THE UNITED REPUBLIC OF TANZANIA

PRESIDENT'S OFFICE,

REGIONAL ADMINISTRATION AND LOCAL GOVERNMENT

NJOMBE REGION

FORM SIX PRE – NATIONAL EXAMINATION

CODE: 142/1 ADVANCED MATHEMATICS 1

(For Both School and Private Candidates)

TIME: 03: 00 Hrs **Wednesday , 13th March 2024 a.m**

INSTRUCTIONS

1. This paper consists of Ten (10) questions, each carrying ten marks.
2. Answer all questions
3. All necessary working and answers for each question must be shown clearly.
4. Mathematical tables and non-programmable calculators may be used.
5. Cellular phones and any unauthorized materials are **not** allowed in the examination room
6. Write your **Examination Number** on every page of your answer booklet(s).

1. Use scientific calculator find the following.

(a) If $s = \dfrac{e^{-\left(\frac{x-m}{2s^2}\right)^2}}{\sqrt[y]{2p}}$. Find y if $x = m$, $s = 3.08$ and $p = 8$

(b) Find the mean and variance of the following data in four decimal places

$$14.3,\quad 8,\quad 4\tan^{-1}(1),\quad \binom{4}{2},\quad 3!,\quad {}^{5}p_{2},\quad 4\log\left({}^{7}p_{3}\right)$$

(c) Change 52 as a decimal number into

(i) Binary number (ii) Hexadecimal number

2. (a) Given that, $4\cosh^2 x = 3\sinh x + p$ where p is a constant.

(i) Find the range of values of p for which the equation has real solution.

(ii) If $p = 4$,find the exact solution of the equation and leave your answer in natural logarithm.

(b) (i) Evaluate the value of k *if* $\sinh^{-1} 7 = k\sinh^{-1} 1$ where $k \in Z^{+}$

(ii) Given $y = \ln\left[\tan\left(\dfrac{\pi}{4} + \dfrac{x}{2}\right)\right]$, evaluate $\sinh y$

(c) Given that $y = \tanh^{-1}\left(3e^{2x}\right)$. Show that $\dfrac{dy}{dx} = \dfrac{-3}{5\cosh 2x + 4\sinh 2\mathrm{x}}$.

3. (a) The value of the objective function at these points are given in the following table

Corner Points	**VALUE :** $f(x,y) = ax + by + c$
$A(4500, 2500)$	$5550/=$
$B(4000, 3000)$	$5450/=$
$C(500, 3000)$	$4400/=$

Obtain the objective function used to minimize the cost.

(b) A merchant plans to sell two types of personal computers, a desktop model and a portablel model that will cost 25,000/= Tsh and 40,000/= Tsh respectively. He estimates that the total monthly demand of computers will not exceed 250 units. Determine the number of units of each types of computers which the merchant should stock to get maximum profit, if he does not want to invest more that 7,000,000 Tsh and if his profit on the desktop model is 45000 Tsh and portable model is 5,000 Tsh.

4. (a) The mean weight of 35 students is 45 kg. If the weight of the teacher is included , the mean weight increases by 500 grams. Find the weight of the teacher.

(b) An incomplete frequency distribution of 228 frequencies is given below.

Interval	10-19	20-29	30-39	40-49	50-59	60-69	70-79
Frequency	12	30	m	50	25	30	n

(i) If the median is 47.5, find the value of missing frequency m and n

(ii) Determine mean deviation from the median.

5. (a) Use the appropriate laws of algebra of set to simplify

$[A-(A\cap B')]\cap(A\cap B)$

(b) Two set A and B are said to be equivalent if $n(A)=n(B)$. If A, B and C are equivalent sets of which A and C are disjoint sets. Prove that

$$n(A\cap B\cup C)=3n(A)-n(A\cap B)-n(B\cap C)$$

(c) In a class of a certain school there are 15 girls who are like chocolate, 5 girls are white and 6 girls, who are intelligent. Every white girls like chocolate, 3 intelligent girls do not like chocolate. If two girls are both white and intelligent.

(i) Present the above information in the Venn diagram.

(ii) Find the number of girls in a class.

(iii) Find the number of girls who are white but not intelligent.

6. (a) If $f(x)=2x^2+x+4$ *and* $fog(x)=2x^2-8x+10$, find $g(x)$

(b) Sketch the graph of $f(x)=\dfrac{2x}{x^2-x-2}$,hence state domain and range.(a)

7. (a) Show that the roots of equation $2x-3\cos\left(\dfrac{x}{2}\right)=0$ lies between 1 and 2.

Hence find roots using Newton Raphson method with three iterations. Give your answer correct to two decimal places.

(b) Use Newton Raphson's to derive secant formula.

(c) Use both Trapezoidal and Simpson's rules with 11 ordinates to approximate the value of $\int_0^1 \frac{x}{(1+x^2)} dx$

8. (a) Find the value of C such that the line $x+y=c$ should be tangent to the circle $x^2+y^2-4x+5y+2=0$ for each value of c, find the coordinates the point of contact.

(b) Find the area of triangle whose vertices are the points with polar coordinates $(1,30^0),(2,60^0)$ and $(3,90^0)$

(c) Find the equations of the lines, which pass through the point $(4,\ 5)$ and make equal angles with the lines $5x-12y=-6$ *and* $3x-4y=7$.

9. (a) Prove that $\int e^{ax}\cos bx\ dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx)+C$.

Hence deduce $\int e^{-\frac{2}{5}x}\cos\left(\frac{3}{10}x\right)dx$

(b) Evaluate $\int_1^{10} x\log_{10}x\,dx$ leave your answer in terms of natural logarithms.

(c) Evaluate $\int \frac{1}{1+\sin^2 x}dx$ use $\tan x = t$

10. (a) Use the definition of differentiation to find $\frac{dy}{dx}$ of $y=\frac{1}{\sqrt{x+1}}$

(b) The parametric equations of a curve are $x=\cos t+t\sin t$, $y=\sin t-t\cos t$ show that $\frac{d^2y}{dx^2}=\frac{\sec^3 t}{t}$.

(c) Apply Taylor's theorem, expand $\tan\left(x+\frac{\pi}{4}\right)$ as far as the term x^4 and evaluate $\tan 46.5^0$ to four significant digits.