement}$$
$$\equiv(A\cap B)\cap(A\cap B) \qquad \text{Identity}$$
$$\equiv (A\cap B) \qquad \text{Idempotent} \qquad (03\ marks)$$

(b)

$$n(A)=n(B)=n(C) \qquad \text{Equivalent set}$$
$$n(A\cup B\cup C)=n(A)+n(B)+n(C)-n(A\cap B)-n(A\cap C)$$
$$-n(B\cap C)+n(A\cap B\cap C) \qquad (01 marks)$$
$$n(A\cup B\cup C)=n(A)+n(A)+n(A)-n(A\cap B)-n(B\cap C)+0+0 \qquad (01 marks)$$
$$n(A\cup B\cup C)=n(A)+n(A)+n(A)-n(A\cap B)-n(B\cap C)+0$$
$$n(A\cup B\cup C)=3n(A)-n(A\cap B)-n(B\cap C) \qquad (01 marks)$$

Hence shown

5 (c) (i)



$(02marks)$

(ii) $\quad No\ of\ girls\ in\ the\ class = 9+3+2+1+3$ $\quad (01\ mark)$

(iii) $\quad 3\ girls\ are\ white\ but\ not\ in-telligent$ $\quad (01\ mark)$

$6(a)$ $\quad f(x) = 2x^2 + x + 4$

$$fog(x) = 2x^2 - 8x + 10$$

$$fog(x) = f(g(x))$$

$$fog(x) = 2(g(x))^2 + g(x) + 4 \qquad (01mark)$$

$$2x^2 - 8x + 10 = 2(g(x))^2 + g(x) + 4$$

$$let\ \ g(x) = m$$

$$2x^2 - 8x + 10 = 2m^2 + m + 4$$

$$2m^2 + m + 4 - \left(2x^2 - 8x + 10\right) = 0$$

$$2m^2 + m + 4 - 2x^2 + 8x - 10 = 0 \qquad (01mark)$$

$$2m^2 + m - 2x^2 + 8x - 6 = 0$$

$$2m^2 + m - \left(2x^2 - 8x + 6\right) = 0$$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$m = \frac{-1 \pm \sqrt{1 - 4(2)\left(-2x^2 + 8x - 6\right)}}{2 \times 2}$$

$$m = \frac{-1 \pm \sqrt{1 - 8\left(-2x^2 + 8x - 6\right)}}{4}$$

$$m = \frac{-1 \pm \sqrt{1 + 16x^2 - 64x + 48}}{4} \qquad (01mark)$$

$$m = \frac{-1 \pm \sqrt{16x^2 - 64x + 49}}{4}$$

$$but\ m = g(x)$$

$$g(x) = \frac{-1 \pm \sqrt{16x^2 - 64x + 49}}{4} \qquad (01mark)$$

6 (b) $f(x)=\dfrac{2x}{x^2-x-2}$

$V.A$

$x^2-x-2=0$

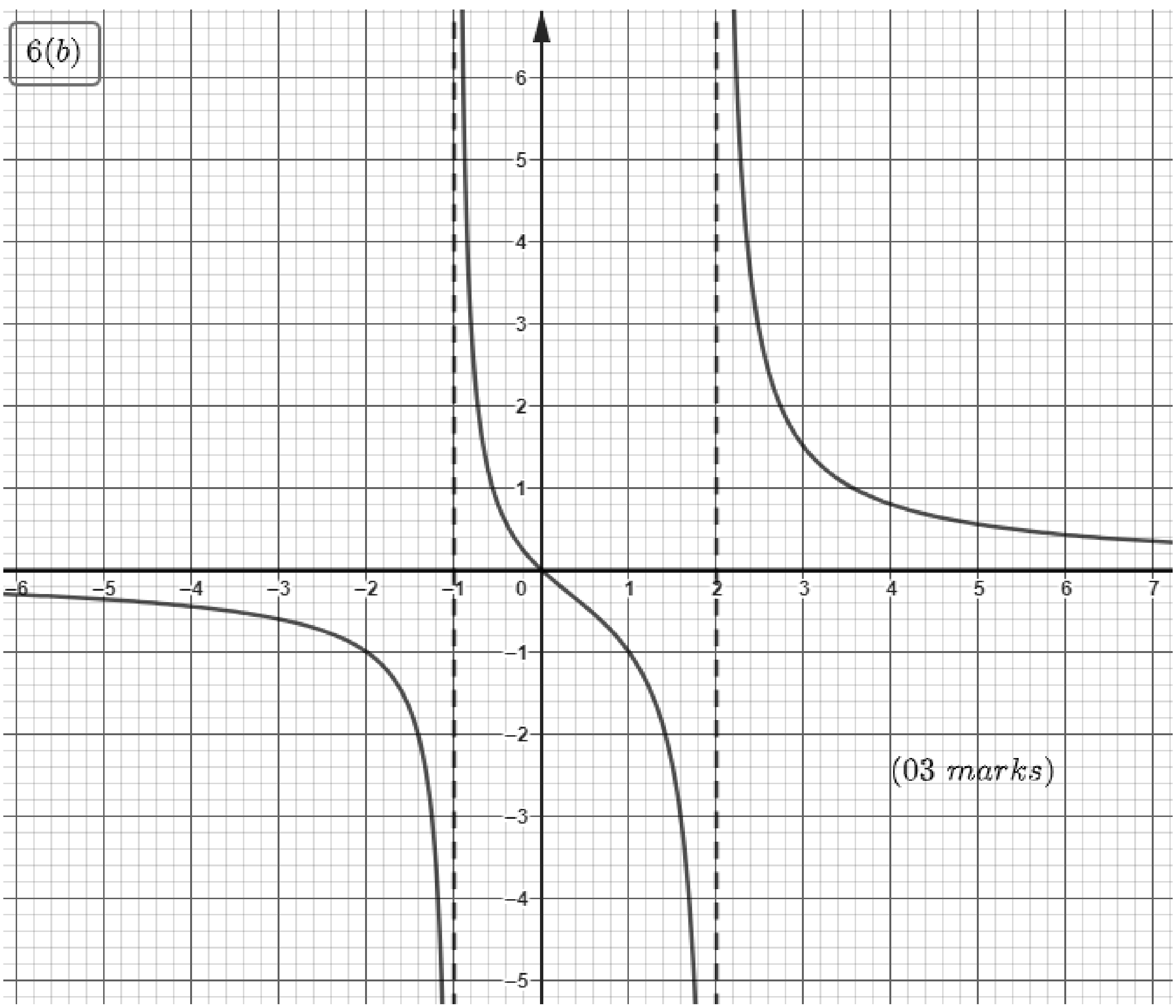
$x=-0 \quad and \quad x=-2$ $\left(00\frac{1}{2}\ marks\right)$

$H..A$

$$y=\dfrac{\frac{2x}{x^2}}{\frac{x^2}{x^2}-\frac{x}{x^2}-\frac{2}{x^2}} \Rightarrow y=\dfrac{\frac{2}{x}}{1-\frac{1}{x}-\frac{2}{x^2}}=0$$ $\left(00\frac{1}{2}\ marks\right)$

$x-\text{int}\,ercept \quad (0,\ 0)$ $\left(00\frac{1}{2}\ marks\right)$

$y-\text{int}\,ercept \quad (0,\ 0)$ $\left(00\frac{1}{2}\ marks\right)$



$Domain = \{x : x \in lR \ \ except \ \ x = -1 \ \ and \ \ x = 2\}$ $\left(00\frac{1}{2} marks\right)$

$Range = \{y : y \in lR \ \ exccept \ \ y = 0\}$ $\left(00\frac{1}{2} marks\right)$

7 (a) $f(x_n) = 2x_n - 3\cos\left(\frac{x_n}{2}\right)$

$But \quad x_1 = 1 \quad and \quad x_2 = 2$

$f(x_1) = 2(1) - 3\cos\left(\frac{1}{2}\right) = -0.633$ $\left(00\frac{1}{2} marks\right)$

$f(x_2) = 2(2) - 3\cos\left(\frac{2}{2}\right) = 2.379$

$f(x_1) \times f(x_2) = -ve$

$f(x_1) \times f(x_2) < 0$ $\left(00\frac{1}{2} marks\right)$

$The \ root \ lies \ btn \ 1 \ and \ 2$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$f'(x_n) = 2 - \frac{3}{2}\sin\left(\frac{x_n}{2}\right)$ $\left(00\frac{1}{2} marks\right)$

$x_{n+1} = x_n - \frac{\left(2x_n - 3\cos\left(\frac{x_n}{2}\right)\right)}{\left(2 + \frac{3}{2}\sin\left(\frac{x_n}{2}\right)\right)} = \frac{3x_n \sin\left(\frac{x_n}{2}\right) + 6\cos\left(\frac{x_n}{2}\right)}{4 + 3\sin\left(\frac{x_n}{2}\right)}$ $\left(00\frac{1}{2} marks\right)$

$x_0 = \frac{x_1 + x_2}{2} = \frac{1+2}{2} = 1.5$ $\left(00\frac{1}{2} marks\right)$

1^{st} $iteration \ n = 0$

$x_1 = \frac{3x_0 \sin\left(\frac{x_0}{2}\right) + 6\cos\left(\frac{x_0}{2}\right)}{4 + 3\sin\left(\frac{x_0}{2}\right)} = \frac{3(1.5)\sin\left(\frac{1.5}{2}\right) + 6\cos\left(\frac{1.5}{2}\right)}{4 + 3\sin\left(\frac{1.5}{2}\right)} = 1.2337$ $\left(00\frac{1}{2} marks\right)$

2^{nd} $iteration \ n = 1$

$x_2 = 1.2266$ $\left(00\frac{1}{2} marks\right)$

3^{rd} $iteration \ n = 2$

$x_3 = 1.2266$ $\left(00\frac{1}{2} marks\right)$

7 (b) $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$n = n+1$

$x_{n+1} = x_{n+1} - \frac{f(x_{n+1})}{f'(x_{n+1})}$ $\left(00\frac{1}{2}\ marks\right)$

$but \quad f'(x_{n+1}) = slope$

$f'(x_{n+1}) = \frac{f(x_{n+1}) - f(x_n)}{x_{n+1} - x_n}$

$x_{n+2} = x_{n+1} - \frac{f(x_{n+1})}{\left(\frac{f(x_{n+1}) - f(x_n)}{x_{n+1} - x_n}\right)}$ $\left(00\frac{1}{2}\ marks\right)$

$x_{n+2} = x_{n+1} - \left(\frac{x_{n+1} - x_n}{f(x_{n+1}) - f(x_n)}\right) f(x_{n+1})$ $\left(00\frac{1}{2}\ marks\right)$

7 (c) $h = \frac{b-a}{n-1} = \frac{1--0}{10} = 0.1$ $\left(00\frac{1}{2} marks\right)$

n	x	$\frac{x}{1+x^2}$	1^{st} & $last$	$Middle$	Odd	$Even$
0	0	0	0			
1	0.1	0.09901		0.09901	0.09901	
2	0.2	0.19231		0.19231		0.192308
3	0.3	0.27523		0.27523	0.27523	
4	0.4	0.34483		0.34483		0.344828
5	0.5	0.4		0.4	0.4	
6	0.6	0.44118		0.44118		0.441176
7	0.7	0.4698		0.4698	0.4698	
8	0.8	0.4878		0.4878		0.487805
9	0.9	0.49724		0.49724	0.49724	
10	1	0.5	0.5			
$\sum$	$--$	**3.70739**	**0.5**	**3.20739**	**1.74128**	**1.466117**

$\left(01\frac{1}{2} marks\right)$

$$Trapezium\ \ Rule$$

$$\int_a^b f(x)dx = \frac{h}{2}\left(1^{st} + last + 2\sum middle\right)$$

$$\int_0^1 \frac{x}{(1+x^2)} = \frac{0.1}{2}(0.5 + 2\times 3.20739) \qquad \left(00\frac{1}{2}\ \ marks\right)$$

$$\int_0^1 \frac{x}{(1+x^2)} = 0.3457 \qquad \left(00\frac{1}{2}\ \ marks\right)$$

$$Simpson\ Rule$$

$$\int_a^b f(x)dx = \frac{h}{3}\left(1^{st} + last + 4\sum odd + 2\sum even\right)$$

$$\int_0^1 \frac{x}{(1+x^2)} = \frac{0.1}{3}(0.5 + 4\times 1.7413) + 2\times 1.4661 \qquad \left(00\frac{1}{2}\ \ marks\right)$$

$$\int_0^1 \frac{x}{(1+x^2)} = 0.3466 \qquad \left(00\frac{1}{2}\ \ marks\right)$$

8(a) $x^2 + y^2 - 4x + 5y + 2 = 0$

$$x + y = c \Rightarrow y = c - x$$

$$x^2 + (c-x)^2 - 4x + 5(c-x) + 2 = 0$$

$$x^2 + c^2 - 2cx + x^2 - 4x + 5c - 5x + 2 = 0$$

$$2x^2 - 2cx - 9x + c^2 + 5c + 2 = 0 \qquad \left(00\frac{1}{2}\ \ mark\right)$$

$$2x^2 - (2c+9)x + c^2 + 5c + 2 = 01$$

$$b^2 = 4ac$$

$$(-(2c+9))^2 = 4\times 2\times(c^2 + 5c + 22) \qquad \left(00\frac{1}{2}\ \ mark\right)$$

$$4c^2 + 36c + 81 = 8(c^2 + 5c + 2)$$

$$4c^2 + 36c + 81 = 8c^2 + 40c + 16$$

$$4c^2 + 4c - 65 = 0$$

$$c = -4.562 \qquad or \qquad c = 3.562 \qquad (01\ \ mark)$$

$$2x^2 - 2(2(-4.562) + 9)x + (-4.562)^2 + 5(-4.562) + 2 = 0$$

$$2x^2 + 18.248x - 1.844E^{-3} = 0$$

$x = -9.124 \ or \ \ x = 1E^{-4}$

$$y = 3.562 - x \qquad \left(00\frac{1}{2} \ \ mark\right)$$

$$2x^2 - 2(2(3.562)+9)x + (3.562)^2 + 5(3.562) + 2 = 0$$

$$2x^2 + 14.248x + 32.498 = 0$$

$$x = -3.562$$

$$y = -4.563 - x \qquad \left(00\frac{1}{2} \ \ mark\right)$$

8 (b)

$$A = \frac{1}{2}\begin{vmatrix} r_1\cos\theta_1 & r_1\sin\theta_1 & 1 \\ r_2\cos\theta_2 & r_2\sin\theta_2 & 1 \\ r_3\cos\theta_3 & r_1\sin\theta_3 & 1 \end{vmatrix} \qquad \left(00\frac{1}{2}marks\right)$$

$$A = \frac{1}{2}\begin{vmatrix} \cos30^0 & \sin30^0 & 1 \\ 2\cos60^0 & 2\sin60^0 & 1 \\ 3\cos90^0 & 3\sin90^0 & 1 \end{vmatrix} \qquad \left(00\frac{1}{2}marks\right)$$

$$A = \frac{1}{2}\begin{vmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} & 1 \\ 1 & \sqrt{3} & 1 \\ 0 & 3 & 1 \end{vmatrix} \qquad \left(00\frac{1}{2}marks\right)$$

$$A = \frac{1}{2}\left(\frac{\sqrt{3}}{2}\begin{vmatrix} \sqrt{3} & 1 \\ 3 & 1 \end{vmatrix} - \frac{1}{2}\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} + 1\begin{vmatrix} 1 & \sqrt{3} \\ 0 & 3 \end{vmatrix}\right) \qquad \left(00\frac{1}{2}marks\right)$$

$$A = \frac{1}{2}\left(\frac{\sqrt{3}}{2}\left(\sqrt{3}-3\right) - \frac{1}{2}(1-0) + 1(3-0)\right)$$

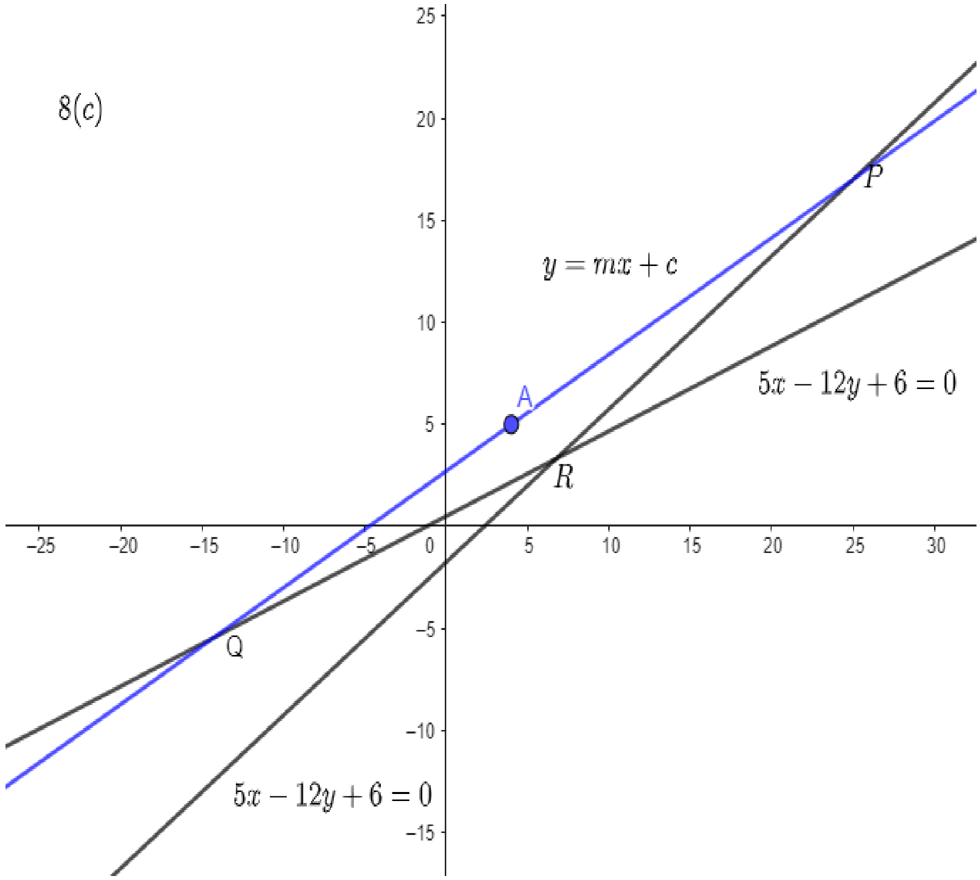
$$A = \frac{1}{2}\left(\frac{3}{2} - 3\frac{\sqrt{3}}{2} - \frac{1}{2} + 3\right) \qquad \left(00\frac{1}{2}marks\right)$$

$$A = \frac{1}{2}\left(4 - 3\frac{\sqrt{3}}{2}\right)$$

$$A = 2 - \frac{3\sqrt{3}}{4} \quad square \ \ units \qquad \left(00\frac{1}{2}marks\right)$$

$$A = 0.7010 \qquad square\,units$$

$8(c)$



$P\hat{Q}R = Q\hat{P}R$

$\tan\left(P\hat{Q}R\right) = \tan\left(Q\hat{P}R\right)$

$5x - 12y = -6$

$12y = 5x + 6$

$y = \frac{5}{12}x + \frac{6}{12}$

$m_1 = \frac{5}{12}$ $\left(00\frac{1}{2}marks\right)$

$\tan\left(P\hat{Q}R\right) = \dfrac{m - \frac{5}{12}}{1 + \frac{5}{12}m}$

$3x - 4y = 7$

$4y = 3x - 7$

$y = \frac{3}{4}x - \frac{7}{4}$

$m_2 = \frac{3}{4}$ $\left(00\frac{1}{2}marks\right)$

$$\tan\left(Q\hat{P}R\right)=\frac{\frac{3}{4}-m}{1+\frac{3}{4}m}$$

$$\frac{m-\frac{5}{12}}{1+\frac{5}{12}m}=\frac{\frac{3}{4}-m}{1+\frac{3}{4}m} \qquad \left(00\frac{1}{2}marks\right)$$

$$\frac{12m-5}{12+5m}=\frac{3-4m}{4+3m}$$

$$48m+36m^2-20-15m=36-48m+15m-20m^2$$

$$56m^2+66m-56=0$$

$$m=-\frac{7}{4} \quad or \quad m=\frac{4}{7}$$

$$When \quad m=-\frac{7}{4} \qquad at \quad A(4,\ 5) \qquad \left(00\frac{1}{2}marks\right)$$

$$y=mx+c$$

$$5=-\frac{7}{4}(4)+c$$

$$c=5+7$$

$$c=12$$

$$y=-\frac{7}{4}x+12$$

$$7x+4y-48=0 \qquad \left(00\frac{1}{2}marks\right)$$

$$When \quad m=\frac{4}{7} \qquad at \quad A(4,\ 5)$$

$$5=\frac{4}{7}(4)+c$$

$$35=16+7c \qquad \left(00\frac{1}{2}marks\right)$$

$$c=\frac{19}{7}$$

$$y=\frac{4}{7}x+\frac{19}{7}$$

$$4x-7y+19=0 \qquad \left(00\frac{1}{2}marks\right)$$

9 (a) $\int e^{ax}\cos bx\,dx$

$\int u\,dv = uv - \int v\,du$

$let \quad u = \cos bx$

$\frac{du}{dx} = -b\sin bx \quad du = -b\sin bx\,dx$

$dv = e^{ax}dx \qquad v = \int e^{ax}dx = \frac{1}{a}e^{ax}$

$\int e^{ax}\cos bx\,dx = \frac{1}{a}e^{ax}\cos bx - \int \frac{1}{a}e^{ax}(-b\sin bx)dx$ $\left(00\frac{1}{2}marks\right)$

$\int e^{ax}\cos bx\,dx = \frac{1}{a}e^{ax}\cos bx + \int \frac{1}{a}e^{ax}b\sin bx\,dx$

$\int e^{ax}\cos bx\,dx = \frac{1}{a}e^{ax}\cos bx + \frac{b}{a}\int e^{ax}\sin bx\,dx$ $\left(00\frac{1}{2}marks\right)$

$let \quad u = \sin bx$

$\frac{du}{dx} = b\cos bx \quad du = b\cos bx\,dx$

$dv = e^{ax}dx \qquad v = \int e^{ax}dx = \frac{1}{a}e^{ax}$ $\left(00\frac{1}{2}marks\right)$

$\int e^{ax}\cos bx\,dx = \frac{1}{a}e^{ax}\cos bx + \frac{b}{a}\left(uv - \int v\,du\right)$

$\int e^{ax}\cos bx\,dx = \frac{1}{a}e^{ax}\cos bx + \frac{b}{a}\left(\frac{1}{a}e^{ax}(b\cos bx\ ) - \int \frac{1}{a}e^{ax}(b\cos bx)dx\right)$

$\int e^{ax}\cos bx\,dx = \frac{1}{a}e^{ax}\cos bx + \frac{b^2}{a^2}e^{ax}\cos bx - \frac{b^2}{a^2}\int e^{ax}\cos bx\,dx$ $\left(00\frac{1}{2}marks\right)$

$\int e^{ax}\cos bx\,dx + \frac{b^2}{a^2}\int e^{ax}\cos bx\,dx = \frac{1}{a}e^{ax}\cos bx + \frac{b^2}{a^2}e^{ax}\cos bx\,dx$

$\int e^{ax}\cos bx\,dx\left(1 + \frac{b^2}{a^2}\right) = \frac{1}{a}e^{ax}\cos bx + \frac{b^2}{a^2}e^{ax}\cos bx\,dx$

$\left(\frac{a^2+b^2}{a^2}\right)\int e^{ax}\cos bx\,dx = \frac{1}{a}e^{ax}\cos bx + \frac{b^2}{a^2}e^{ax}\cos bx\,dx$ $\left(00\frac{1}{2}marks\right)$

$\int e^{ax}\cos bx\,dx = \frac{ae^{ax}\cos bx + be^{ax}\sin bx}{a^2+b^2} + c$

$\int e^{ax}\cos bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx) + c$ $\left(00\frac{1}{2}marks\right)$

$Hence \ \ shown$

$$\int e^{ax}\cos bx dx=\frac{e^{ax}}{a^2+b^2}\left(a\cos bx+b\sin bx\right)+c$$

$$\int e^{-\frac{2}{5}x}\cos\frac{3}{10}xdx=\frac{e^{-\frac{2}{5}x}}{\left(-\frac{2}{5}\right)^2+\left(\frac{3}{10}\right)^2}\left(-\frac{2}{5}\cos\left(\frac{3}{10}\right)x+\frac{3}{10}\sin\left(\frac{3}{10}\right)x\right)+c$$

$$\int e^{-\frac{2}{5}x}\cos\frac{3}{10}xdx=\frac{e^{-\frac{2}{5}x}}{\left(-\frac{2}{5}\right)^2+\left(\frac{3}{10}\right)^2}\left(-\frac{2}{5}\cos\left(\frac{3}{10}\right)x+\frac{3}{10}\sin\left(\frac{3}{10}\right)x\right)+c$$

$$\int e^{-\frac{2}{5}x}\cos\frac{3}{10}xdx=\frac{e^{-\frac{2}{5}x}}{\frac{4}{25}+\frac{9}{100}}\left(-\frac{2}{5}\cos\left(\frac{3}{10}\right)x+\frac{3}{10}\sin\left(\frac{3}{10}\right)x\right)+c \qquad \left(00\frac{1}{2}\ marks\right)$$

$$\int e^{-\frac{2}{5}x}\cos\frac{3}{10}xdx=4e^{-\frac{2}{5}x}\left(-\frac{2}{5}\cos\left(\frac{3}{10}\right)x+\frac{3}{10}\sin\left(\frac{3}{10}\right)x\right)+c$$

$$\int e^{-\frac{2}{5}x}\cos\frac{3}{10}xdx=e^{-\frac{2}{5}x}\left(-\frac{8}{5}\cos\left(\frac{3}{10}\right)x+\frac{12}{10}\sin\left(\frac{3}{10}\right)x\right)+c$$

$$\int e^{-\frac{2}{5}x}\cos\frac{3}{10}xdx=e^{-\frac{2}{5}x}\left(\frac{6}{5}\sin\left(\frac{3}{10}\right)x-\frac{8}{5}\cos\left(\frac{3}{10}\right)\right)+c \qquad \left(00\frac{1}{2}\ marks\right)$$

9 (b) $\int_1^{10} x\log_{10} xdx$

$$\int_1^{10} x\log_{10} xdx = \int_1^{10} x\frac{\ln x}{\ln 10}dx$$

$$\int_1^{10} x\log_{10} xdx = \frac{1}{\ln 10}\int_1^{10} x\ln xdx \qquad \left(00\frac{1}{2}marks\right)$$

$$u = \ln x \quad \frac{du}{dx} = \frac{1}{x} \quad du = \frac{dx}{x}$$

$$dv = xdx \quad v = \frac{x^2}{2}$$

$$\int udv = uv - \int vdu$$

$$\int_1^{10} x\log_{10} xdx = \frac{1}{\ln 10}\left(\frac{x^2}{2}\ln x - \int\left(\frac{x^2}{2}.\frac{1}{x}\right)dx\right)$$

$$\int_1^{10} x\log_{10} xdx = \frac{1}{\ln 10}\left(\frac{x^2}{2}\ln x - \int\left(\frac{x}{2}\right)dx\right) \qquad \left(00\frac{1}{2}marks\right)$$

$$\int_1^{10} x\log_{10} xdx = \frac{1}{\ln 10}\left(\frac{x^2}{2}\ln x - \frac{x^2}{4}\right)_1^{10}$$

$$\int_1^{10} x\log_{10} xdx = \frac{1}{\ln 10}\left(\left(\frac{100}{2}\ln 10 - \frac{100}{4}\right) + \left(\frac{1}{4}\right)\right) \qquad \left(00\frac{1}{2}marks\right)$$

$$\int_1^{10} x\log_{10} xdx = \frac{1}{\ln 10}\left(50\ln 10 - \frac{99}{4}\right)$$

$$\int_1^{10} x\log_{10} xdx = 50 - \frac{99}{4\ln 10} \qquad \left(00\frac{1}{2}marks\right)$$

9 (c) $$\int \frac{1}{1+\sin^2 x}dx = \int \frac{\frac{1}{\cos^2 x}}{\frac{1}{\cos^2 x}+\frac{\sin^2 x}{\cos^2 x}}dx$$

$$\int \frac{1}{1+\sin^2 x}dx = \int \frac{\sec^2 x}{\sec^2 x+\tan^2 x}dx \qquad \left(00\frac{1}{2}marks\right)$$

$$\int \frac{1}{1+\sin^2 x}dx = \int \frac{\sec^2 x}{1+\tan^2 x+\tan^2 x}dx$$

$$\int \frac{1}{1+\sin^2 x}dx = \int \frac{\sec^2 x}{1+2\tan^2 x}dx$$

$$t=\tan x \quad \frac{dt}{dx}=\sec^2 x \Rightarrow dx = \frac{dt}{\sec^2 x} \qquad \left(00\frac{1}{2}marks\right)$$

$$\int \frac{1}{1+\sin^2 x}dx = \int \frac{\sec^2 x}{1+2t^2}\frac{dt}{\sec^2 x}$$

$$\int \frac{1}{1+\sin^2 x}dx = \int \frac{dt}{2t^2+1}$$

$$\int \frac{1}{1+\sin^2 x}dx = \int \frac{dt}{\left(\sqrt{2}t\right)^2+1} \qquad \left(00\frac{1}{2}marks\right)$$

$$\sqrt{2}t=\tan\theta \Rightarrow \sqrt{2}\frac{dt}{d\theta}=\sec^2\theta,\ \ dt=\frac{1}{\sqrt{2}}\sec^2\theta\, d\theta$$

$$\int \frac{1}{1+\sin^2 x}dx = \int \frac{\frac{1}{\sqrt{2}}\sec^2\theta\, d\theta}{(\tan\theta)^2+1} = \int \frac{\frac{1}{\sqrt{2}}\sec^2\theta\, d\theta}{\sec^2\theta} \qquad \left(00\frac{1}{2}marks\right)$$

$$\int \frac{1}{1+\sin^2 x}dx = \frac{1}{\sqrt{2}}\int d\theta = \frac{1}{\sqrt{2}}\theta + c \qquad \left(00\frac{1}{2}marks\right)$$

$$\int \frac{1}{1+\sin^2 x}dx = \frac{1}{\sqrt{2}}\int d\theta = \frac{1}{\sqrt{2}}\tan^{-1}\sqrt{2}t + c$$

$$\int \frac{1}{1+\sin^2 x}dx = \frac{1}{\sqrt{2}}\int d\theta = \frac{1}{\sqrt{2}}\tan^{-1}\left(\sqrt{2}\tan x\right) + c \qquad \left(00\frac{1}{2}marks\right)$$

10 (a) $f(x)=\dfrac{1}{\sqrt{x+1}}$

$$f(x+h)=\frac{1}{\sqrt{(x+h)+1}}$$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{\dfrac{1}{\sqrt{(x+h)+1}}-\dfrac{1}{\sqrt{x+1}}}{h} \qquad \left(00\frac{1}{2}\ marks\right)$$

$$=\lim_{h\to 0}\frac{\dfrac{\sqrt{x+1}-\sqrt{x+h+1}}{\left(\sqrt{x+h+1}\right)\sqrt{x+1}}}{h}$$

$$=\lim_{h\to 0}\frac{\sqrt{x+1}-\sqrt{x+h+1}}{\left(\sqrt{x+h+1}\right)h\sqrt{x+1}} \qquad \left(00\frac{1}{2}\ marks\right)$$

$$=\lim_{h\to 0}\frac{\sqrt{x+1}-\sqrt{x+h+1}}{\left(\sqrt{x+h+1}\right)h\sqrt{x+1}}\times\frac{\sqrt{x+1}+\sqrt{x+h+1}}{\sqrt{x+1}+\sqrt{x+h+1}}$$

$$=\lim_{h\to 0}\frac{x+1-(x+h+1)}{h\left((x+1)\sqrt{x+h+1}+(x+h+1)\sqrt{x+1}\right)} \qquad \left(00\frac{1}{2}\ marks\right)$$

$$=\lim_{h\to 0}\frac{x+1-x-h-1}{h\left((x+1)\sqrt{x+h+1}+(x+h+1)\sqrt{x+1}\right)}$$

$$=\lim_{h\to 0}\frac{-\cancel{h}}{h\left((x+1)\sqrt{x+h+1}+(x+h+1)\sqrt{x+1}\right)} \qquad \left(00\frac{1}{2}\ marks\right)$$

$$=\lim_{h\to 0}\frac{-1}{\left((x+1)\sqrt{x+h+1}+(x+h+1)\sqrt{x+1}\right)}$$

$$=\frac{-1}{\left((x+1)\sqrt{x+0+1}+(x+0+1)\sqrt{x+1}\right)} \qquad \left(00\frac{1}{2}\ marks\right)$$

$$=\frac{-1}{\left((x+1)\sqrt{x+1}+(x+1)\sqrt{x+1}\right)}$$

$$=\frac{-1}{2(x+1)\sqrt{x+1}}$$

$$\frac{dy}{dx}=\frac{-1}{2(x+1)\sqrt{x+1}} \qquad \left(00\frac{1}{2}\ marks\right)$$

10 (b) $x=\cos t+t\sin t$

$y=\sin t-t\cos t$

$\frac{d^2y}{dx^2}$ *is required*

$\frac{dx}{dt}=-\sin t+t\cos t+\sin t$ $\left(00\frac{1}{2}marks\right)$

$\frac{dx}{dt}=t\cos t$

$\frac{dy}{dt}=\cos t+t\sin t-\cos t$ $\left(00\frac{1}{2}marks\right)$

$\frac{dy}{dt}=t\sin t$

$\frac{dy}{dx}=\frac{dy}{dt}\div\frac{dx}{dt}$ $\left(00\frac{1}{2}marks\right)$

$\frac{dy}{dx}=\frac{t\sin t}{t\cos t}$

$\frac{dy}{dx}=\tan t$ $\left(00\frac{1}{2}marks\right)$

$\frac{d}{dt}\left(\frac{dy}{dx}\right)=\sec^2 t$

$\frac{d^2y}{d^2x}=\frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$ $\left(00\frac{1}{2}marks\right)$

$\frac{d^2y}{d^2x}=\frac{\sec^2 t}{t\cos t}$

$\frac{d^2y}{d^2x}=\frac{1}{t\cos^3 t}$

$\frac{d^2y}{d^2x}=\frac{\sec^3 t}{t}$ $\left(00\frac{1}{2}marks\right)$

10 (c) $f(x) = \tan x$

$$f\left(\frac{\pi}{4}\right) = \tan\frac{\pi}{4} = 1$$

$$f'(x) = \sec^2 x$$

$$f'\left(\frac{\pi}{4}\right) = \sec^2\frac{\pi}{4} = 1 + \tan^2\frac{\pi}{4} = 2 \qquad \left(00\frac{1}{2}\,marks\right)$$

$$f''(x) = 2\tan x\sec^2 x = 2\tan x\left(1 + \tan^2 x\right)$$

$$f''\left(\frac{\pi}{4}\right) = 2\tan\frac{\pi}{4}\sec^2\frac{\pi}{4} = 2\tan\frac{\pi}{4}\left(1 + \tan^2\frac{\pi}{4}\right) = 4 \qquad \left(00\frac{1}{2}\,marks\right)$$

$$f'''(x) = 2\sec^2 x + 6\sec^2 x\tan^2 x$$

$$f'''(x) = 2\left(1 + \tan^2 x\right) + 6\left(1 + \tan^2 x\right)\tan^2 x$$

$$f'''\left(\frac{\pi}{4}\right) = 2\left(1 + \tan^2\frac{\pi}{4}\right) + 6\left(1 + \tan^2\frac{\pi}{4}\right)\tan^2\frac{\pi}{4}$$

$$f'''\left(\frac{\pi}{4}\right) = 2(2) + 6(2) = 4 + 12 = 16 \qquad \left(00\frac{1}{2}\,marks\right)$$

$$f^{iv}(x) = 2 + 16\sec^2 x\tan x + 24\sec^2 x\tan^3 x$$

$$f^{iv}(x) = 2 + 16\left(1 + \tan^2 x\right)\tan x + 24\left(1 + \tan^2 x\right)\tan^3 x$$

$$f^{iv}\left(\frac{\pi}{4}\right) = 2 + 16\left(1 + \tan^2\frac{\pi}{4}\right)\tan\frac{\pi}{4} + 24\left(1 + \tan^2\frac{\pi}{4}\right)\tan^3\frac{\pi}{4} \qquad \left(00\frac{1}{2}\,marks\right)$$

$$f^{iv}\left(\frac{\pi}{4}\right) = 2 + 16(2) + 24(2) = 2 + 32 + 48 = 82 \qquad \left(00\frac{1}{2}\,marks\right)$$

$$f(a+h) = f(a) + \frac{hf'(a)}{1!} + \frac{h^2 f''(a)}{2!} + \frac{h^3 f'''(a)}{3!} + \frac{h^4 f^{iv}(a)}{4!}\ldots\ldots$$

$$f\left(\frac{\pi}{4} + x\right) = f\left(\frac{\pi}{4}\right) + \frac{xf'\left(\frac{\pi}{4}\right)}{1!} + \frac{x^2 f''\left(\frac{\pi}{4}\right)}{2!} + \frac{x^3 f'''\left(\frac{\pi}{4}\right)}{3!} + \frac{x^4 f^{iv}\left(\frac{\pi}{4}\right)}{4!}\ldots \qquad .\left(00\frac{1}{2}\,marks\right)$$

$$f\left(\frac{\pi}{4} + x\right) = 1 + 2x + 2x^2 + \frac{8}{3}x^2 + \frac{10}{3}x^4$$

$$45^0 + x = 46.5 \Rightarrow x = 1.5^0 = 0.02618 \qquad \left(00\frac{1}{2}\,marks\right)$$

$$\tan\left(45^0 + 1.5^0\right) = 1 + 2(0.02618) + 2(0.02618)^2 + \frac{8}{3}(0.02618)^3 + \frac{10}{3}(0.02618)^4$$

$$\tan 46.5^0 = 1.054 \qquad \left(00\frac{1}{2}\,marks\right)$$