

THE UNITED REPUBLIC OF TANZANIA
PRESIDENT'S OFFICE
REGIONAL ADMINISTRATION AND LOCAL GOVERNMENT
DARES SALAAM REGION
FORM SIX MOCK EXAMINATION
ADVANCED MATHEMATICS
Code: 142/1
Time 3:00 Hours
Wednesday 5th February, 2025 A.M

INSTRUCTIONS

1. This paper consists of **ten (10) compulsory** questions.
2. Answer **all** questions and each question carries **ten (10)** marks.
3. **All** necessary working and answers for each question done must be shown clearly.
4. Mathematical tables and non-programmable calculator may be used.
5. Cellular phones and any unauthorized materials are **not** allowed in the examination room.
6. Write your **Examination Number** on every page of your answer sheet(s).

This paper consists of 4 printed pages

1. (a) By using non-programmable calculator evaluate the following in four significant figures

(i) $\int_0^{\frac{\sqrt{2}}{2}} \frac{x \sin^{-1}(x^2)}{\sqrt{1-x^4}} dx$

(ii) Find $(\log_7 46)^3 + \sum_{x=1}^5 \frac{\ln(1+x)}{\tan^{-1} x}$ Correct to 4 decimal places

- b) Use a non-programmable scientific calculator to evaluate each of the following

(i) $P(Z \leq -1.53)$

(ii) $P(-2 \leq Z \leq 0.27)$

- c) Given the following frequency distribution table, calculate the mean and standard deviations using a non-programmable scientific calculator correct to 2 decimal places.

Class interval	11- 20	21 - 30	31 - 40	41 - 50	51 - 60	61 - 70
frequency	15	9	7	11	6	8

2. (a) If $y = \frac{e^x - 1}{e^x + 1}$ prove that $\frac{dy}{dx} = \frac{1}{2} \operatorname{sech}^2\left(\frac{x}{2}\right)$

- (b) solve the simultaneous equation $\begin{cases} 10 \cosh y + 6 \sinh x = 17 \\ 3 \cosh x - 5 \sinh y = 0 \end{cases}$ giving your answer in logarithm form

- (c) (i) Show that $\int \operatorname{sech} x dx = 2 \tan^{-1}(e^x) + c$

(ii) Given that $\coth^{-1}(m) - \coth^{-1}(n) = \frac{1}{2} \ln 8$ prove that $m = \frac{9n+7}{7n+9}$

3. (a) A 100kg powder mixture is to be made from ingredient, P, Q, and R which cost Tsh. 600/=, 300/= and 400/= per kg respectively. Find the least and greatest cost of powder to be made if the quantity of P exceed that of Q by at least 10kg and must not exceed that of R by more than 40kg and if the quantity of R cannot exceed that of Q

4. (a) A set of values of variable X has mean 5 and standard deviation 2. Values of a new variable are obtained by using the formula $Y = 4X - 3$. Find the mean and standard deviation of the set of values of Y

- (b) Use coding method to find the class interval of the following distribution if its arithmetic mean and assumed mean are 33 and 35 respectively

Step deviation	-3	-2	-1	0	1	2
Frequency	5	10	25	30	20	10

- (c) Prove that the standard deviation of the first n natural numbers is given by $\delta = \sqrt{\frac{n^2 - 1}{12}}$

5. (a) (i) If $M = \left\{x: \frac{x}{x-2} \geq 0, x \in R\right\}$ Find all member of M
 (iii) Verify that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ by venn diagram

- (b) Use laws of algebra of sets to simplify
 $(A \cap B \cap C) \cup (A^c \cap B \cap C) \cup (B^c \cup C^c)$
- (c) A form six class at Kisutu secondary school contain 40 boys and 36 girls. A Certain research confirmed that 48 of the students are taking mathematics, 45 are taking physics, 30 boys area taking physics, 16 girls are taking mathematics only, 3 boys and 5 girls are taking neither of the two subjects and 17 boys are taking both mathematics and physics.
- The number of girls taking both mathematics and physics
 - The total number of students taking mathematics only
 - The total number of students taking physics only
6. (a) Find $g(-4)$ if $f(x) = \frac{x+9}{3x}$ and $f \circ g(x) = \frac{2(x+5)}{3(2x+1)}$
- (b) Using suitable table of value, Draw the graph of $f(x) = 2 + e^{2x}$ if $-3 \leq x \leq 1.2$ and $g(x) = 1 - e^x$ if $-3 \leq x \leq 2.7$ on the same xy - plane
- (c) Given $f(x) = \frac{x^3 - 9x}{3x^2 - 6x - 9}$ describe the discontinuities and the intersect. Hence sketch the graph
7. (a) (i) Give two limitation of newton Raphson method in finding the roots of the function
- (ii) If $f(x) = \tan x - 3x$ show that the newton raphsons method for approximating the roots of $f(x)$ leads to the recurrence formular $x_{n+1} = \frac{\sin(2x_n) - 2x_n}{6\cos^2(2x_n) - 2}$ starting with $x_0 = \frac{2\pi}{5}$ perform five iteration find the zeros of $f(x)$ accurately to the eight decimal places
- (b) (i) The value $A = \int_a^b f(x)dx$ represent the area under the graph of $y = f(x)$ between $x = a$ and $x = b$. Derive the trapezium rule with 6 ordinate to find the ap proximation of
- (ii) Using the trapezium rule obtained in 7(b)(i) above to estimatimate the area under the curve
- $y = f(x)$ between $x = 0$, and $x = 1$ strictly use $h = 0.2$
- | | | | | | | | | | | | |
|------------|---|---------|--------|---------|---------|-------|-------|--------|--------|--------|-----|
| x | 0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| $y = f(x)$ | 1 | 0.90909 | 0.8333 | 0.76903 | 0.71409 | 0.666 | 0.625 | 0.5882 | 0.5536 | 0.5263 | 0.5 |
8. (a) Find the equations of two orthogonal lines through which makes an angle of with the line $y = 2x + 1$
- (b) Find the values of such that the line $X + Y - C = 0$ shall be a tangent to the circle $x^2 + y^2 - 4x + 2 = 0$, for each value of c find the coordinates of the point of contact
- (c) A circle through the origin is orthogonal to both circles $x^2 + y^2 - 4x + 8y = 0$ and $x^2 + y^2 + 2x + 6y + 1 = 0$, find the equation of the circle.

9. (a) (i) Find the Value of b if $\int_2^3 \frac{bx}{(x+1)(x-1)} dx = 2\ln\left(\frac{8}{3}\right)$

(ii) Evaluate $\int x \sqrt{\frac{1-x^2}{1+x^2}} dx$

(c) Show that area enclosed by curve $y = x(4-x)$ and straight line $y = 2x$ is $\frac{4}{3}$ square units. Hence find the volume of the solid formed when this area makes a complete revolution about x -axis (give your answer in terms of π)

10. (a) Differentiate $f(x) = \cos\left(\frac{1}{x}\right) - 2x$ from first principle

(b) The parametric equation of a curve is $x = \cos t + t \sin t$, $y = \sin t - t \cos t$ show that

$$\frac{d^2y}{dx^2} = \frac{\sec^3 t}{t}$$

(c) Use Taylor's series to expand $\sin\left(\frac{\pi}{6} + h\right)$ in ascending power of h as far as the term of h^4 and hence forms into four significant figure.