BASIC MATH - F IV

MIDTERM

MARKING GUIDE.

1 (a) Find the LCM of 6min, 8min, and 10min and 15min.

2	6	8	10	15
2	3	4	5	15
2	3	2	5	15
3	3	1	5	15
5	1	1	5	5
1	1	1	1	1
	1	1	1	1

(01).

$LCM = 2 \times 2 \times 2 \times 3 \times 1 \times 5 = 120$ minutes

$LCM = 120$ min $= 2$ hours. (01)

$8:00$ am $+ 2$ hours $= 10:00$ am

The time will ring together again at 10:00am (01)

(b) (i) Amount paid to each worker $= \dfrac{\text{Total amount}}{\text{Number of workers}}$ (01)

$= \dfrac{27,000,000}{200}$ (01)

$= 135,000$

$\therefore$ Each worker get 135,000 Tshs per day

(ii) The Constructor will not remain with money. (01)

2. (a) (i) Given $\log\sqrt{x} = \frac{1}{2}\log x = \sqrt{\log x}$ Squaring both sides (01)

$\left(\frac{1}{2}\log x\right)^2 = \left(\sqrt{\log x}\right)^2$

$\frac{1}{4}(\log x)(\log x) = \log x$

$\dfrac{\left(\frac{1}{4}\right)(\log x)(\log x)}{\log x} = \dfrac{\log x}{\log x}$ (01)

$\frac{1}{4}\log x = 1$

$\log_{10} x = 4$

$x = 10^4$

$x = 10000$ (01)

UBN COOPERATION

2 (a) (ii) Given $\log x^2 = (\log x)^2$

$$\frac{2\log x}{\cancel{\log x}} = \frac{(\cancel{\log x})(\log x)}{(\log x)} \quad 01$$

$$2 = \log x$$

$$\log_{10} x = 2$$

$$x = 10^2$$

$$x = 100 \quad 01$$

(b) $\frac{1}{2-\sqrt{3}} + \frac{1}{2+\sqrt{3}} = \frac{(2+\sqrt{3})+(2-\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})}$

$$= \frac{-\sqrt{3}+\sqrt{3}+4}{(2)^2-(\sqrt{3})^2} \quad 01$$

$$= \frac{\cancel{2+3}\ 4}{4-3}$$

$$= \frac{4}{1}$$

$\therefore$ Therefore the sum of $\frac{1}{2-\sqrt{3}}$ and $\frac{1}{2+\sqrt{3}}$ is 4 which is a rational number 01

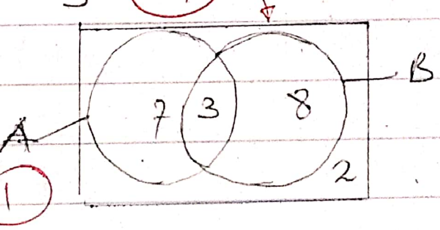
3 (a) $n(A) = 10$, $n(B) = 11$ and $A \cup B = 18$

We can find $n(A \cap B)$ from 01

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$ where ~~of~~

we will get $n(A \cap B) = 3$ 01

from Venn diagram



$n(A \cap B)' = 7+2+8 = 17$ 01

(b) let A : event that a student walk to school

B : The event that a student has blonde hair

We have $P(A) = 0.35$

$P(B) = 0.2$

$P(A \cup B) = 0.45$

3 (b) We need to find $P(A \cap B)$, which is the probability that a student has blonde hair and walks to school. We can use the formula for the union of two events.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \quad 01$$

$$0.45 = 0.35 + 0.2 - P(A \cap B)$$

$$P(A \cap B) = 0.55 - 0.45$$

$$P(A \cap B) = 0.10 \quad 01$$

$\therefore$ The probability that a student has blonde hair and walks to school is 0.1. 01

4 (a) Let $A = (0, y)$
$B = (x, 0)$

Since (2,3) is the midpoint, then we can find values of x and y as follows 0½

$$2 = \frac{x+0}{2}$$

$$x = 4 \quad 0½$$

$$3 = \frac{y+0}{2}$$

$$y = 6 \quad 0½$$

Therefore $A = (0, 4)$, $B = (6, 0)$

$$\text{Slope } (m) = \frac{4-0}{0-6} = -\frac{2}{3} \quad 0½$$

eqn of a line $-\frac{2}{3} = \frac{y-0}{x-6}$ 0½

$$3y = -2x + 12$$

$$y = \frac{-2x}{3} + 4 \quad 0½$$

(b) Let $(x, y) = (6, 8)$

Then magnitude of vector $\underline{a}$ is given by

$$|\underline{a}| = \sqrt{x^2 + y^2}$$

$$|\underline{a}| = \sqrt{6^2 + 8^2}$$

$$|\underline{a}| = \sqrt{100} \quad 01$$

$$\therefore |\underline{a}| = 10.$$

UBH COOPERATION

Suppose $\underline{a}$ makes angle α with the positive X-axis and β with the positive y-axis, then the direction Cosines are given by

$$\cos\alpha = \frac{x}{|\underline{a}|} \quad \text{and} \quad \cos\beta = \frac{y}{|\underline{a}|}$$

$$\cos\alpha = \frac{6}{10} \quad (01) \qquad \cos\beta = \frac{8}{10} \quad (01)$$

$$\cos\alpha = 3/5 \qquad \cos\beta = \frac{4}{5}$$

Therefore direction Cosines of $\underline{a}$ are $3/5$ and $4/5$

5 ⓐ Given the area $A = 720\,m^2$ and $n = 6$

Recall formula for area of a regular polygon

$$A = \frac{1}{2} n r^2 \sin\left(\frac{360}{n}\right) \quad (01)$$

$$720\,m^2 = \frac{1}{2} \times 6 \times r^2 \times \sin\left(\frac{360°}{6}\right)$$

$$720\,m^2 = 3r^2 \sin 60°$$

$$r^2 = \frac{720\,m^2}{3 \times 0.866} \quad (01)$$

$$r^2 = 277.14\,m^2$$

$$r = 16.65\,m \quad (01)$$

ⓑ Given : BD is an angle bisector of $\angle CDA$, $\angle C \cong \angle A$

To prove $\triangle CBD \cong \triangle ABD$

Solution

4BM COOPERATION

Statement	Reason
1. $\angle C \cong \angle A$	Given (03)
2. $\angle CBD \cong \angle ADB$	Definition of an angle bisector
3. $DB \cong DB$	Reflexive property
4 $\triangle CBD \cong \triangle ABD$	AAS

06. let w = number of workers
h = number of hours
d = Number of days
from the equestion,
$w \propto 1/h$ and $w \propto 1/d$ (01)
Joint Variation is given by the relation $w \propto \frac{1}{hd}$ (01)
Variation equation is

$$W = \frac{k}{hd}$$ (01)

Thus $$W_1 = \frac{k}{h_1 d_1}$$

$$W_2 = \frac{k}{h_2 d_2}$$

$$\frac{W_1}{W_2} = \frac{h_2 d_2}{h_1 d_1}$$ (01)

Given $d_1 = 52$ days, $h_1 = 8$ hours, $W_1 = 9$ workers
$d_2 = ?$, If $h_2 = 6$ hours, $W_2 = 13$ workers

$$\frac{9}{13} = \frac{6}{8} \times \frac{d_2}{52}$$ (01)

$d_2 = 48$ days.
∴ It will take 48 days for 13 workers to complete the piece of work. (01)
UBN COOPERATION

7 (a) let the original price be t, a reduction of 5% on t = 0.95t (01)

Thus $0.95t = 6650$

$$t = \frac{6650}{0.95} = 7000/=$$ (01)

Therefore, the original price was 7000/=

(b) Balance sheet as at Dec 2005

LIABILITIES		AMOUNT	ASSETS		AMOUNT
Capital	70,000		Intangible assets		
Net loss	5,000		Goodwill		5000
	65,000				
Drawing	5,000	60000	Fixed assets		
Long term liability			Building	45,000	
Loan from bank		20,000	motor vehicle	25,000	70000
Short term liability			CURRENT ASSETS		
Creditor		10,000	closing stock	5,000	
		(01)	Debtors	2,500	(01)
			Bank	6,000	
(01)			Cash	1,500	15,000
		90,000	(01)		90,000

8 (a)(i) For arithmetic progression

$d = A_2 - A_1 = A_3 - A_2 = A_4 - A_3$ (0½)

Given $A_1 = 3$, $A_2 = x$, $A_3 = y$, $A_4 = -24$

$d = x - 3 = y - x = -24 - y$

$x - 3 = y - x$

$2x - y = 3$ —— (i) (0½)

and $y - x = -24 - y$

$2y - x = -24$ —— (ii)

Solve simultaneously to get $x = -6$

and $y = -15$ (0½)

UBN COOPERATION

2 (a) (ii) For Geometric Progression

$G_1 = 3, G_2 = x, G_3 = y, G_4 = -24$

$r = \frac{G_2}{G_1} = \frac{G_3}{G_2} = \frac{G_4}{G_3}$

$\frac{x}{3} = \frac{y}{x} = \frac{-24}{y}$ (01/2)

$\frac{x}{3} = \frac{y}{x} \rightarrow x^2 = 3y$ — (i)

$\frac{y}{x} = \frac{-24}{y} \rightarrow y^2 = -24x$ — (ii)

Take $(x^2)^2 = (3y)^2$.

$x^4 = 9y^2$.

$x^4 = 9(-24x)$

$x^3 = 9(-24)$

$x^3 = -216$

$x = \sqrt[3]{-216} = -6$ (01/2)

$y = \frac{x^2}{3} = \frac{(-6)^2}{3} = 12$.

$\therefore x = -6, y = 12$ (01/2)

(b) Let the total number of rows be, n If a = first term

d = Common difference.

we have $a_n = a + (n-1)d$ starting from the bottom row, the number of logs decrease by 2, thus $d = -2$

$a = 60$ (Number of Logs on first Layer) and $a_n = 2$.

To find the number of rows, we use the above equation, thus. (01)

UBM COOPERATION

8 (b) $2 = 60 + (n-1)(-2)$ (01)

$n = 30.$

Total number of logs

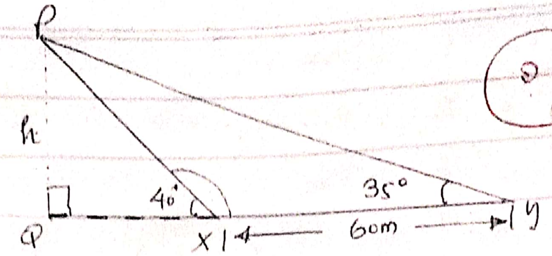
$S_n = \frac{n}{2}(2a + (n-1)d)$

$S_{30} = \frac{30}{2}[2(60) + (30-1)(-2)]$

$S_{30} = 930$ (01)

MBM COOPERATION

09. (a) (i)



01/2

$P\hat{X}Y = 180° - 40° = 140°$

Hence $X\hat{P}Y = 180° - (140° + 35°) = 5°$

In $\Delta\ PXY$

$$\frac{\overline{PY}}{\sin 140°} = \frac{\overline{XY}}{\sin 5°}$$ 01/2

$$\overline{PY} = \frac{\overline{XY} \sin 140°}{\sin 5°}$$

$$= \frac{60m \sin 140°}{\sin 5°}$$

$\therefore\ \overline{PY} = 442.5m$ 01/2

(ii) Using a right angled triangle PQY

where $\hat{Y} = 35°$, $\overline{PY} = 442.5m$ 01/2

$$\frac{\overline{PQ}}{\sin 35°} = \frac{442.5m}{\sin 90°}$$ 01/2

Then $\overline{PQ} = \frac{442.5m \times \sin 35°}{\sin 90°}$ 01/2

Therefore, the Height of the Kite is 253.8m

(b) Given $\sin x = \sin 2x$

but $\sin 2x = 2\sin x \cos x = \sin x$. 01

Therefore $\sin x - 2\sin x \cos x = 0$

$\sin x(1 - 2\cos x) = 0$

either $\sin x = 0$, where $x = 0°, 180°, 360°$ 01/2

oR $1 - 2\cos x = 0$, $\cos x = \frac{1}{2}$, where

$x = 60°, 300°$ 01/2

$\therefore$ The truth set is $\{0°, 60°, 180°, 300°, 360°\}$ 01

UBM COOPERATION

10 (a) $4x^2 + Q + 9y^2$

For a perfect square

$b^2 = 4ac$

$Q^2 = 4(9y^2)$

$Q^2 = (2\times3y)^2$ (02)

$Q = 6y$.

(b) (i) from the graph.

$x = 2$ and $x = 5$

$x - 2 = 0$ $x - 5 = 0$

$(x-2)(x-5) = 0$

$x^2 - 5x - 2x + 10 = 0$

$x^2 - 7x + 10 = 0$ (02)

(ii) factors of $x^2 - 7x + 10 = 0$ are $x - 2 = 0$ and $x - 5 = 0$

Therefore $x^2 - 7x + 10 = (x-2)(x-5)$ (02)

SECTION B

11 (a) (i) Frequency distribution table

Scores	Class Marks	Frequency	Cummulative frequency	fx
39-49	42	14	14	588
50-60	55	20	33	1100
61-71	66	10	44	660
72-82	77	5	49	385
83-93 (0½)	88 (0½)	1	50	88
Total		50 (01)		2821

(01)

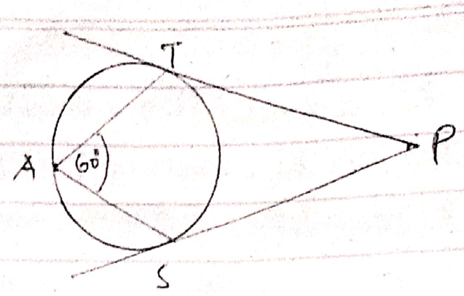
(ii) Mean $(\bar{x}) = \frac{\Sigma fx}{N}$ (01)

$\bar{x} = \frac{2821}{50} = 56.42$

$\therefore$ The mean score was 56.42 (01)

UBN COOPRATION

11 (b)



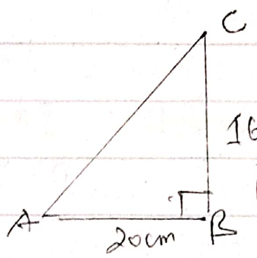
$m\angle TPS = \frac{1}{2}\left[m(\widehat{TAS}) - m(\widehat{TS})\right]$ (01)

$m(\text{arc } TS) = 2\times 60° = 120°$ (01)

$m(\text{arc } TAS) = 360° - 120° = 240°$ (01)

$\therefore m\angle TPS = \frac{1}{2}(240° - 120°) = 60°$ (02)

12 (a) Consider a right angled triangle ACG as shown in figure below. To find $\overline{AG}$, first find $\overline{AC}$ from ΔABC



(01/2)

Pythagoras theorem

$\overline{AC}^2 = \overline{AB}^2 + \overline{BC}^2$

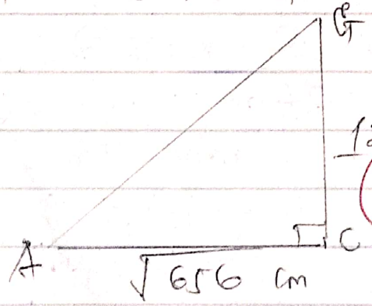
$= (20\,cm)^2 + (16\,cm)^2$

$\overline{AC}^2 = 656\,cm^2$

$\overline{AC} = \sqrt{656\,cm^2}$

$\overline{AC} = \sqrt{656}\,cm$ (01)

from ΔACG By using Pythagoras theorem



(01/2)

$\overline{AG}^2 = \overline{AC}^2 + \overline{CG}^2$

$\overline{AG}^2 = (\sqrt{656}\,cm)^2 + (12\,cm)^2$

$\overline{AG} = \sqrt{800\,cm^2}$

$\overline{AG} = 28.28\,cm.$

$\therefore$ The length of $\overline{AG}$ is 28.8 cm. (01)

UBN COOPERATION

12 (a) (ii) The projection of $\overline{AG}$ on the plane ABCD is $\overline{AC}$. The angle between $\overline{AG}$ and the plane ABCD is $C\hat{A}G$. Calculation of $C\hat{A}G$ is done as follows.

since $\overline{AG} = 28.3$ cm, then

$\sin(C\hat{A}G) = \dfrac{\overline{CG}}{\overline{AG}}$, from ΔACG (01)

$\sin(C\hat{A}G) = \dfrac{12\text{ cm}}{28.3\text{ cm}} = 0.4243$

$C\hat{A}G = \sin^{-1}(0.4243)$

$C\hat{A}G = 25°6'$

$\therefore$ The angle $\overline{AG}$ makes with the plane ABCD is $25°6'$. (01)

(b) An aeroplane flies north, hence its longitude is unchanged. It starts south of the equator, and flies north. It may cross the equator and so end up north of the equator. Suppose the plane flies along θ latitudes.

length of an arc $= \dfrac{\theta}{360°} \times 2\pi R$, $\theta = x$ (01)

length of an arc $= 4000$ km

$4000 = \dfrac{2 \times 3.14 \times 6370 \times \theta}{360°}$ (01)

$\theta = \dfrac{4000 \times 360°}{2 \times 3.14 \times 6370}$

$\theta = 35.99° \approx 36°$ (01)

since $36° > 20°$, then its new position is at north of the equator.

12 (b) that is $(yN, 30°E)$, therefore

$$y + 20° = 36°$$
$$y = 36° - 20° \quad 01$$
$$y = 16°$$

Hence new latitude is $16°N$.

Therefore, the new position is $(16°N, 30°E)$ 01

13 (a) Matrix $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ is used to

perform the reflection of any point (x,y) about the line $y = -x$,

Thus the image of (x,y) is $(-y,-x)$

from $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -y \\ -x \end{pmatrix}$

choose any two points from the line $y = 2x - 1$ say $(1,1)$ and $(2,3)$. Then find the image of these points when reflected about the line $y = -x$, images are $(-1,-1)$ and $(-3,-2)$

equation of the image of the line will be: $m\ (\text{slope}) = \dfrac{-2-(-1)}{-3-(-1)} = \dfrac{1}{2}$

$$\frac{1}{2} = \frac{y-(-1)}{x-(-1)}$$

$$2y = x + 1$$

$$y = \frac{x}{2} + \frac{1}{2}$$

UBN COOPERATION

13 (b) let the Present ages of father and his son be x and y respectively. Thus

$$\frac{x}{y} = \frac{8}{3} \rightarrow 3x = 8y$$

$$3x - 8y = 0 \quad \text{--- (i)}$$

Ten years ago

$$\frac{x-10}{y-10} = \frac{6}{1}$$

$$x - 10 = 6y - 60$$

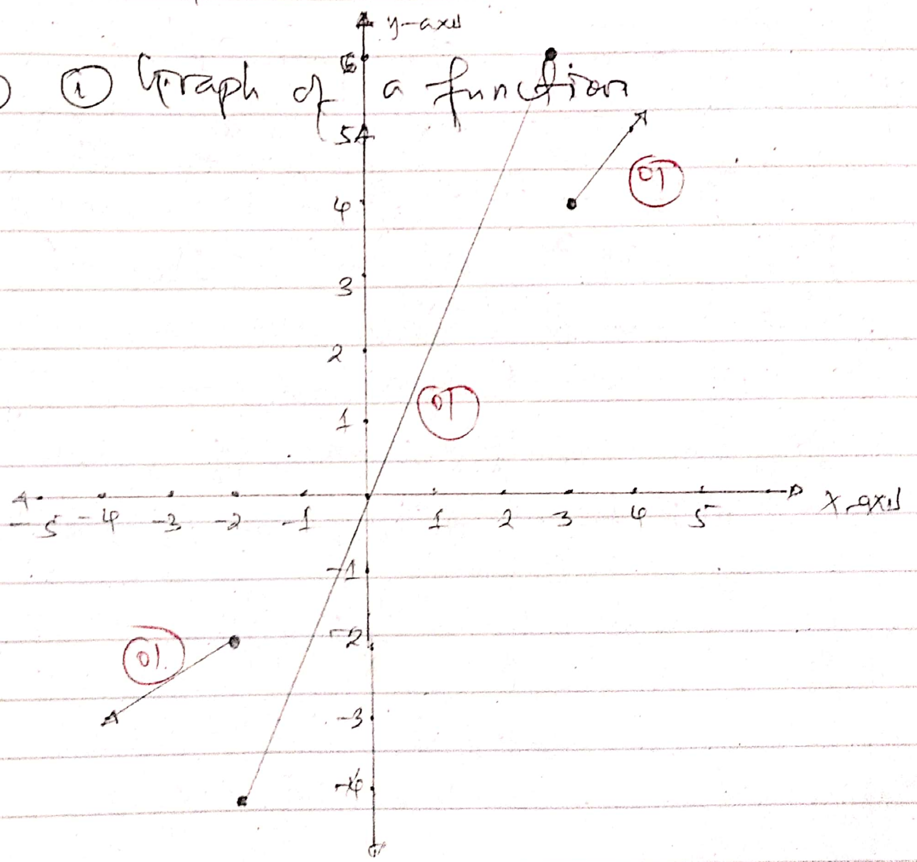
$$x = 6y - 50$$

$$x - 6y = -50 \quad \text{--- (ii)}$$

Solve simultaneous equation and get $x = 40$ and $y = 15$

∴ The fathers age is 40 years and son is 15

14 (a) (i) Graph of a function



4BN COOPERATION

14 (a) (ii) Domain = $x : x \in R$ (01)
Range = $y : y \in R$ (01)

(b) (i) Given $y = \frac{1}{x-1}$

Domain let $x-1 \neq 0$, $x \neq 1$.

Domain = $\{x : x \in R, x \neq 1\}$ (01)

(ii) Range: make x the subject

$\frac{1}{y} = x-1$

$x = \frac{1}{y} + 1$

let $y \neq 0$

Range = $\{y : y \in R, y \neq 0\}$ (01)

(iii) Inverse

Take $y = \frac{1}{x-1}$

$x = \frac{1}{y-1}$

$\frac{1}{x} = y-1$

$y = \frac{1}{x} + 1$

$R^{-1} = \{(x, y) : y = \frac{1}{x} + 1\}$ (01)

(iv) Domain of R^{-1}

let $x \neq 0$

Domain = $\{x : x \in R, x \neq 0\}$ (01)

(v) Range of R^{-1} $y = \frac{1}{x} + 1$

$y-1 = \frac{1}{x}$

$\frac{1}{y-1} = x$

let $y-1 \neq 0$, $y \neq 1$

Range = $y : \{y \in R, y \neq 1\}$ (01)

UBM COOPERATION