

**THE UNITED REPUBLIC OF TANZANIA
PRESIDENT'S OFFICE,
REGIONAL ADMINISTRATION AND LOCAL GOVERNMENT
NJOMBE REGION**



FORM SIX PRE – NATIONAL EXAMINATION

CODE: 142/2 ADVANCED MATHEMATICS 2

(For Both School and Private Candidates)

TIME: 03: 00 HOURS

MONDAY, 18TH March 2024 A.M

INSTRUCTIONS

- ❖ This PAPER consists of section A and B with eight (8) questions.
- ❖ Answer all questions in section A and two (2) questions from section B
- ❖ All work done and answer must be shown clearly
- ❖ Mathematical tables and non-programmable calculators may be used.
- ❖ Cellular phones and other unauthorized materials are not allowed in the examination room
- ❖ Write your examination number on every page of your answer booklet(s)

SECTION A (60 MARKS)

1. (a) A certain eruptible pandemic disease occur 1% of the population. A sample screening procedure is available and in 8 out of 10 cases where patient has a disease, if 0.05 chance that the test will give a positive result.
- Find the probability that a randomly selected individual does not have disease but gives a positive result in the screening test
 - Find the probability that a randomly selected individual gives a positive result on screening test
 - If danken has taken the disease test and her result is positive, what is the probability that he has disease.

(b) Given that $P(A') = \frac{1}{2}$, $P(B') = \frac{7}{10}$ and $P(A \cup B)' = \frac{2}{5}$. Find the value of;

(i) $P(A \cap B')$ (ii) $P(A \cap B)$

(c) The random variable x has probability distribution as follows.

x	2	3	5	7	11
$P(x)$	0.17	0.33	0.25	x	y

Find the numerical of value of x and y , if $E(x) = \frac{14}{3}$

2. (a) (i) Fact 1: Islands are surrounded by water
Fact 2: Maui is an Island
Fact 3: Maui was formed by a volcano.

If the first three statements are facts, which of the following statements must also be a fact?

- Maui is surrounded by water
- All Islands are formed by volcanoes
- All volcanoes are on Islands.

(ii) Simplify $r \vee [(\sim p \rightarrow q) \wedge p]$ using laws of propositions, hence draw an electrical network for the resulting simplified statement.

b) Given the statement $p \rightarrow (q \rightarrow \sim p)$. Construct the truth table for the contrapositive

c) Use the laws of algebra of proposition to show that the statement $[(p \rightarrow \sim q) \wedge (r \rightarrow q)] \rightarrow (p \rightarrow \sim r)$ is a tautology.

d) Test whether or not the following argument is valid

“If I am illiterate, then I can’t read and write. I cannot read but I can write. Thus I am not illiterate”

3.a) If $\mathbf{U} = 4\mathbf{i} + \mathbf{Mj} + 3\mathbf{k}$ and $\mathbf{v} = -4\mathbf{i} + 3\mathbf{j}$ and the projection of $\mathbf{U}$ onto $\mathbf{v}$ is $\frac{2}{3}$. Find the value of m

b) If Q is an angle between two-unit vectors a and b show that

$$\frac{1}{2}[\mathbf{a} + \mathbf{b}] = \cos\left(\frac{Q}{2}\right)$$

c) At a time t second, the position vector $\mathbf{r}$ of a particle $\mathbf{p}$ is with respect to the origin (o) is $\mathbf{r}(t) = (3 \cos 2t)$

- i. Show that the velocity of a particle is always at the right angle to op
- ii. Find the speed of the particle at $t = 0$
- iii. Find the accretion of the particle $\mathbf{P}$ at $t = 0$

4.(a) Find the complex number $z = x + iy$, satisfy the equation $\frac{1+z^2}{1-z^2} = i$

(b) Prove that $16 \cos^5 \theta = \cos 5\theta + 5 \cos 3\theta + 10 \cos \theta$

(c) The point P represents a complex number z on an argand diagram such that $|z - 6i| = 2|z - 3|$, show that as z varies, the locus of p is a circle, state the radius and coordinates of the centre of the circle.

SECTION B: (40 MARKS)

Answer only two (2) questions.

5. a) Show that

(i)
$$\frac{1 + \cos x + \cos 2x}{\sin x + \sin 2x} = \cot x$$

(ii)
$$\cos^2(\alpha + \beta) + \cos^2(\alpha - \beta) = \cos 2\alpha \cos 2\beta + 1.$$

b) Find the general solution in degree for the equation

$$9\sin^2 x + 10\sin x \cos x - 2\cos^2 x = 1$$

c) Find x if $\tan^{-1}(2x + 1) - \tan^{-1}(2x - 1) = \tan^{-1}\left(\frac{1}{8}\right)$

d) Express $\cos 2\theta - \sin 2\theta$ in form $R \cos(2\theta + \alpha)$, Hence or otherwise find the general solution in radians for the equation $\sin 2\theta = \cos 2\theta - 1$.

e) Find the expression involving θ that the following approximate to very small value of θ

$$\frac{1 + \sin \theta}{5 + 3 \tan \theta - 4 \cos \theta}$$

6. a) (i) If one of the roots of the equation $ax^2 + bx + c = 0$ is twice the other root, show that $2b^2 = 9ac$

(ii) Given that $(2x + 1)$ is a factor of $2x^3 + ax^2 + 16x + 6$, find the value of a and the real quadratic factor of $2x^3 + 9x^2 + 16x + 6$

b) (i) Given that a is a constant, x is a variable and the fourth term in the expansion of

$$\left(ax + \frac{1}{x}\right)^n \text{ is } \frac{5}{2}. \text{ Find the value of } a \text{ and } n.$$

(ii) Find x if $\log_x 8 - \log_x 2^{16} = 1$

c) Solve the system of equation by determinant method

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$$

$$\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$$

$$\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$$

d) Express $\frac{16x}{x^4-16}$ into partial fraction

7. a) (i) Solve the differential Equation $x \frac{dy}{dx} - \frac{y^2}{x} = y$

(ii) Find the general solution of the differential equation $y'' - 2y' - 3 = 0$

b) From the differential equation given that $y = A \cos 2x + B \sin 2x$

c) A copper ball is heated to a temperature of 100°C , then at time $t = 0$ it is placed in water which is maintained at a temperature of 30°C . At the end of 3 minutes the temperature of the ball is reduced to 70°C . Find the time at which the temperature of the ball is reduced to 31°C .

8. a) Show that the equation of the tangent to the parabola $y^2 = 4ax$ at the

point (p, q) is $qy = 2a(x + p)$

b) A man running a race course, notes that the sum of the distances from the two flag posts from him is always 10 metres and the distance between the flag posts is 8. Find the equation of the path traced by a man.

c) The equation of hyperbola is given by

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

(i) Find the length of its latus rectum

(ii) The line $y = Mx + c$ is a tangent to the curve, show that $C = \pm\sqrt{M^2a^2 - b^2}$

d) (i) Prove that the Cartesian of $r = \frac{4}{1+\cos\theta}$ represents the translated parabola

(ii) Draw the graph of $r = 2 + 4 \cos \theta$