THE UNITED REPUBLIC OF TANZANIA

PRESIDEN

ADMINISTRATION

INSTRUCTIONS

1. This paper consists of section **A** and **B** with a total of **eight (08)** questions
2. Answer **all** question in section A and only two (2) questions in section B
3. All work done and answers for each question done must be shown clearly.
4. NECTA's Mathematical table and non-programmable calculator may be used
5. Cellular phones and un authorized material are **not** allowed in the examination room.
6. Write your **Examination Number** on every page of your answer booklet (s)

This paper consists of 4 printed pages

Answer A (60 Marks)
Answer **all** questions in this section

1. (a) A certain medical disease occurs in 5% of the population. A simple screening procedure is available and in 3 out of 10 cases where the patient has the disease it provides a positive result. If the patient does not have the disease there is still 0.05 chance that the test will give a positive result. Draw a tree diagram to represent this.
   (i) Find the probability that a randomly selected individual does not have the disease but gives a positive result on the test.
   (ii) James has taken the test and his results is positive. Find the probability that he has the disease.

   (b) The probability density function of a discrete random variables x is given by $P(X = x) = a\left(\frac{3}{4}\right)^x$ $for\ x = 0,1,2,3 \ldots \ldots \ldots$ Find the value of the constant a.

   (c) The poison probability distribution is defined

$$p(X = x)\begin{cases} \dfrac{e^{-x}\ \lambda^x}{x!} & for\ x = 0,1,2,, \\ 0 & elsewhere \end{cases}$$

   Show that for the following distribution
   (i) $P(x = x) = 1$ The mathematical expectation is λ
   (iii) The variance is λ

2. (a) Without using the truth tables prove that the proposition $[(p \rightarrow q)\Lambda{\sim}q] \rightarrow {\sim}p$ is tautology

   (b) Draw the electrical Newton of statement $[(p \rightarrow q) \rightarrow (q \rightarrow (p \rightarrow {\sim}q)]$ with five switches only.

   (c) Using the truth table proves that the conditional operation distributes one conjunction.
   (d) Test the validity of the following argument using the truth table. "if water was boiled then it will be free of germs. If water is not being infected, water is not boiled and is free of germs. Therefore, people will not be infected".

3. (a) The vector $\vec{a}$ and $\vec{b}$ are of equal magnitude k $(k \neq 0)$ the angle between them is 60° if $\vec{c} = 3a - b$ and $\vec{d} = 2a - 10b$ show that $\vec{c}$ and $\vec{d}$ are perpendicular

   (b) The scalar product of the vector $i + j + k$ with unit vector along the sum of the vector $2i + 4j - 5k$ and $xi + 2j + 3k$ is equal to 1 find the value of x

   (c) The vertices *A*,*B* and *C* of the triangle are at the points with position vector $\vec{a}, \vec{b}$ and $\vec{c}$ respectively show that the area of the triangle is equal to $\frac{1}{2}|a \times b + b \times c + c \times a|$

4. (a) Given that $Z = x + iy$ Express the Complex number $\frac{z+i}{iz+2}$ in polynomial form.

(b) Use De Moivre's theorem to Prove that $\sin^6\theta = \frac{1}{32}(10 - 15\cos 2t - \cos 3t)$ where $t = 2\theta$

(c) Find in form where r and θ are exact, the roots of the equation $1 + 2\sqrt{3}Z^4 + 2iZ^4 = 0$

(d) Describes the locus of $arg\left(\frac{z-1}{z+1}\right) \leq \frac{\pi}{4}$

SECTION B (40 Marks)

Answer **two (2)** questions from this section

5. (a) If $\sin\theta = \frac{\sqrt{2}}{2}$ find the value of $\frac{5\cos\theta + \tan\theta - \frac{1}{2}\operatorname{cosec}\theta}{3\cot\theta + \sec\theta - 2\operatorname{cosec}\theta}$ in simple surd form.

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(b) If $\sin(2\tan^{-1}x) + \tan(\tan^{-1}x) = \frac{4x}{1-x^2}$.

(c) Given that $24\cos\left(\frac{\pi x}{3}\right) + 10\sin\left(\frac{\pi x}{3}\right) = 13$ without using $t - formula$ show that

$x = (6n \pm 1) + \frac{3}{\pi}\tan^{-1}\left(\frac{5}{12}\right)$

(d) Using $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$ with an appropriate value of θ

(i) Show that $\tan\frac{\pi}{8} = \sqrt{2} - 1$

(ii) Using the result in 5(b)(i) to find the value of $\tan\left(\frac{3\pi}{8}\right)$ in form of $\sqrt{a} + b$

6. (a) Given that $a = \log_{12} 18$ and $\log_{24} 54$ prove that $ab + 5(a - b) = 1$

(b) (i) Express in partial fraction $\frac{3x^2 - x - 1}{(x-1)^5}$

(ii) Using the partial fraction above by substituting $(x-1)^3 = 5$ obtain the series. (The decimal terms should be excluded in series)

(c) Prove that $\sum_{k=1}^{n} \frac{1}{(3k-2)(3k++1)} = \frac{n}{3n+1}$

(d) The roots of the equation $x^3 + Px^2 + Qx + 30 = 0$ are in the ratio 2:3:5 find the value of P and Q

7. (a) Formulate the differential equation of the solution

(i) $y = Ae^{bx+1}$

(ii) $y = A\sin x + B\cos x$

(b) Solve the differential equation of $(x^2 + y^2) + 2xy\frac{dy}{dx} = 0$

(c) Find the general solution of differential equation $\frac{d^2y}{dx^2} + \frac{2dy}{dx} - 8y = 0$ .Hence find the solution which satisfies the condition $y(0) = 1$ *and* $y'(0) = 3$

(d) According to Newton's law of cooling the rate at which the temperature of the body falls in proportional to the amount by which its temperature exceeds that of its surrounding. Suppose that the temperature of an objects falls from 200°C to 100°C in 40 minutes in a surrounding temperature of 10°. Prove that after minutes, the temperature degree of the body is given by $T = 10 + 190e^{-kt}$. Hence show that $k = \frac{1}{40}\ln\left(\frac{19}{9}\right)$

8. (a) Show whether the equation of a normal to the parabola at point (x, y) *is* $(x - x_1)y_1 + 2a(y - y_1) = 0$

(b) (i) The line $y = x - c$ touches the ellipse $9x^2 + 16y^2 = 144$. Find the the value of c and the coordinate of the point of contact.

(ii) An arch is in form of a semi ellipse, it is 8M wide and 3M high at the centre. Find height of the arch at the point P which is 1M from one of the end. Hence find the distance from the centre to point P

(c) Find the coordinate of the Centre, Vertices and Equation of the Asymptotes of the the equation $36x^2 - 25y^2 + 100x + 288y + 424 = 0$.Hence sketch the graph.

(e) (i) If $r^2 = tan2\theta$ then show that $2xy = x^4 + y^4$

(ii) Draw the graph of $r = 2(1 + \cos\theta)$