THE PRESIDENT'S OFFICE

REGIONAL ADMINISTRATION AND LOCAL GOVERNMENT

FORM SIX COMPETENCE EXAMINATIONS (FOSCE) - 2025

CODE: 142/2 **ADVANCED MATHEMATICS 2**

TIME: 3 HOURS **26^{TH} FEBRUARY 2025, AM**

Instruction

1. This paper consists of section A and B with a total of eight (8) questions.
2. Answer all question in Section A and two (2) questions from section B.
3. Section A carries sixty (60) marks and section B carries forty(40) marks
4. All work done in answering each question must be shown clearly.
5. NECTA's mathematical table and non-programmable calculators may be used.
6. Cellular phones and any unauthorized materials are not allowed in examination room.
7. Write your Examination number on every page of your answer booklet(s)

1. a) Over a long period of time it has been observed that a given rifleman can hit a target on a single trial with probability equal to 0.8. Suppose he fires 4 shots at the target. Find the following probabilities.

   i. He will hit the target exactly two times.

   ii. He will hit the target at least once.

   b) A continuous random variable X is defined by $f(x) = kx$ for $0 \leq x \leq 4$.find the value of k and $p(1 \leq x \leq 2\frac{1}{2})$

   c) The following table lists the probability distribution of the number of the defective per hour of a machine production.

Defective per hour				
Probability	0.33	0.25	0.17	0.10

   Find the probability that the number of defectives for this machine production through a given hour.

   i. At least 2

   ii. 1 to 3

2. a). Write the contrapositive and construct truth table of the converse of the conditional

   $(p \to q) \to \sim q$

   b) Use the laws of algebra of preposition to simplify the statements $q \vee (p \wedge \sim q) \vee (r \vee q)$ and hence draw the corresponding simple electric network

   c) Write the statement A in most simplified form from $x, y \ and \ z$

x	T	T	T	T	F	F	F	F
y	T	T	F	F	T	T	F	F
z	T	F	T	F	T	F	T	F
A	F	F	T	T	F	F	T	T

   d) Draw a simple electrical network corresponding to the compound statement

   $P \wedge [Q \wedge R \vee \sim Q] \vee [\sim P \wedge \sim R]$

3. a) The vector $\underline{p} = (-5,7,1)$ and $\underline{q} = (S,S,S)$ where $S > 0$ are such that $\underline{p} + \underline{q}$ and $\underline{p} - \underline{q}$ are Orthogonal. Find the value of S.

   b) Find the points of intersection between the plane $3\boldsymbol{x} - \boldsymbol{y} + 2\boldsymbol{z} = 6$ l, $\frac{x-4}{2} = \frac{y+2}{-1} = \frac{5-z}{2}$

c) Prove that the component of a vector $\underline{a}$ in direction $\underline{b}$ is given by $\frac{\underline{a}.\underline{b}}{|\underline{b}|}$

d) Derive formula for the Area of the triangle by using vector product technique hence find the area of triangle whose its vertices are $A(-2,3,1), B(2,4,-1)$ and $C(1,1,1)$.

4. a) If $(x+iy)(p+iq) = (x^2+y^2)i$,then show that $x = q, y = p$

b) Write the forth roots of $z = -8$

c) Use De moivre's theorem to simplify $\frac{\cos 4\theta + i \sin 4\theta}{\cos 5\theta - \text{isin } 5\theta}$

d) Given the Real part of $\frac{Z-2i}{Z+4}$ is zero prove that the locus of Zisthe crcle. Find the center and show that the radius is $\sqrt{5}\ units$

5. (a) Show that $2\left[\tan^{-1}(\frac{1+y}{1-y}) - \tan^{-1} y\right] = \frac{\pi}{2}$

b) If $A + B + C = 180°$,prove that $\cos A + \cos B + \cos C = 1 + 4 \sin\frac{1}{2}A \sin\frac{1}{2}B \sin\frac{1}{2}C$

c) If $2cos\theta = a + \frac{1}{a}$, prove that $2cos3\theta = a^3 + \frac{1}{a^3}$

d) Solve the equation $3\cos 2\theta + \sin\theta = 1$ for values of θ from $0°$ to $360°$ inclusive.

6. a) Express $\frac{2}{n(n+1)(n+2)}$ in partial fraction and use it to deduce the sum of $\sum_{i=1}^{n} \frac{2}{k(k+1)(k+2)}$ and find its sum when $n \to \infty$

b) Expand $\frac{1}{(1+2x)^3}$ in ascending powers of x as far as x^3, using the binomial series.

c) Solve the following simultaneous equations by Cramer's Rule.
$$\begin{aligned} 2x + 3y - z &= -7 \\ -3x + y + 2z &= 1 \\ 3x - 4y - 4z &= -1 \end{aligned}$$

7. a) i. Solve the differential equation $x\frac{dy}{dx} - \frac{y^2}{x} = y$

ii. Find the general solution of the differential equation $y'' - 2y' = 0$

b) Form the differential equation given that $y = A\cos 2x + B\sin 2x$

c) For a period of about 15 years, the growth of Tanzania gross domestic product (GDP) was predicted to vary between -1% and$+5\%$. This variation was modeled by $\left((4\cos\frac{1}{2}t\ ) + 5\right)\%$, Where t is time in years? Find a formula for GDP during the 15 years.

8. a) i. Define the term Conic section

ii. Find the equation of normal to the parabola $64x = y^2m^{-1}$ at the parametric coordinates $(\frac{mt^2}{4}, 2mt^2)$ where m is constant

b) Show that the equation $y^2 + 4y - 8x - 4 = 0$ represent a parabola hence find focus, Vertex and symmetrical axis

c) If $y^2 + (x-1)^2 = 4$ is an equation of circle, Verify whether its radius is $\cos\theta \pm \sqrt{3 + cos^2\theta}$ where θ is an angle made by the radius of the polar axis.

d) Sketch the graph of $r = -(2\cos x - 1)$ such that $0 \leq x \leq 2\pi$