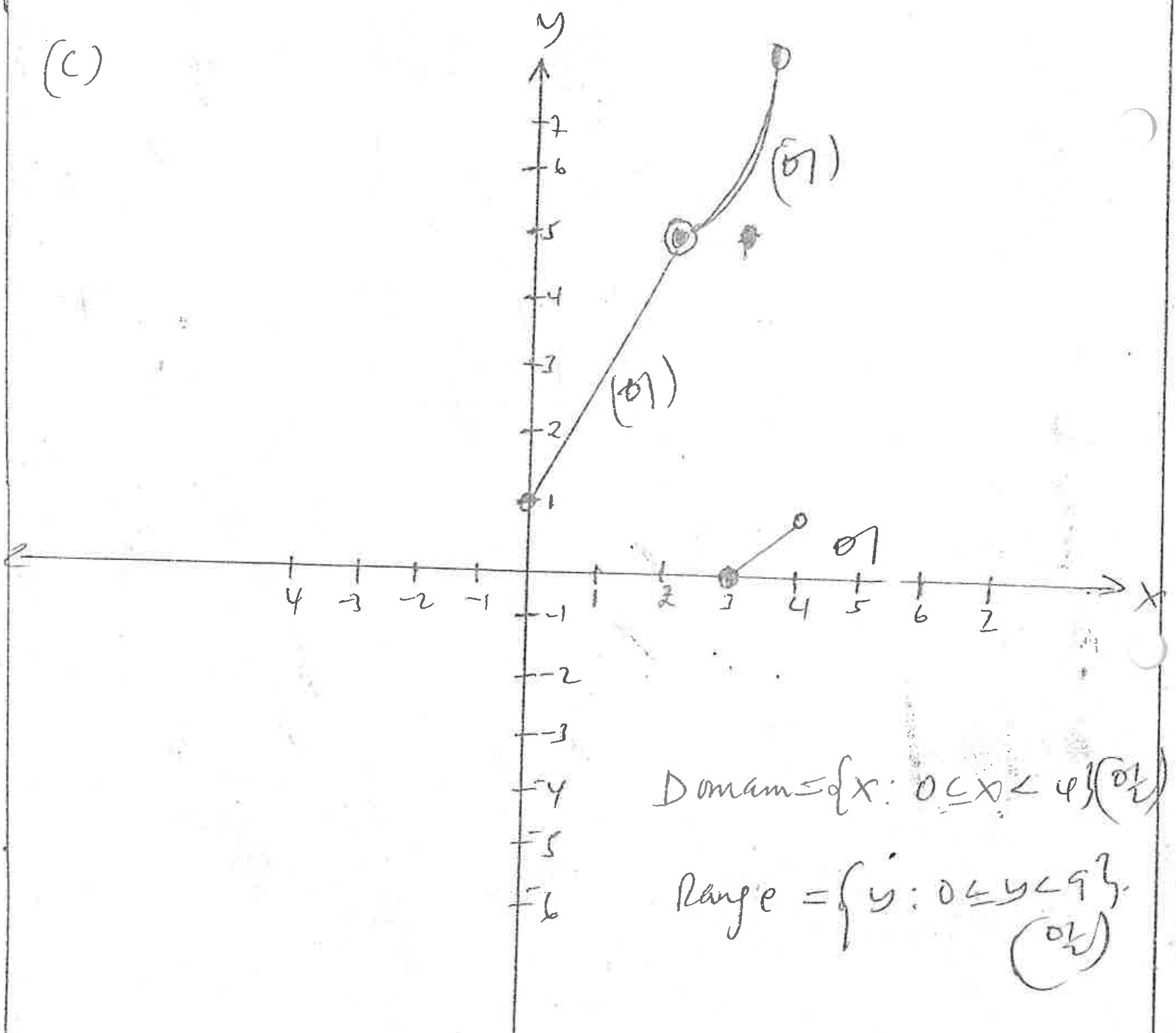
2 (b) turning point is (h, k)

$\therefore$ turning point is $\left(\frac{-5}{2}, \frac{3}{4}\right)$ (01)

its minimum point since $a > 0$ $(0\frac{1}{2})$

(c)



Domain $= \{x : 0 \leq x < 4\}$ $(0\frac{1}{2})$

Range $= \{y : 0 \leq y < 9\}$ $(0\frac{1}{2})$

2 (a) let $f(x) = ax^2 + bx + c$

$f(2) = 8$

$4a + 2b + c = 8$ --(i) $(0\frac{1}{2})$

$f(-1) = 3$

$a - b + c = 3$ ---(ii) $(0\frac{1}{2})$

$f(3) = 13$

$9a + 3b + c = 13$ ---(iii) $(0\frac{1}{2})$

$4a + 2b + c = 8$

$a - b + c = 3$ $(0\frac{1}{2})$

$9a + 3b + c = 13$

$a = 0.83$ $b = 0.83$ $c = 3$ $(0\frac{1}{2})$

$\therefore f(x) = 0.83x^2 + 0.83x + 3$ $(0\frac{1}{2})$

(b) $f(x) = x^2 + 5x + 7$

$f(x) = x^2 + 5x + \left(\frac{5}{2}\right)^2 + \frac{3}{4}$ (0)

comparing with general form

$f(x) = a(x+h)^2 + k$ $(0\frac{1}{2})$

$a = 1$ $h = -\frac{5}{2}$ $k = \frac{3}{4}$.

1 (a) $X = -6.7$
$y = 1.79$ (1 mark @)
$Z = 4$

(b) 555.32. (2 marks)

(c) (i) $\overline{X} = 6.2$

(ii) $Var(X) = 5.189$

(iii) 9306 (1 marks @)

(iv) $\sum X^2 = 6550.575$.

PRESIDENTS OFFICE

REGIONAL ADMINISTRATION AND
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DAR ES SALAAM REGION.

FORM SIX MOCK EXAMINATION

BASIC APPLIED MATHEMATICS (BAM)

MARKING GUIDE

3 (b) $3x - y = 9 \quad --(i)$

$x^2 + xy + 2 = 0 \quad -(ii)$

$y = 3x - 9$ from (1)

Substitute into $x^2 + xy + 2 = 0$

$$x^2 + x(3x - 9) + 2 = 0$$

$$x^2 + 3x^2 - 9x + 2 = 0$$

$$4x^2 - 9x + 2 = 0 \quad \text{(01 marks)}$$

$$x = 2 \quad \text{or} \quad x = \frac{1}{4} \quad 0\tfrac{1}{2}$$

when $x = 2$

$$y = 3(2) - 9$$

$$y = -3 \quad 0\tfrac{1}{2}$$

when $x = \frac{1}{4}$

$$y = 3\left(\tfrac{1}{4}\right) - 9 = 7\tfrac{1}{4}$$

$$(x, y) = (2, -3) \text{ and } \left(\tfrac{1}{4}, \tfrac{29}{4}\right) \quad (01)$$

3. (c) (i) $1 + 1.543 + 3.76$ $\quad 01\tfrac{1}{2}$

(ii) 6.303 $\quad 01\tfrac{1}{2}$

3 (a) $G_n = G_1 r^{n-1}$

$G_3 = G_1 r^2$

$G_5 = G_1 r^4$ (0½)

$G_3 + G_5 = 90$

$G_1 r^2 + G_1 r^4 = 90$ (0½)

$G_1(r^2 + r^4) = 90$

but $G_1 = 1$

$r^4 + r^2 = 90$

let $r^2 = m$

$m^2 + m = 90$

$m = 9$ ~~or~~

$r^2 = 9$

$r = \pm 3$ (01)

(ii) $S_n = \frac{G_1(r^n - 1)}{r-1}$ (1 marks) $G_1 = 1$, $r = 3$, $n = 4$

$S_4 = \frac{3^4 - 1}{(4)-1} = 26.7$ (1 marks)

$\frac{81-1}{2} = 2$

21

1. (a)

(b) 1066.8 (04)

(c) (i) $\bar{x} = 6.2$

$V = 5.189$

(ii) $\sum x = 930.5$

(iv) $\sum x^2 = 6550.575$

$01\frac{1}{2}$

Penelope 02

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5 (a) (i) $\int \frac{x}{(\sqrt{1-x})^3}\, dx$.

let $u = \sqrt{1-x}$

$u^2 = 1-x$

$2u\,du = -dx$ $\left(0\frac{1}{2} \text{ marks}\right)$

$x = u^2 - 1$

$= \int \frac{u^2-1}{u^3} \cdot -2u\, du$

$= -2\int \frac{u^2-1}{u^2}\, du$ $\left(0\frac{1}{2}\right)$

$= -2\int (1-u^{-2})\, du$

$= -2\left(u - \frac{u^{-1}}{-1}\right) + c$

$= -2\left(u + \frac{1}{u}\right) + c$ $\left(0\frac{1}{2} \text{ marks}\right)$

$= \left[-2\left(\sqrt{1-x} + \frac{1}{\sqrt{1-x}}\right) + c\right]_0^{\frac{1}{2}}$

$= -4 + \frac{1}{2}$

$= \underline{\underline{-3\frac{1}{2}}} = -\frac{7}{2}$ $\left(0\frac{1}{2}\right)$

(ii) $\int x^3 \sqrt{x^4+5}\, dx$

let $u = x^4 + 5$

$du = 4x^3 dx$

$dx = \dfrac{du}{4x^3}$ $\quad 0\frac{1}{2}$

$\int x^3 \cdot u^{\frac{1}{2}} \cdot \dfrac{du}{4x^3}$ $\quad 0\frac{1}{2}$

$$\frac{1}{4}\int u^{\frac{1}{2}}\, du = \frac{1}{4}\,\frac{u^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$= \frac{2}{12}\, u^{\frac{3}{2}} + c$$

(01 mark)

$$= \frac{1}{6}\left(x^4+5\right)^{\frac{3}{2}} + c$$

5 (b) Area enclosed between $y = x^2+2$ and $y = 2x+2$

$y = x^2 + 2$ (table as written: $y = 2x+2$)

x	-3	-2	-1	0	1	2
y	11	6	3	2	3	6

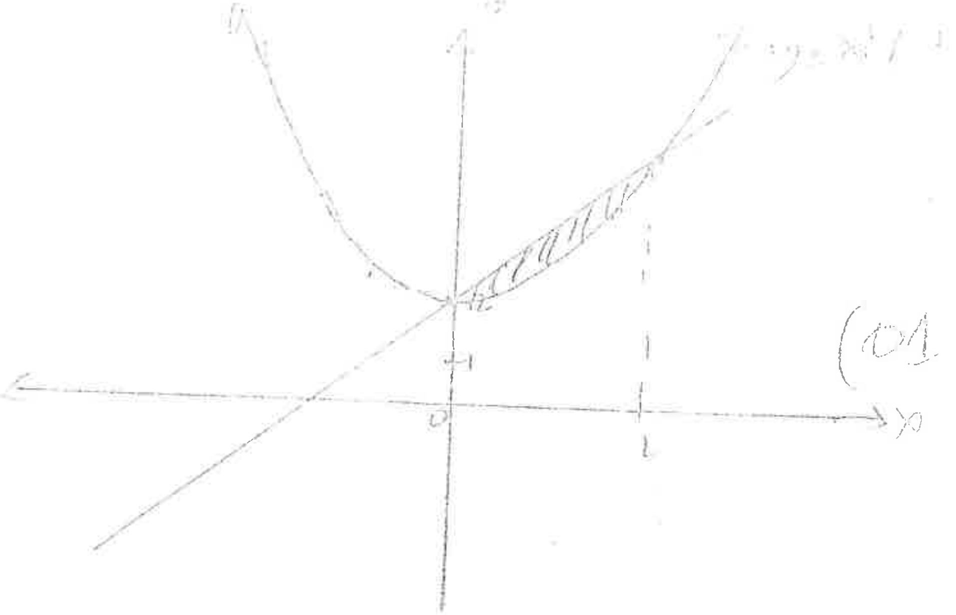
$y = 2x+2$

x	-3	-2	-1	0	1
y	-4	-2	0	2	4

$y = 2x+2$ and $y = x^2+2$

$2x + 2 = x^2 + 2$

$x^2 = 2x \qquad x = 0,\ x = 2$



(01 mark)

$$A = \int_a^b (y_1 - y_2)\, dx$$

$$A = \int_0^2 \left((2x+2) - (x^2+2)\right) dx$$

$$A = \int_0^2 (2x - x^2)\, dx \quad (01)$$

$$A = \left[x^2 - \frac{x^3}{3}\right]_0^2 = \left(\frac{8}{3} - 3\right)$$

(01)

A = 0.3 Square unit

5 (c) If $\int_{-2}^{2}(-x^2+m)\,dx = \frac{32}{3}$

$$\left[\frac{-x^3}{3} + mx\right]_{-2}^{2} = \frac{32}{3} \quad (01\text{ marks})$$

$$\left(\frac{-8}{3} + 2m\right) - \left(\frac{8}{3} - 2m\right) = \frac{32}{3} \quad (01\text{ marks})$$

$$\frac{-16}{3} + 4m = \frac{32}{3}$$

$$4m = \frac{32}{3} + \frac{16}{3}$$

$$4m = \frac{48}{3}$$

$$4m = 16$$

$$m = 4 \quad (01\text{ marks})$$

6 (a) 3, 5, 7, 8, 12, 13, 14, 18, 21

$N = 9$

$Q_1 = \left(\frac{N+1}{4}\right)^{th}$ value (0½ marks)

$Q_1 = \left(\frac{10}{4}\right)^{th} = 2.5^{th}$ value

$Q_1 = \frac{5+7}{2} = 6$ (0½ marks)

$Q_3 = \frac{3}{4}(N+1)^{th}$ value

$= \frac{3}{4}(10)^{th}$ value

$= 7.5^{th}$ value (½ marks)

$= \frac{14+18}{2}$

$Q_3 = 16$ (0½ marks)

(b) Table $A = 25.5$

Class Interval	f	X	d=X−A	u=d/c	fu	fu²
					0	0
1 – 10	0	5.5	−20	−2		
					−9	9
11 – 20	9	15.5	−10	−1		
21 – 30	03	25.5	0	0	0	0
31 – 40	06	35.5	10	1	6	6
41 – 50	03	45.5	20	2	6	12
Total	21	.	.	.	03	27

(3 marks (0½ @)

(i) $\bar{x} = A + c\frac{\Sigma fu}{\Sigma f}$

$\bar{x} = 25.5 + 10\left(\frac{3}{21}\right)$

$\bar{x} = 26.92857$ ($1\frac{1}{2}$ marks)

(ii) $\text{mode} = L + \left(\frac{t_1}{t_1 + t_2}\right) c$

$= 10.5 + \left(\frac{9}{9+6}\right) 10$

$\text{Mode} = 16.5$ ($1\frac{1}{2}$ marks)

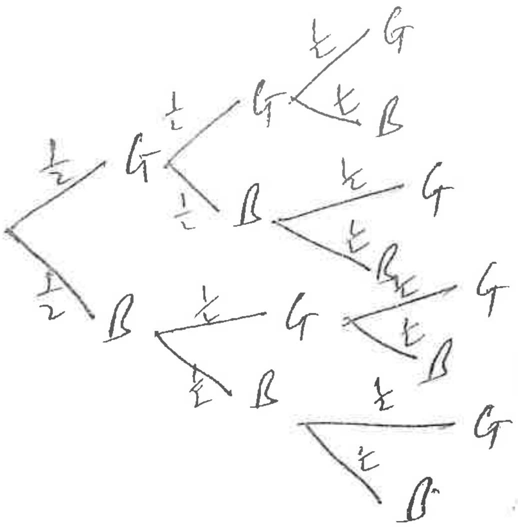
(iii) $\text{Standard deviation} = c\sqrt{\frac{\Sigma fu^2}{\Sigma f} - \left(\frac{\Sigma fu}{\Sigma f}\right)^2}$

$= 10\sqrt{\frac{27}{21} - \left(\frac{3}{21}\right)^2}$

$= 10\sqrt{1.0817}$

$\sigma = \underline{10.4}$ (2 marks)

7 (a)



(01 marks)

Probability that second child is a boy if the first is girl:

$= P(GBG) + P(GBB)$ (01 marks)

$= P(G) \times P(B) \times P(G) + P(G) \times P(B) \times P(B)$

$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

$= \frac{1}{4}$ ($0\frac{1}{2}$ marks)

(b) $n = 25$

$n(E) = {}^{15}C_3 \times {}^{10}C_2$ ($0\frac{1}{2}$ marks)

$n(S) = {}^{25}C_5$ ($0\frac{1}{2}$ marks)

$P(E) = \frac{nE}{n(S)} = \frac{{}^{15}C_3 \times {}^{10}C_2}{{}^{25}C_5}$ (01 marks)

$P(E) = 0.385375$ ($0\frac{1}{2}$ marks)

7 (c) (i) For independent event $P(A \cap B) = P(A) \times P(B)$.

$$P(A) \times P(B) = 0.3 \times 0.5$$

$$P(A) \times P(B) = 0.15 \quad (01 \text{ marks})$$

but $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$0.65 = 0.3 + 0.5 - P(A \cap B)$$

$$P(A \cap B) = 0.8 - 0.65$$

$$P(A \cap B) = 0.15 \quad (01 \text{ marks})$$

Since $P(A \cap B) = P(A) \times P(B)$ then event A and B are independent events (01 marks)

(ii)

Position	1	2	3	4
Possibilities	5	7	6	5

(01 marks)

$$= 5 \times 7 \times 6 \times 5$$

$$= 1050 \quad (01 \text{ marks})$$

8 (a) $\tan^2\theta - \sec\theta = 0$

But $\sec^2\theta = 1 + \tan^2\theta$ (01 marks)

$\tan^2\theta = \sec^2\theta - 1$

$\sec^2\theta - 1 - \sec\theta = 0$

$\sec^2\theta - \sec\theta - 1 = 0$ (01 marks)

$\sec\theta = -0.618$ and $\sec\theta = 1.618$

$\sec\theta = \cancel{0}.618$

$\cos\theta = -1.618$ $\quad\quad$ $\cos\theta = 0.\cancel{76}618$

$\underline{\underline{\theta = 51.82}}$ (01 marks)

(b) To prove that

$\cos 3x = 4\cos^3 x - 3\cos x$

Consider L.H.S

$\cos 3x = \cos(2x + x)$ (01 marks)

$= \cos 2x \cos x - \sin 2x \sin x$

$= (\cos^2 x - \sin^2 x)\cos x - 2\sin x \cos x \sin x$

8 $\cos^3 x - \sin^2 x \cos x - 2\cos x(1-\cos^2 x)$

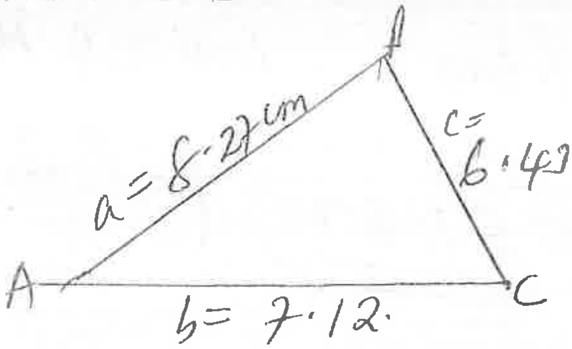
$\cos^3 x - \cos x(1-\cos^2 x) - 2\cos x + 2\cos^3 x$ (01 marks)

$\cos^3 x - \cos x + \cos^3 x - 2\cos x + 2\cos^3 x$

$4\cos^3 x - 3\cos x$.

$\therefore \cos 3x = 4\cos^3 x - 3\cos x$ (1 marks)

(c) Given Δ ABC



(01 marks)

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos A = \frac{7.12^2 + 6.43^2 - 8.27^2}{2(7.12) \times 6.43}$$ (01 marks)

$$A = \cos^{-1}(0.2582) = \underline{75.04}$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$B = \cos^{-1}\left(\frac{8.27^2 + 6.43 - 7.12}{2(8.27 \times 6.43)}\right)$$

$$B = \cos^{-1}(\cancel{56.4}\ 0.5548) = 56.3$$ (01 marks)

for $\hat{C}$

$$\hat{A} + \hat{B} + \hat{C} = 180$$

$$56.3 + 75.04 + C = 180$$

$$C = 180 - 56.3 - 75.04$$

$$C = 75.04$$ (01 marks)

9 (a) To solve the eqn:

$6 + 3^x = 3^{2x}$.

let $m = 3^x$

$m^2 = 6 + m$ (1 marks) since $3^{2x} = (3^x)^2$

$m^2 - m - 6 = 0$

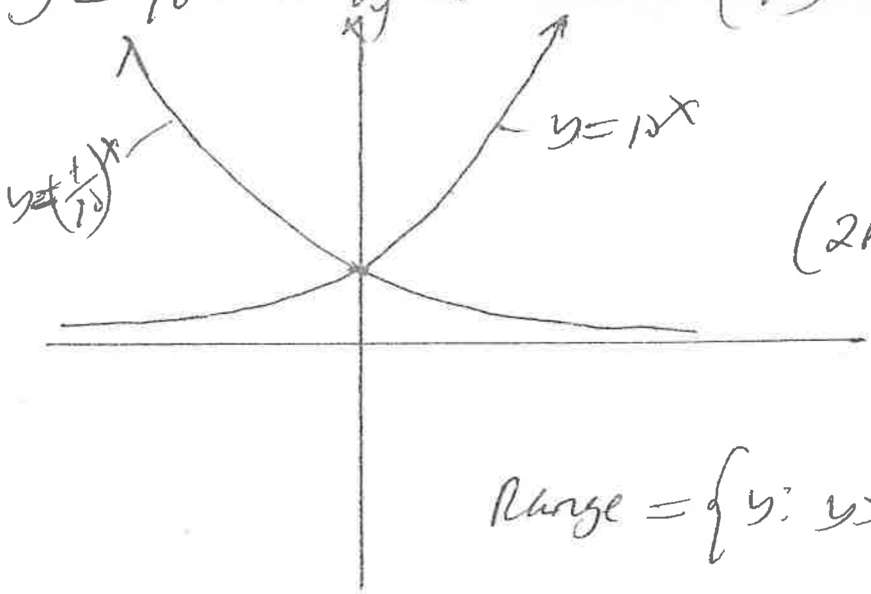
$m = 3$ or $m = -2$ (1 marks)

$3^x = 3$

$x = 1$ (1 marks)

when $3^x = -2$ (does not exis

(b) If $y = 10^x$ and $y = \left(\frac{1}{10}\right)^x$



(2 marks)

Range $= \{y: y > 0\}$ (01 marks)

(i) If $y = f(x) = 10^{2x}$ show that

$$\frac{dy}{dx} = 10^{2x} \ln 100$$

Soln:

$$y = 10^{2x}$$

$$\ln y = 2x \ln 10 \quad \text{(01 marks)}$$

$$\frac{d}{dx}(\ln y) = \frac{d}{dx}(2x \ln 10)$$

$$\frac{1}{y}\frac{dy}{dx} = 2 \ln 10 \quad \text{(1 marks)}$$

$$\frac{1}{y}\frac{dy}{dx} = \ln 10^2$$

$$\frac{dy}{dx} = y \ln 100 \quad \text{(01 marks)}$$

(01 marks)

$$\frac{dy}{dx} = 10^{2x} \ln 100$$ hence shown.

10 (a) We need to show that $A^2 - 5A = 14I$

Consider L.H.S

$$A^2 - 5A = \begin{pmatrix} 3 & -5 \\ -4 & 2 \end{pmatrix}\begin{pmatrix} 3 & -5 \\ -4 & 2 \end{pmatrix} - 5\begin{pmatrix} 3 & -5 \\ -4 & 2 \end{pmatrix}$$

(1 marks)

$$= \begin{pmatrix} 9+20 & -15-10 \\ -12-8 & 20+4 \end{pmatrix} - \begin{pmatrix} 15 & -25 \\ -20 & 10 \end{pmatrix}$$

$$= \begin{pmatrix} 14 & 0 \\ 0 & 14 \end{pmatrix}$$ (1 marks)

$$= 14\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

but $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$\therefore$ $\underline{A^2 - 5A = 14I}$ (1 marks)

(b) Let x be number of ordinary curtains
y be number of deluxe curtains $(0\frac{1}{2}$ marks)

	Time in (hrs)	material (m)	Profit (Tsh)
ordinary	1.5	12	18,500
deluxe	3	14	25,500
Total available	30	180	

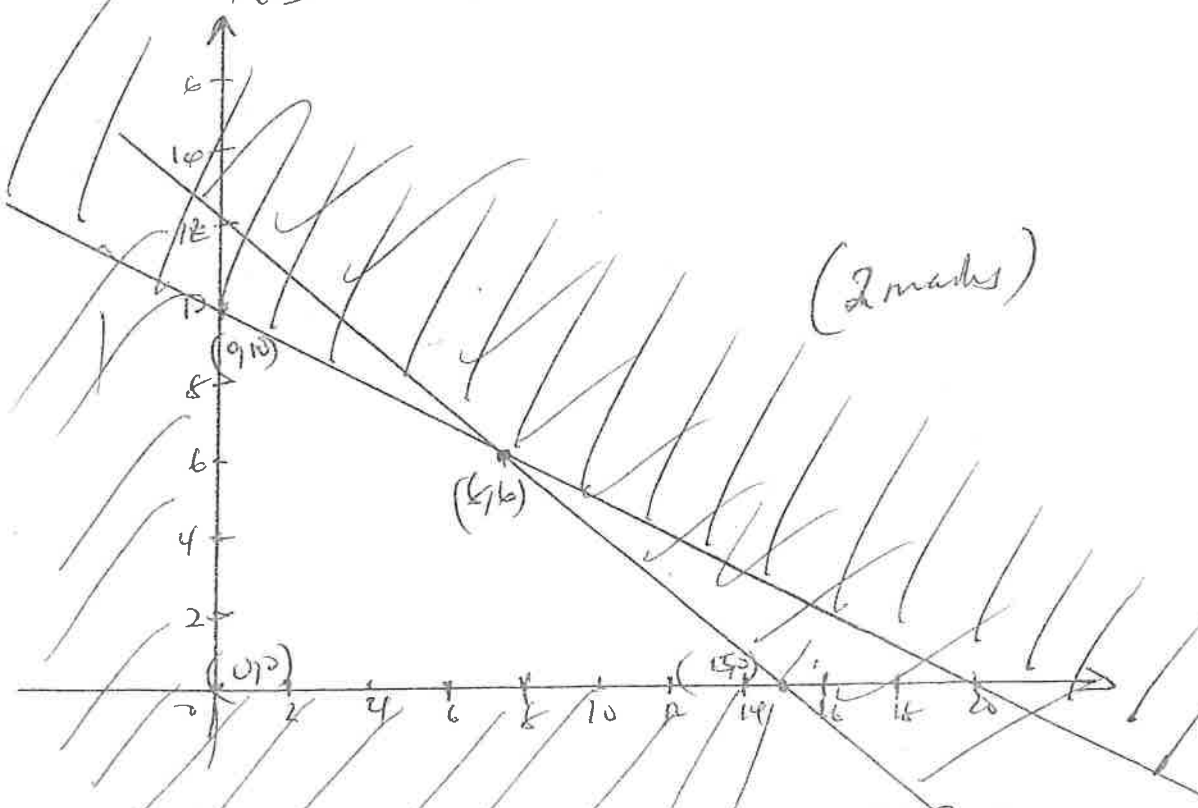
Objective function $f(x) = 18500x + 25,500y$
$(0\frac{1}{2}$ marks).

Constraints

$1.5x + 3y \leq 30$
$x + 2y \leq 20$ (if divided by $1\frac{1}{2}$)

$12x + 14y \leq 180$ (2 marks)

$x \geq 0 \quad y \geq 0$



(2 marks)

Corner points	$f(x,y) = 18500x + 25500y$
$O(0,0)$	0
$A(0,10)$	255,000/=
$B(8,6)$	301,000/=
$C(15,0)$	277,500

(01 marks)

Hence the maximum profit is Tsh 301,000, Therefore, the firm should make 8 ordinary curtains and 6 deluxe curtains so as to get the maximum profit

(1 marks)